Gravitation

INTRODUCTION

Newton observed that an object, an apple, when released near the earth surface is accelerated towards the earth. As acceleration is caused by an unbalanced force, there must be a force pulling objects towards the earth. If someone throws a projectile with some initial velocity, then instead of that object moving off into space in a straight line, it is continously acted on by a force pulling it back to earth. If we throw the projectile with greater velocity then the path of projectile would be different as well and its range is also increased with initial velocity. If the projection velocity is further increased until at some initial velocity, the body would not hit the earth at all but would go right around it in an orbit. But at any point along its path the projectile would still have a force acting on it pulling it toward the surface of earth.

Newton was led to the conclusion that the same force that causes the apple to fall to the earth also causes the moon to be pulled to the earth. Thus the moon moves in its orbit about the earth because it is pulled toward the earth. But if there is a force between the moon and the earth, why not a force between the sun and the earth or why not a force between the sun and the other planets ? Newton proposed that the same force, named gravitational force which acts on objects near the earth surface also acts on all the heavenly bodies. He proposed that there was a force of gravitation between each and every mass in the universe.

Section A - Newton's law of Gravitation & Gravitational Field, Potential & Potential energy

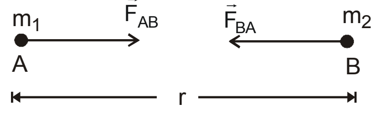
1.1 Newtons's Law of Universal Gravitation

All physical bodies are subjected to the action of the forces of mutual gravitational attraction. The basic law describing the gravitational forces was stated by Sir Issac Newton and it is called Newton's Law. of Universal gravitation.

The law is stated as : "Between any two particles of masses m_1 and m_2 at separation r from each other there exist attractive forces $\vec{F}_{AB}$ and $\vec{F}_{BA}$ directed from one body to the other and equal in magnitude which is directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between the two". Thus we can write

$$F_{AB} = F_{BA} = G\frac{m_1 m_2}{r^2} \qquad ...(1)$$

Where G is called universal gravitational constant. Its value is equal to 6.67×10^{-11} Nm^2/kg. The law of gravitation can be applied to the bodies whose dimensions are small as compared to the separation between the two or when bodies can be treated as point particles.



If the bodies are not very small sized, we can not directly apply the expression in equation-(1) to find their natural gravitational attraction. In this case we use the following procedure to find the same. The bodies are initially split into small parts or a large number of point masses.

Now using equation-(1) the force of attraction exerted on a particle of one body by a particle of another body can be obtained. Now we add all forces vectorially which are exerted by all independent particles of second body on the particle of first body. Finally the resultants of these forces is summed over all particles of the first body to obtain the net force experinced by the bodies. In general we use integration or basic summation of these forces.

⇒ Gravitational force is a conservative force.

⇒ Gravitational force is a central force.

⇒ Gravitational force is equal in magnitude & opposite in direction

⇒ Gravitational forces are action - reaction pair.

⇒ Gravitational force acts along the line joining the two masses.

⇒ Gravitational force doesn't depend upon the medium

⇒ Gravitational force is an attractive force.

$$\vec{F} = \frac{-Gm_1m_2\vec{r}}{|\vec{r}|^3}$$

[Head of $\vec{r}$ is placed at that position where we have to evaluate force]

2. GRAVITATIONAL FIELD

We can state by Newton's universal law of gravitation that every mass M produces, in the region around it, a physical situation in which, whenever any other mass is placed, force acts on it, is called gravitational field. This field is recognized by the force that the mass M exerts another mass, such as *m*, brought into the region.

2.1 Strength of Gravitational Field

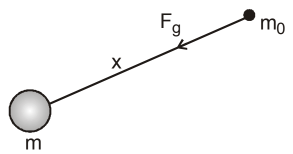
We define gravitational field strength at any point in space to be the gravitational force per unit mass on a test mass (mass brought into the field for experimental observation). Thus for a point in space if a test mass m_0, experiences a force $\vec{F}$, then at that point in space, gravitational field strength which is denoted by $\vec{g}$, is given as $\vec{g} = \frac{\vec{F}}{m_0}$

Gravitational field strength $\vec{g}$ is a vector quantity and has same direction as that of the force on the test mass in field.

Generally magnitude of test mass is very small such that its gravitational field does not modify the field that is being measured. It should be also noted that gravitational field strength is just the acceleration that a unit mass would experience at that point in space.

2.2 Gravitational Field Strength of Point Mass

As per our previous discussion we can state that every point mass also produces a gravitational field in its surrounding. To find the gravitational field strength due to a point mass, we put a test mass m_0 at a point P at distance x from a point mass m then force on m_0 is given as



$$F_g = \frac{Gmm_0}{x^2}$$

Now if at point P, gravitational field strength due to m is g_p then it is given as

$$g_p = \frac{F_g}{m_0} = \frac{Gm}{x^2}$$

The expression written in above equation gives the gravitational field strength at a point due to a point mass. It should be noted that the expression in equation written above is only applicable for gravitational field strength due to point masses. It should not be used for extended bodies.

However, the expression for the gravitational field strength produced by extended masses has already been derived in electrostatics section.

[Just replace k by –G & Q by M in those expression]

So we will just revise the expression of gravitational field strength at points due to various extended masses. Gravitational field strength :

1. At a point on the axis of Ring = $\dfrac{-GMx}{(x^2+R^2)^{3/2}}$

2. At a point on the axis of disc

$$= \frac{2GM}{R^2}\left[1-\frac{x}{\sqrt{R^2+x^2}}\right]$$

3. At an axial point of a rod = $\dfrac{-GM}{L}\left[\dfrac{1}{x}-\dfrac{1}{x+L}\right]$

4. Due to a circular Arc = $\dfrac{-2GM\sin(\phi/2)}{\phi R^2}$

5. Due to a long infinite thread = $\dfrac{-2G\lambda}{x}$

6. Due to long solid cylinder

(a) at an outer point $= \dfrac{-2G\rho\pi R^2}{x}$

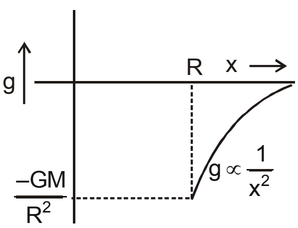
(where ρ is mass density per volume)

(b) at an inner point = $-2G\rho\pi x$

7. Due to hollow sphere :

(a) for outer points = $\dfrac{-GM}{x^2}$ (Behaving as a point mass)

(b) for points on surface = $\dfrac{-GM}{R^2}$ (Behaving as a point mass)

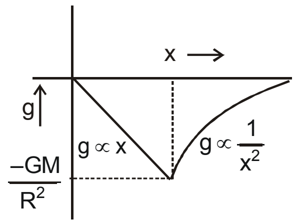


(c) for inner points = 0 (As no mass is enclosed within it)

8. Due to solid sphere

(a) For outer points = $\dfrac{-GM}{x^2}$ (Behaving as a point mass)

(b) For points on surface = $\dfrac{-GM}{R^2}$ (Behaving as a point mass)



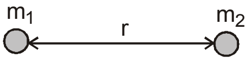
(c) For inner points = $\dfrac{-GMx}{R^3}$

3. INTERACTION ENERGY

This energy exists in a system of particles due to the interaction forces between the particles of system. Analytically this term is defined as the work done against the interaction of system forces in assembling the given configuration of particles. To understand this we take a simple example of interaction energy of two points masses.

Figure (a) shows a system of two point masses m_1 and m_2 placed at a distance r apart in space. here if we wish to find the interaction potential energy of the two masses, this must be the work done in bringing the two masses from infinity (zero interaction state) to this configuration. For this we first fix m_1 at its position and bring m_2 slowly from infinity to its location. If in the process m_2 is at a distance x from m_1 then force on it is

$$F = -\frac{Gm_1m_2}{x^2}\hat{i}$$



(a)

(b)

This force is applied by the gravitational field of m_1 to m_2. If it is further displaced by a distance dx towards m_1 then work done by the field is

$$dW = \vec{F}.\vec{dx} = \frac{Gm_1m_2}{x^2}dx$$

Now in bringing m_2 from infinity to a position at a distance r from m_1, the total work done by the field is

$$W = \int dW - \int_{\infty}^{r} \frac{Gm_1m_2}{x^2}dx$$

$$= -G\,m_1\,m_2\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$W = +\frac{Gm_1m_2}{r}$$

Thus during the process field of system has done $\frac{Gm_1m_2}{r}$ amount of work. The work is positive because the displacement of body is in the direction of force.

Initially when the separation between m_1 and m_2 was very large (at infinity) there was no interaction between them. We conversely say that as a reference when there is no interaction the interaction energy of the system is zero and during the process system forces (gravitational forces) are doing work so system energy will decrease and becomes negative (as initial energy was zero). As a consequence we can state that in general if system forces are attractive, in assembling a system of particles work will be done by the system and it will spend energy in assembling itself. Thus finally the interaction energy of system will be negative. On the other hand if in a given system of particles, the system forces are repulsive, then in assembling a system some external forces have to be work against the system forces and in this case some work must be done by external forces on the system hence finally the interaction energy of the system of particles must be positive.

In above example as work is done by the gravitaional forces of the system of two masses, the interaction energy of system must be negative and it can be given as

$$U_{12} = -\frac{Gm_1m_2}{r} \qquad ...(1)$$

As gravitational forces are always attractive, the gravitational potential energy is always taken negative.

3.1 Interaction Energy of a System of Particles

If in a system there are more than two particles then we can find the interaction energy of particle in pairs using equation (1) and finally sum up all the results to get the total energy of the system. For example in a system of N particles with masses m_1, m_2........m_n separated from each other by a distance r_{12}........ where r_{12} is the separation between m_1 and m_2 and so on.

In the above case the total interaction energy of system is given as

$$U = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{Gm_im_j}{r_{ij}}$$

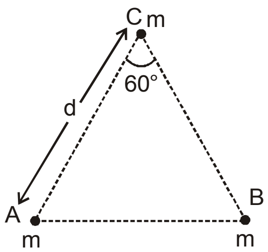
In this expression the factor $\frac{1}{2}$ is taken because the interaction energy for each possible pair of system is taken twice during summation as for mass m_1 and m_3

$$U = -\frac{Gm_1m_3}{r_{13}} = -\frac{Gm_3m_1}{r_{31}}$$

Now to understand the applications of interaction energy we take few examples.

EXAMPLE 1

Three particles each of the mass m are placed at the corners of an equilateral triangle of side d and shown in figure. Calculate (a) the potential energy of the system, (b) work done on this system if the side of the triangle is changed from d to 2d.



Sol. (a) As in case of two-particle system potential energy is given by $(-Gm_1m_2/r)$, so

$$U = U_{12} + U_{23} + U_{31}$$

or $$U_i = -3\frac{Gmm}{d} = -\frac{3Gm^2}{d}$$

(b) When d is changed to 2d,

$$U_f = -\frac{3Gm^2}{2d}$$

Thus work done in changing in potential energy is given as

$$W = U_f - U_i = \frac{3Gm^2}{2d}$$

EXAMPLE 2

Two particles m_1 and m_2 are initially at rest at infinite distance. Find their relative velocity of approach due to gravitational atraction when their separation is d.

Sol. Initiallly when the separation was large there was no interaction energy and when they get closer the system gravitational energy decreases and the kinetic energy increases.

When separation between the two particles is d, then according to energy conservation we have

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} = 0$$

As no other force is present we have according to momentum conservation

$$m_1v_1 = m_2v_2$$

From equations written above

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}\frac{m_1^2}{m_2}v_1^2 = \frac{Gm_1m_2}{d}$$

or $$v_1 = \sqrt{\frac{2Gm_2^2}{d(m_1+m_2)}} = \sqrt{\frac{2G}{d(m_1+m_2)}}m_2$$

And on further solving we get

$$v_2 = \sqrt{\frac{2G}{d(m_1+m_2)}}m_1$$

Thus approach velocity is given as

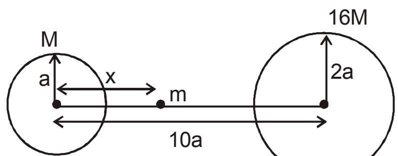
$$v_{ap} = v_1 + v_2 = \sqrt{\frac{2G(m_1+m_2)}{d}}$$

EXAMPLE 3

If a particle of mass 'm' is projected from a surface of bigger sphere of mass '16M' and radius '2a' then find out the minimum velocity of the paticle such that the particle reaches the surface of the smaller sphere of mass M and radius 'a'. Given that the distance between the centres of two spheres is 10 a.

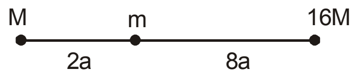
Sol. When the particle is at the surface of bigger sphere it is attracted more by the bigger sphere and less by the smaller sphere. As it is projected the force of attraction from bigger sphere decreases and that from smaller sphere increase and thus the particle reaches the state of equilibrium at distance x

from the centre smaller sphere



$$\frac{GMm}{x^2} = \frac{G(16M)m}{(10a-x)^2}$$

$$(10a - x)^2 = 16x^2$$



$$10a - x = 4x \Rightarrow x = 2a$$

After this point the attraction on the particle from the smaller sphere becomes more than that from the bigger sphere and the particle will automatically move towards the smaller sphere. Hence the minimum velocity to reach the smaller sphere is the veloicty required to reach the equilibrium state according to energy conservation, we have,

$$-\frac{G(16M)m}{2a} - \frac{GMm}{8a} + \frac{1}{2}mv^2$$

$$= \frac{-G(16M)m}{8a} - \frac{GMm}{2a}$$

$$v^2 = \frac{45GM}{4a}$$

$$\Rightarrow \qquad v = \sqrt{\frac{45GM}{4a}}$$

4 GRAVITATIONAL POTENTIAL

The gravitational potential at a point in gravitational field is the gravitional potenial energy per unit mass placed at that point in gravitational field. Thus at a certain point in gravitational field, a mass m_0 has a potential energy U then the gravitational potential at that point is given as

$$V = \frac{U}{m_0}$$

or if at a point in gravitational field gravitational potential V is known then the interaction potential energy of a point mass m_0 at that point in the field is given as

$$U = m_0 v$$

Interaction energy of a point mass m_0 in a field is defined as work done in bringing that mass from infinity to that point. In the same fashion we can define gravitational potential at a point in field, alternatively as "*Work done in bringing a unit mass from infinity to that point against gravitational forces.*"

When a unit mass is brought to a point in a gravitational field, force on the unit mass is $\vec{g}$ at a point in the field. Thus the work done in bringing this unit mass from infinity to a point P in gravitational field or gravitational potential at point P is given as

$$V_P = -\int_{\infty}^{P} \vec{g} \cdot d\vec{x}$$

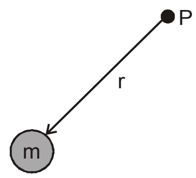
Here negative sign shown that V_P is the negative of work done by gravitation field or it is the external required work for the purpose against gravitational forces.

4.1 Gravitational Potential due to a Point Mass

We know that in the surrounding of a point mass it produces its gravitational field. If we wish to find the gravitational potential at a point P situated at a distance r from it as shown in figure, we place a test mass m_0 at P and we find the interaction energy of m_0 with the field of m, which is given as

$$U = -\frac{Gmm_0}{r}$$

Now the gravitational potential at P due to m can be written as



$$V = \frac{U}{m_0} = -\frac{Gm}{r}$$

The expression of gravitational potential in equation is a standard result due to a point mass which can be used as an elemental form to find other complex results, we'll see later. The same thing can also be obtained by using equation

$$V_P = \int_{\infty}^{P} \vec{g} \cdot d\vec{x}$$

$$\text{or} \qquad V_P = \int_{\infty}^{r} \frac{Gm}{x^2} dx \quad \text{or} \quad V_P = -\frac{Gm}{r}$$

4.2 Gravitational potential

1. Due to a rod at an a xial point = $-G\lambda \ln\left(\frac{a+l}{a}\right)$

2. Due to ring at an axial point = $\frac{-GM}{\sqrt{R^2+x^2}}$

3. Due to ring at the centre = $\frac{-GM}{R}$

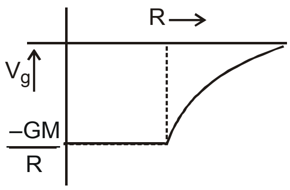
4. Due to Disc = $-G\sigma 2\pi[\sqrt{R^2+x^2}-x]$

   (where σ is mass density per unit area)

5. Due to hollow sphere

for outer points = $\frac{-GM}{r}$

For surface points = $\frac{-GM}{R}$

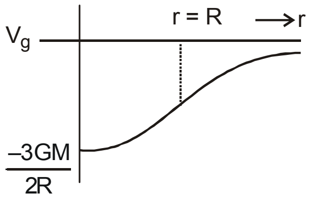


For inner points = $-\frac{GM}{R}$

6. Due to solid sphere

For outer points = $\frac{-GM}{r}$

For surface points = $\frac{-GM}{R}$



For inner points = $\frac{-GM}{2R^3}(3R^2-r^2)$

Potential energy of hollow sphere = $\frac{-GM^2}{2R}$

Potential energy of solid sphere = $\frac{-3GM^2}{5R}$

Note

The student can now attempt section A from exercise.

Section B - Variations in 'g'

5. GRAVITATIONAL LINES OF FORCES

Gravitational field can also be represented by lines of force. A line of force is drawn in such a way that at each point the direction of field is tangent to line that passes through the point. Thus tangent to any point on a line of force gives the direction of gravitational field at that point. By convention lines of force are drawn in such a way that their density is proportional to the strength of field. Figure shown shows the field of a point mass in its surrounding. We can see that the lines of force are radially inward giving direction of field and as we go closer to the mass the density of lines is more which shows that field strength is increasing.

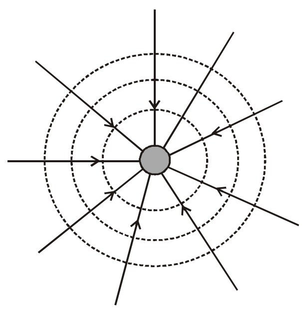
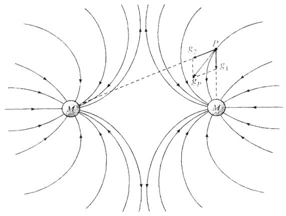


Figure shown shows the configuration of field lines for a system of two equal masses separated by a given distance.



Here we can see that there is no point where any two lines of force intersects or meet. The reason is obvious that at one point in space there can never be two direction of gravitational fields. It should be noted that a line of force gives the direction of net gravitational field in the region. Like electric field gravitational field never exists in closed loops.

- **Gravitational Flux :** $\phi = \int \vec{g} \cdot \vec{ds}$

- **Gravitational Gauss law :** $\oiint \vec{g} \cdot \vec{ds} = -4\pi G M_{in}$

Here $\vec{g}$ is the gravitational field due to all the masses.

M_{in} is the mass inside the assumed gaussian surface.

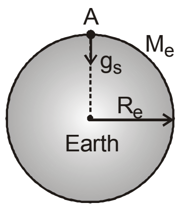
5.1 Gravitational Field Strength of Earth:

We can consider earth to be a very large sphere of mass M_e and radius R_e. Gravitational field strength due to earth is also regarded as acceleration due to gravity or gravitational acceleration. Now we find values of g at different points due to earth.

- Earth behaves as a non conducting solid sphere

5.2 Value of g on Earth's Surface :

If g_s be the gravitational field strength at a point A on the surface of earth, then it can be easily obtained by using the result of a solid sphere. Thus for earth, value of g_s can be given as



$$g_s = \frac{GM_e}{R_e^2} \qquad \text{...(1)}$$

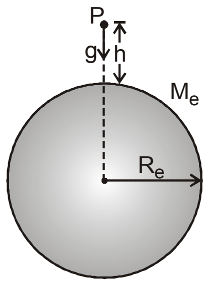
5.3 Value of g at a Height *h* Above the Earth's Surface:

If we wish to find the value of g at a point P as shown in figure shown at a height h above the Eath's surface. Then the value can be obtained as

$$g_s = \frac{GM_e}{(R_e + h)^2}$$

or

$$g_s = \frac{GM_e}{R_e^2\left(1+\frac{h}{R_e}\right)^2} = \frac{g_s}{\left(1+\frac{h}{R_e}\right)^2}$$



If point P is very close to Earth's surface then for h $<< R_e$ we can rewrite the expression in given equation as

$$g_h = g_s\left(1+\frac{h}{R_e}\right)^{-2} \simeq g_s\left(1-\frac{2h}{R_e}\right)$$

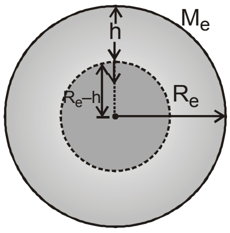
[Using binomial approximation] ...(2)

5.4 Value of g at a Depth *h* Below the Earth's Surface

If we find the value of g inside the volume of earth at a depth *h* below the earth's surface at point P as shown in figure, then we can use the result of *g* inside a solid sphere as

$$g_{in} = \frac{GM_e x}{R_e^3}$$

Here x, the distance of point from centre of earth is given as



$x = R_e - h$

Thus we have

$$g_h = \frac{GM_e(R_e - h)}{R_e^3} = g_s\left(1 - \frac{h}{R_e}\right) \quad ...(3)$$

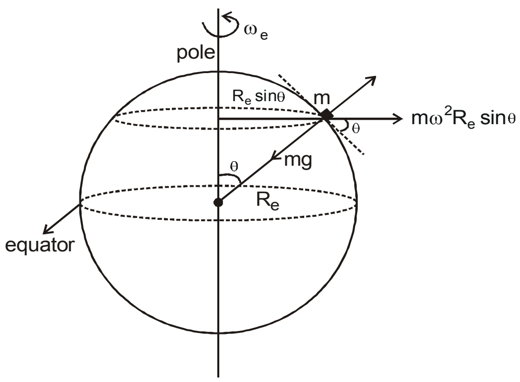
From equation (1), (2) and (3) we can say that the value of *g* at eath's surface is maximum and as we move above the earth's surface or we go below the surface of earth, the value of *g* decrease.

5.5 Effect of Earth's Rotation on Value of *g*

Let us consider a body of mass *m* placed on Earth's surface at a latitude θ as shown in figure. This mass experiences a force mg_s towards the centre of earth and a centrifugal force $mw_e^2 R_e \sin\theta$ relative to Earth's surface as shown in figure. If we consider g_{eff} as the effective value of *g* on earth surface at a latitude θ then we can write

$$g_{eff} = \frac{\vec{F}_{net}}{m} =$$

$$g_{eff} = \sqrt{(\omega_e^2 R_e \sin\theta)^2 + g^2 + 2\omega_e^2 R_e \sin\theta . g\cos(90+\theta)}$$



ω_e^4 is very very small

So we can write

$$g_{eff} = \sqrt{g^2 - 2\omega_e^2 R_e \sin^2\theta g}$$

$$g_{eff} = g\left(1 - \frac{2\omega_e^2 R_e \sin^2\theta}{g}\right)^{1/2}$$

$$\approx g - \omega_e^2 R \sin^2\theta \quad ...(i)$$

From equation (1) we can find the value of effective gravity at poles and equatorial points on Earth as

At poles $\theta = 0 \Rightarrow g_{poles} = g_s = 9.83$ m/s^2

At equator $\theta = \frac{\pi}{2}$

$\Rightarrow g_{equator} = g_s - \omega^2 R_e = 9.78$ m/s^2

Thus we can see that the body if placed at poles of Earth, it will only have a spin, not circular motion so there is no reduction in value of *g* at poles due to rotation of earth. Thus at poles value of g on Earth surface is maximum and at equator it is minimum. But an average we take 9.8 m/s^2, the value of g everywhere on earth's surface.

5.6 Effect of Shape of Earth on Value of g

Till now we considered that earth is spherical in its shape but this is not actually true. Due to some geological and astromonical reasons, the shape of earth is not exact spherical. It is ellipsoidal.

As we've discussed that the value of *g* at a point on earth surface depends on radius of Earth.It is observed that the approximate difference in earth's radius at different points on equator and poles is $r_e - r_p \simeq 21$ to 34 km. Due to this the difference in value of g at poles and equatorial points is approximately $g_p - g_e \simeq 0.02$ to 0.04 m/s^2, which is very small. So for numerical calculations, generally, we ignore this factor while taking the value of *g* and we assume Earth is spherical in shape.

EXAMPLE 4

Calculate the mass and density of the earth. Given that Gravitational constant G = 6.67 × 10^{-11} Nm^2/kg^2, the radius of the earth = 6.37 × 10^6 m and g = 9.8 m/s^2.

Sol. The acceleration due to gravity on earth surface is given as

$$g_e = \frac{GM_e}{R_e^2}$$

or $$M_e = \frac{g_s R_e^2}{G} = \frac{9.8 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$= 6 \times 10^{24}$ kg

If ρ be the density of earth, then

$$M = \frac{4}{3}\pi R^3 \times \rho$$

or $$\rho = \frac{3M}{4\pi R^3}$$

$$= \frac{3 \times (6 \times 10^{24})}{4 \times 3.14 \times (6.37 \times 10^6)^3}$$

$= 5.5 \times 10^3$ kg/m^3

EXAMPLE 5

If the radius of the earth were to shrink by one percent, its mass remaining the same, what would happen to the acceleration due to gravity on the earth's surface?

Sol. Consider the case of body of mass *m* placed on the earth's surface (mass of the earth M and radius R). If *g* is acceleration due to gravity, then we known that

$$g_s = \frac{GM_e}{R_e^2} \qquad ...(1)$$

Now, when the radius is reduced by 1%, i.e. radius becomes 0.99 R, let acceleration due to gravity be g', then

$$g' = \frac{GM}{(0.99R)^2} \qquad ...(2)$$

From equation (1) and (2), we get

$$\frac{g'}{g} = \frac{R^2}{(0.99R)} = \frac{1}{(0.99)^2}$$

or $$g' = g \times \left(\frac{1}{0.99}\right)^2$$

or $g' = 1.02$ g

Thus, the value of *g* is increased by 2%.

EXAMPLE 6

At what rate should the earth rotate so that the apparent g at the equator becomes zero? What will be the length of the day in this situation ?

Sol. At earth's equator effective value of gravity is

$$g_{eq} = g_s - \omega^2 R_e$$

If g_{eff} at equator is zero, we have

$$g_s = \omega^2 R_e$$

or $$\omega = \sqrt{\frac{g_s}{R_e}}$$

Thus length of the day will be

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R_e}{g_s}}$$

$$= 2 \times 3.14\sqrt{\frac{6.4 \times 10^6}{9.8}} = 5074.77\,s$$

$\simeq 84.57$ min.

EXAMPLE 7

Calculate the acceleration due to gravity at the surface of Mars if its diameter is 6760 km and mass is one-tenth that of earth. The diameter of earth is 12742 km and acceleration due to gravity on earth is 9.8 m/s^2.

We know that $g = \left(\frac{GM}{R^2}\right)$

So, $$\frac{g_M}{g_E} = \left(\frac{M_M}{M_E}\right)\left(\frac{R_E}{R_M}\right)^2 = \left(\frac{1}{10}\right)\left(\frac{12742}{6760}\right)^2$$

$\Rightarrow$ $\frac{g_M}{g_E} = 0.35$ or $g_M = 9.8 \times 0.35 = 3.48$ m/s^2

EXAMPLE 8

Calculate the apparent weight of a body of mass m at a latitude λ when it is moving with speed v on the surface of the earth from west to east at the same latitude.

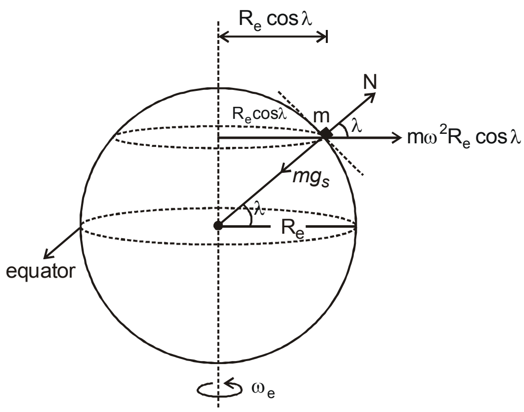
Sol. If W be the apparent weight of body at a latitude λ then from figure shown, we have

$$W = mg - m\omega^2 R\cos^2\lambda \quad ...(1)$$

When body moves at speed v from west to east relative to earth, its net angular speed ω can be given as

$$\omega = \omega_e + \frac{v}{R\cos\lambda} \quad [\omega_e \rightarrow \text{earth's angular velocity}]$$

Now from equation (1) we have



$$W = mg - m\left[\omega_e + \frac{v}{R\cos\lambda}\right]^2 R\cos^2\lambda$$

or

$$W = mg - m\left[\omega_e^2 + \frac{v^2}{R^2\cos^2\lambda} + \frac{2\omega_e v}{R\cos\lambda}\right] R\cos^2\lambda$$

$$= mg - m\omega_e^2 R\cos^2\lambda - \frac{mv^2}{R} - 2m\omega_e v\cos\lambda$$

$$= mg\left(1 - \frac{w_e^2 R\cos^2\lambda}{g} - \frac{2\omega_e v\cos\lambda}{g}\right)$$

$$\left[\text{Neglecting } \frac{mv^2}{R} \text{ as being very small}\right]$$

Note

The student can now attempt section B from exercise.

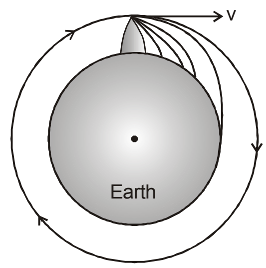
Section C - Kepler's law, Orbital velocity, Escape velocity, Geo - Stationary Satellites

6. SATELLITE AND PLANETARY MOTION

6.1 Motion of a Satellite in a Circular Orbit

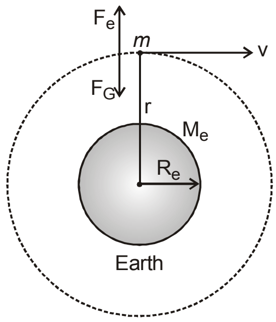
To understand how a satellite continously moves in its orbit, we consider the projection of a body horizontally from the top of a high mountain on earth as shown in figure. Here till our discussion ends we neglect air friction. The distance the projectile travels before hitting the ground depends on the launching speed. The greater the speed, the greater the distance.

The distance the projectile travels before hitting the ground is also affected by the curvature of earth as shown in figure shown. This figure was given by newton in his explanantion of laws of gravitation. it shows different trajectories for diferent launching speeds. As the launching speed is made greater, a speed is reached where by the projectile's path follow the curvature of the earth. This is the launching speed which places the projectile in a circular orbit. Thus an object in circular orbit may be regarded as falling, but as it falls its path is concentric with the earth's spherical surface and the object maintains a fixed distance from the earth's centre. Since the motion may continue indefinitely, we may say that the orbit is stable.



Let's find the speed of a satellite of mass *m* in a circular orbit around the earth. Consider a satellite revolving around the earth in a circular orbit of radius r as shown in figure.

If its orbit is stable during its motion, the net gravitational force on it must be balanced by the centrifugal force on it relative to the rotating frame as



$$\frac{GM_e m}{r^2} = \frac{mv^2}{r}$$

or $$v = \sqrt{\frac{GM_e}{r}}$$

Expression in above equation gives the speed of a statellite in a stable circular orbit of radius r.

6.2 Energies of a Satellite in a Circular Orbit

When there is a satellite revolving in a stable circular orbit of radius *r* around the earth, its speed is given by above equation. During its motion the kinetic energy of the satellite can be given as

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{GM_e m}{r} \qquad ...(1)$$

As gravitational force on satellite due to earth is the only force it experiences during motion, it has gravitational interaction energy in the field of earth, which is given as

$$U = -\frac{GM_e m}{r} \qquad ...(2)$$

Thus the total energy of a satellite in an orbit of radius r can be given as

Total energy E = Kinetic energy K + Potential Energy U

$$= \frac{1}{2}\frac{GM_e m}{r} - \frac{GM_e m}{r}$$

or $$E = -\frac{1}{2}\frac{GM_e m}{r} \qquad ...(3)$$

From equation (1), (2) and (3) we can see that

$$|k| = \left|\frac{U}{2}\right| = |E|$$

The above relation in magnitude of total, kinetic and potential energies of a satelline is very useful in numerical problem so it is advised to keep this relation in mind while handing satellite problems related to energy.

Now to understand satellite and planetary motion in detail, we take few example.

EXAMPLE 9

Estimate the mass of the sun, assuming the orbit of the earth round the sun to be a circle. The distance between the sun and earth is 1.49×10^{11} m and $G = 6.66 \times 10^{-11}$ Nm^2/kg^2.

Sol. Here the revolving speed of earth can be given as

$$v = \sqrt{\frac{GM}{r}} \qquad \text{[Orbital speed]}$$

Where M is the mass of sun and r is the orbit radius of earth.

We known time period of earth around sun is T = 365 days, thus we have

$$T = \frac{2\pi r}{v} \quad \text{or} \quad T = 2\pi r\sqrt{\frac{r}{GM}} \quad \text{or } M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4 \times (3.14)^2 \times (1.49 \times 10^{11})^3}{(365 \times 24 \times 3600)^2 \times (6.66 \times 10^{-11})}$$

$= 1.972 \times 10^{22}$ kg

EXAMPLE 10

If the earth be one-half of its present distance from the sun, how many days will be in one year ?

Sol. If orbit of earth's radius is R, in previous example we've discussed that time period is given as

$$T = 2\pi r\sqrt{\frac{r}{Gm}} = \frac{2\pi}{\sqrt{GM}} r^{3/2}$$

If radius changes or $r' = \frac{r}{2}$, new time period becomes

$$T' = \frac{2\pi}{\sqrt{GM}} r'^{3/2}$$

From above equations, we have

$$\frac{T}{T'} = \left(\frac{r}{r'}\right)^{3/2}$$

or $$T' = T\left(\frac{r'}{r}\right)^{3/2}$$

$$= 365\left(\frac{1}{2}\right)^{3/2} = \frac{365}{2\sqrt{2}} \text{days}$$

EXAMPLE 11

A satellite revolving in a circular equatorial orbit of radius $r = 2.00 \times 10^4$ km from west to east appear over a certain point at the equator every t = 11.6 hours. Using this data, calculate the mass of the earth. The gravitational constant is supposed to be known.

Sol. Here the absolute angular velocity of satellite is given by

$$\omega = \omega_s + \omega_E$$

Where ω_E is the angular velocity of earth, which is from west to east.

or $$\omega = \frac{2\pi}{t} + \frac{2\pi}{T}$$

[Where t = 11.6 hr. and T = 24 hr.]

From Kepler's III law, we have $\omega = \frac{\sqrt{GM}}{r^{3/2}}$

Thus we have $\frac{\sqrt{GM}}{r^{3/2}} = \frac{2\pi}{t} + \frac{2\pi}{T}$

or $$M = \frac{4\pi^2 r^3}{G}\left[\frac{1}{t} + \frac{1}{T}\right]^2$$

$$= \frac{4\pi^2 (2\times 10^7)^3}{(6.67\times 10^{-11})}\left[\frac{1}{11.6\times 3600} + \frac{1}{24\times 3600}\right]^2$$

$= 6.0 \times 10^{24}$ kg

EXAMPLE 12

A satellite of mass m is moving in a circular orbit of radius r. Calculate its angular momentum with respect to the centre of the orbit in terms of the mass of the earth.

Sol. The situation is shown in figure

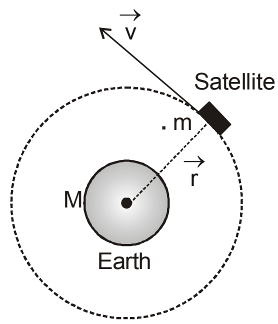
The angular momentum of the satellite with respect to the centre of orbit is given by

$$\vec{L} = \vec{r} \times m\vec{v}$$

Where $\vec{r}$ is the position vector of satellite with respect to the centre

of orbit and $\vec{v}$ is its velocity vector of satellite.

In case of circular orbit, the angle between $\vec{r}$ and $\vec{v}$ is 90°. Hence



$$L = m v r \sin 90° = m v r \qquad ...(1)$$

The direction is perpendicular to the plane of the orbit.

We know orbital speed of satellite is

$$v = \sqrt{\frac{GM}{r}} \qquad ...(2)$$

From equation (1) and (2), we get

$$L = m\sqrt{\frac{GM}{r}} \quad \Rightarrow L = (GMm^2 r)^{1/2}$$

Now we will understand the concept of ***double star system*** through an example.

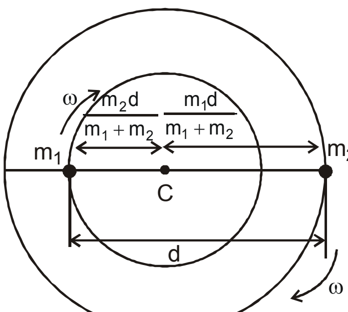
EXAMPLE 13

In a double star, two stars of masses m_1 and m_2. distance d apart revolve about their common centre of mass under the influence of their mutual gravitational attraction. Find an expression for the period T in terms of masses m_1, m_2 and d. Find the ratio of their angular momenta about centre of mass and also the ratio of their kinetic energies.

Sol. The centre of mass of double star from mass m_1 is given by

$$r_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{m_1 \times 0 + m_2 d}{m_1 + m_2} = \frac{m_2 d}{m_1 + m_2}$$

$\therefore$ Distance of centre of mass from m_2 is



$$r'_{cm} = d - r_{cm} = d - \frac{m_2 d}{m_1 + m_2} = \frac{m_1 d}{m_1 + m_2}$$

Both the stars rotate around centre of mass in their own circular orbits with the same angular speed ω. the gravitational force acting on each star provides the necessary centripetal force. if we consider the rotation of mass m_1, then

$$m_1 (r_{cm})\omega^2 = \frac{Gm_1 m_2}{d^2}$$

or $$m_1\left(\frac{m_2 d}{m_1 + m_2}\right)\omega^2 = \frac{Gm_1 m_2}{d^2}$$

This gives $\omega = \dfrac{2\pi}{T} = \sqrt{\left(\dfrac{G(m_1 + m_2)}{d^3}\right)}$

or Period of revolution

$$T = 2\pi\sqrt{\left(\frac{d^3}{G(m_1 + m_2)}\right)}$$

Ratio of Angular Momenta is

$$\frac{J_1}{J_2} = \frac{I_1\omega}{I_2\omega} = \frac{I_1}{I_2} = \frac{m_1\left(\frac{m_2 d}{m_1 + m_2}\right)^2}{m_2\left(\frac{m_1 d}{m_1 + m_2}\right)^2} = \frac{m_2}{m_1}$$

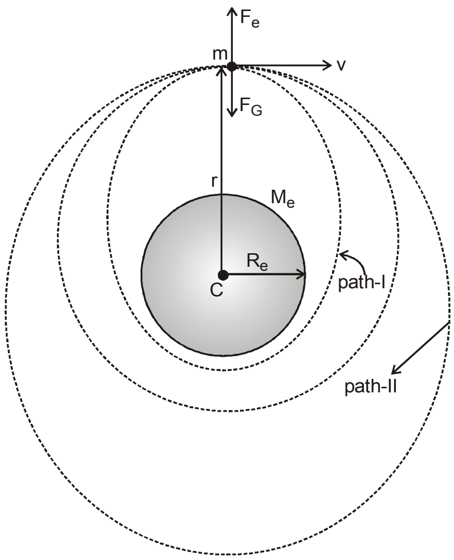
Ratio of kinetic energies is

$$\frac{K_1}{K_2} = \frac{\frac{1}{2} I_1 \omega^2}{\frac{1}{2} I_2 \omega^2} = \frac{I_1}{I_2} = \frac{m_2}{m_1}$$

7. MOTION OF A SATELLITE IN ELLIPTICAL PATH

Whenever a satellite is in a circular or elliptical path, these orbits are called bounded orbits as satellite is moving in an orbit bounded to earth. The bound nature of orbit means that the kinetic energy of satellite is not enough at any point in the orbit to take the satellite to infinity. In equation shown negative total energy of a revolving satellite shows its boundness to earth. Even when a body is in elliptical path around the earth, its total energy must be negative. Lets first discuss how a satellite or a body can be in elliptical path.

Consider a body (satellite) of mass *m* in a circular path of radius r around the earth as shown in figure. we've discussed that in circular path the net gravitational frame on body is exactly balancing the centrifugal force on it in radial direction relative to a rotating frame with the body.

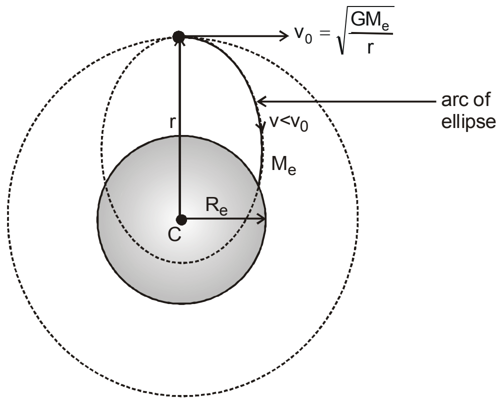


If suddenly the velocity of body decreases then the centrifugal force on it becomes less then the gravitational force acting on it and due to this it can not continue in the circular orbit and will come inward from the circular orbit due to unbalanced force. Mathematical analysis shown that this path-I along which the body is now moving is an ellipse.

The analytical calculations of the laws for this path is beyond the scope of this book. But it should be kept in mind that if velocity of a body at a distance r from earth's centre tangential to the circular orbit is less than $\sqrt{\frac{GM_e}{r}}$ then its path will be elliptical with earth centre at one of the foci of the ellipse.

Similarly if the speed of body exceeds $\sqrt{\frac{GM_e}{r}}$ then it must move out of the circular path due to unbalancing of forces again but this time $F_e > F_g$. Due to this if speed of body is not increased by such a value that its kinetic energy can take the particle to infinity then it will follow in a bigger elliptical orbit as shown in figure in path-II, with earth's at one of the foci of the orbit.

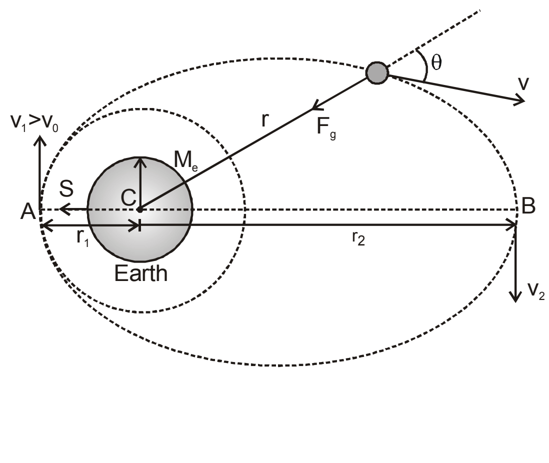
In above case when speed of body was decreased and its value is lesser than $\sqrt{\frac{GM_e}{r}}$ and the speed is decreased to such a value that the elliptical orbit will intersect the earth's surface as shown in figure, then body will follow an arc of ellipse and will fall back to earth.



7.2 Satellite Motion and Angular Momentum Conservation

We've discussed that when a body is in bounded orbit around a planet it can be in circular or elliptical path depending on its kinetic energy at the time of launching. Lets consider a case when a satellite is launched in an orbit around the earth.

A satellite S is first fired away from earth source in vertical direction to penetrate the earth's atmosphere. When it reaches point A, it is imparted a velocity in tangential direction to start its revolution around the earth in its orbit.



This velocity is termed as insertion velocity, if the velocity imparted to satellite is $v_0 = \sqrt{\frac{GM_e}{r_1}}$ then it starts following the circular path shown in figure. If velocity imparted is $v_1 > v_0$ then it will trace the elliptical path shown. During this motion the only force acting on satellite is the gravitational force due to earth which is acting along the line joining satellite and centre of earth.

As the force on satellite always passes through centre of earth during motion, we can say that on satellite there is no torque acting about centre of earth thus total angular momentum of satellite during orbital motion remains constant about earth's centre.

As no external force is involved for earth-satellite system, no external work is being done here so we can also state that total mechanical energy of system also remains conserved.

In the elliptical path of satellite shown in figure if r_1 and r_2 are the shortest distance (perigee) and farthest distance (appogee) of satellite from earth and at the points, velocities of satellite are v_1 and v_2 then we have according to conservation of angular momentum, the angular momentum of satellite at a general point is given as

$$L = mv_1r_1 = mv_2r_2 = mvr \sin \theta$$

During motion the total mechanical energy of satellite (kinetic + potential) also remains conserved. Thus the total energy of satellite can be given as

$$E = \frac{1}{2}mv_1^2 - \frac{GM_e m}{r_1}$$

$$= \frac{1}{2}mv_2^2 - \frac{GM_e m}{r_2}$$

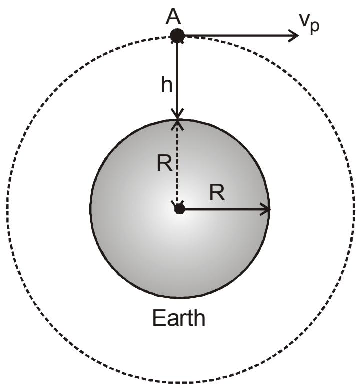
$$= \frac{1}{2}mv^2 - \frac{GM_e m}{r}$$

Using the above relations in equation written above we can find velocities v_1 and v_2 of satellite at nearest and farthest locations in terms of r_1 and r_2.

7.3 Projection of Satellites and Spaceships From Earth

To project a body into space, first it should be taken to a height where no atmopshere is present then it is projected with some initial speed. The path followed by the body also depends on the projection speed. Lets discuss the cases step by step.

Consider the situation shown in figure. A body of mass *m* is taken to a height h above the surface of earth to a point A and then projected with an insertion velocity v_p as shown in figure.



If we wish to launch the body as an earth's satellite in circular path the velocity of projection must be

$$v_p = \sqrt{\frac{GM_e}{R_e + h}} \qquad ...(1)$$

If *h* is small compared to radius of earth, we have

$$v_1 = v_p = \sqrt{\frac{GM_e}{R_e}} = \sqrt{g_s R_e} = 7.93 \text{ km/s.}$$

This velocity v_1 = 7.93 km/s with which, when a body is thrown from earth's surface tangentially so that after projection it becomes a satellite of earth in a circular orbit around it, is called orbital speed or first cosmic velocity.

We've already discussed that if projection speed is lesser the orbital speed, body will start following the inner ellipse and if velocity of projection is increased the body will follow the outer ellipse. If projection speed of the satellite is further increased, the outer ellipse will also become bigger and at a particular higher projection speed, it may also be possible that body will go to infinity and will never come back to earth again.

We have discussed that negative total energy of body shows its boundness. If we write the total energy of a body projected from point A as shown in figure is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_e m}{R_e + h}$$

If after projection body becomes a satellite of earth then it implies it is bounded to earth and its total energy is negative. If at point A, that much of kinetic energy is imparted to the body so that total energy of body becomes zero then it implies that the body will reach to infinity and escape from gravitational field of earth. If v_{II} is such a velocity then we have

$$\frac{1}{2}mv_{II}^2 - \frac{GM_e m}{R_e + h} = 0$$

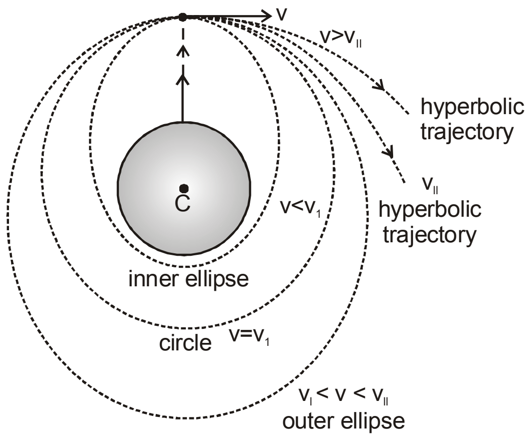
or $$v_{II} = \sqrt{\frac{2GM_e}{R_e + h}} = \sqrt{2}v_I \quad .....(2)$$

For h << R_e, we have

$$v_{II} = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2g_s R_e} = 11.2\,km/s \quad .....(3)$$

Thus from earth's surface a body is thrown at a speed of 11.2 km/s, it will escape from earth's gravitation. If the projection speed of body is less than this value total energy of body is negative and it wil orbit the earth in elliptical orbit. This velocity is referred as the second cosmic velocity or escape velocity. When a body is thrown with this speed, it follows a parabolic trajectory and will become free from earth's gravitational attraction.

When body is thrown with speed more then v_{II} then it moves along a hyperbolic trajectory and also leaves the region where the earth's gravitational attraction acts. Also when it reaches infinity some kinetic energy will be left in it and it becomes a satellite of sun, that is small artificial planet.



All the calculations we've performed till now do not take into account the influence of the sun and of the planets on the motion of the projected body. In other words we have assumed that the reference frame connected with the earth is an inertial frame and the body moves relative to it. But in reality the whole system body and the earth is in a non inertial frame which is permanently accelerated relative to sun.

Lets take some examples to understand some basic concepts related to gravitational energy and projection.

EXAMPLE 14

A spaceship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the spaceship in the orbit of overcome the gravitational pull. (Radius of the earth = 6400 km and g = 9.8 m/sec.)

Sol. In an orbit close to earth's surface velocity of space ship is $v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

We know escape velocity is $v_{II} = \sqrt{2gR}$

Hence additional velocity required to be imparted is $\Delta v = v_{II} - v = (\sqrt{2} - 1)\sqrt{gR}$

$= (\sqrt{2} - 1)\sqrt{9.8 \times 6400 \times 10^3} = 3.28 \times 10^3$ m/s

EXAMPLE 15

A particle is fired vertically upward with a speed of 9.8 km/s. Find the maximum height attained by the particle. Radius of the earth = 6400 km and g at the surface = 9.8 m/s^2. Consider only earth's gravitation.

Sol. Initial energy of particle on earth's surface is

$$E_r = \frac{1}{2}mu^2 - \frac{GMm}{R}$$

If the particle reaches upto a height h above the surface of earth then its final energy will only be the gravitational potential energy.

$$E_f = -\frac{GMm}{R+h}$$

According to energy conservation, we have

$$E_t = E_f$$

or $$\frac{1}{2}mu^2 - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or $$\frac{1}{2}u^2 - gR = -\frac{gR^2}{R+h}$$

or $$h = \frac{2gR^2}{2gR - u^2} - R$$

$$= \frac{2\times 9.8\times(6400\times10^3)^2}{2\times9.8\times6400\times10^3 - (9.8)^2} - 6400\times10^3$$

$$= (27300 - 6400) \times 10^3 = 20900 \text{ km}$$

EXAMPLE 16

A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy slowly at a constant rate C due to friction. If M_e and R_e denote the mass and radius of the earth respectively, show the the satellite falls on the earth in a limit t given by

$$t = \frac{GmM_e}{2C}\left(\frac{1}{R_e} - \frac{1}{r}\right)$$

Sol. Let velocity of satellite in its orbit of radius r be v then we have

$$v = \sqrt{\frac{GM_e}{r}}$$

When satellite approaches earth's surface, if its velocity becomes v', then it is given as

$$v' = \sqrt{\frac{GM_e}{R_e}}$$

The total initial energy of satellite at a distance r is

$$E_{T_f} = K_f + U_r$$

$$= \frac{1}{2}mv^2 - \frac{GM_e m}{R_e}$$

$$= -\frac{1}{2}\frac{GM_e m}{r}$$

The total final energy of satellite at a distance R_e is

$$E_{T_f} = K_f + U_r$$

$$= \frac{1}{2}mv'^2 - \frac{GM_e m}{R_e}$$

$$= -\frac{1}{2}\frac{GM_e m}{R_e}$$

As satellite is loosing energy at rate C, if it takes a time t in reaching earth, we have

$$Ct = E_{T_i} - E_{T_f}$$

$$= \frac{1}{2}GM_e m\left[\frac{1}{R_e} - \frac{1}{r}\right]$$

$\Rightarrow$ $$t = \frac{GM_e m}{2C}\left[\frac{1}{R_e} - \frac{1}{r}\right]$$

EXAMPLE 17

An artifical satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth.

(i) Determine the height of the satellite above earth's surface.

(ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth.

Sol. (i) Let M and R be the mass and radius of the earth respectively. If *m* be the mass of satellite, then escape velocity from earth $v_c = \sqrt{2gR_e}$

velocity of satellite = $\sqrt{\frac{gR_e}{2}}$

Further we know orbital speed of satallite at a height *h* is

$$v_s = \sqrt{\left(\frac{GM_e}{r}\right)} = \sqrt{\left(\frac{R_e^2 g}{R_e + h}\right)}$$

or $$v_s^2 = \frac{R^2 g}{R + h}$$

From equation written above, we get

$$h = R = 6400 \text{ km}$$

(ii) Now total energy at height h = total energy at earth's surface (principle of conservation of energy)

or $$0 - GM_e \frac{m}{R + h} = \frac{1}{2}mv^2 - GM_e \frac{m}{R_e}$$

or $$\frac{1}{2}mv^2 = \frac{GM_e m}{R_e} - \frac{GM_e m}{2R_e}$$

[As h = R]

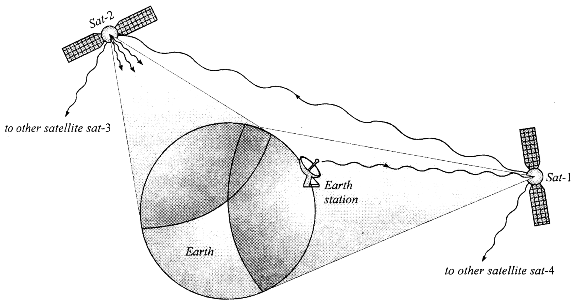
Solving we get $v = \sqrt{gR_e}$

or $$\sqrt{9.8 \times 6400 \times 10^3} = 7.919 \text{ km/s}$$

8. COMMUNICATION SATELLITES

Communication satellite around the earth are used by Information Technology for spreading information through out the globe.

Figure shows as to how using satellites an information from an earth station, located at a point on earth's surface ca be sent throughout the world.



First the information is sent to the nearest satellite in the range of earth station by means of electromanetic waves then that satellite broadcasts the signal to the region of earth exposed to this satellite and also send the same signal to other satellite for broadcasting in other parts of the globe.

8.1 Geostationary Satellite and Parking Orbit

There are so many types of communication satellites revolving around the arth in different orbits at different heights depending on their utility. Some of which are Geostationary satellites, which appears at rest relative to earth or which have same angular velocity as that of earth's rotation i.e., with a time peiod of 24 hr. such satellite must be orbiting in an orbit of specific radius. This orbit is called parking orbit. If a Geostationary satellite is at a height h above the earth's surface then its orbiting speed is given as

$$v_{gs} = \sqrt{\frac{GM_e}{(R_e + h)}}$$

The time period of its revolution can be given Kepler's third law as

$$T^2 = \frac{4\pi^2}{GM_e}(R_e + h)^3$$

or $$T^2 = \frac{4\pi^2}{g_s R_e^2}(R_e + h)^3$$

or $$h = \left(\frac{g_s R_e^2}{4\pi^2} T^2\right)^{1/3} - R_e$$

or $$h = \left[\frac{9.8\times[6.4\times10^6]\times[86400]^2}{4\times(3.14)^2}\right]^{1/3} - 6.4\times10^6$$

$$= 35954.6 \text{ km} \simeq 36000 \text{ km}$$

Thus when a satellite is launched in an orbit at a height of about 36000 km above the quator then it will appear to be at rest with respect to a point on Earth's surface. A Geostationary satellite must have in orbit in equatorial plane due to the geographic limitation because of irregular geometry of earth (ellipsoidal shape.)

In short

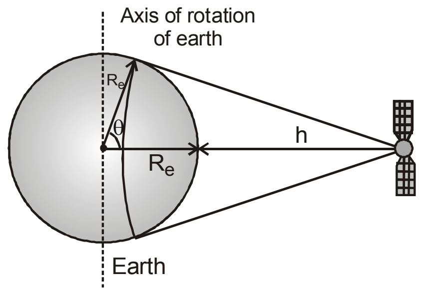
- Plane of the satellite should pass through centre of the planet
- For geostationary satallites plane should be equatorial plane
- Time peirod should be 24 hrs & direction should be west to east
- For any point on the earth, geostationary satellite is stationary.

8.2 Broadcasting Region of a Satellite

Now as we known the height of a geostationary satellite we can easily find the area of earth exposed to the satellite or area of the region in which the comunication can be mode using this satellite. Figure shown earth and its exposed area to a geostationary satellite. Here the angle θ can be given as

$$\theta = \cos^{-1}\left(\frac{R_e}{R_e+h}\right)$$

Now we can find the solid angle Ω which the exposed area subtend on earth's centre as



$$\Omega = 2\pi\,(1 - \cos\theta)$$

$$= 2\pi\left(1-\frac{R_e}{R_e+h}\right) = \frac{2\pi h}{R_e+h}$$

Thus the area of earth's surface to geostationary satellite is

$$S = \Omega R_e^2 = \frac{2\pi h R_e^2}{R_e+h}$$

Lets take some examples to understand the concept in detail.

EXAMPLE 18

A satellite is revolving around the earth in an orbit of radius double that of the parking orbit and revolving in same sense. Find the periodic time duration between two instants when this satallite is closest to a geostationary satellite.

Sol. We know that the time period of revolution of a satellite is given as

$$T^2 = \frac{4\pi^2}{GM_e}r^3 \qquad \text{[Kepler's III law]}$$

For satellite given in problem and for a geostationary satellite we have

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^3 \qquad \text{or} \qquad T_1 = \left(\frac{r_1}{r_2}\right)^3 \times T_2$$

$$= (2)^3 \times 24 = 192 \text{ hr}$$

If Δt be the time between two sucessive instants when the satellite are closed then we must have

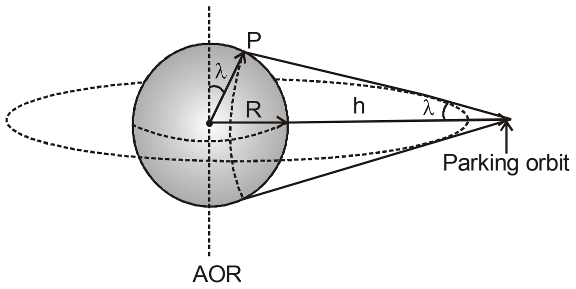
$$\Delta t = \frac{\theta}{\omega_1} = \frac{2\pi+\theta}{\omega_2} = \frac{2\pi}{\omega_2-\omega_1}$$

Where ω_1 and ω_2 are the angular speeds of the two planets

EXAMPLE 19

Find the minimum colatitude which can directly receive a signal from a geostationary satellite.

Sol. The farthest point on earth, which can receive signals from the parking orbit is the point where a length is drawn on earth surface from satellite as shown in figure. The colatitude λ of point P can be obtained from figure as



$$\sin\lambda = \frac{R_e}{R_e + h} \simeq \frac{1}{7}$$

We known for a parking orbit $h \simeq 6R_e$

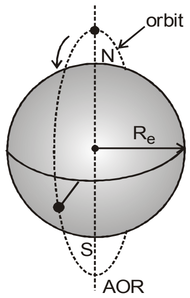
Thus we have $\lambda = \sin^{-1}\left(\frac{1}{7}\right)$

EXAMPLE 20

If a satellite is revolving around the earth in a circular orbit in a plane containing earth's axis of rotation. if the angular speed of satellite is equal to that of earth, find the time it takes to move from a point above north pole of a point above the equator.

Sol. A satellite which rotates with angular speed equal to earth's rotation has an orbit radius 7 R_e and the angular speed of revolution is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{86400} = 7.27 \times 10^{-5}\ \text{rad/s}$$



When satellite moves from a point above north pole to a point above equator, it traverses an angle $\pi/2$, this time taken is

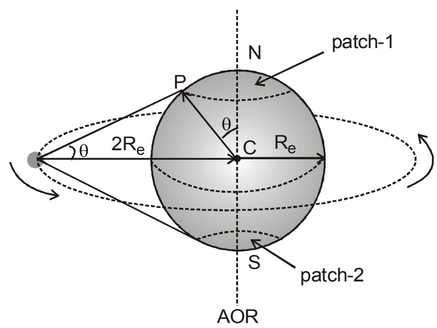
$$t = \frac{\pi/2}{\omega} = 21600\ \text{s} = 6\ \text{hrs.}$$

EXAMPLE 21

A satellite is orbiting around the earth in an orbit in equatorial plane of radius $2R_e$ where R_e is the radius of earth. Find the area on earth, this satellite covers for communication purpose in its complete revolution.

Sol. As shown in figure when statelite S revolves, it covers a complete circular belt on earth's surface for communication. If the colatitude of the farthest point on surface upto which singals can be received (point P) is θ then we have

$$\sin\theta = \frac{R_e}{2R_e} = \frac{1}{2} \qquad \text{or} \qquad \theta = \frac{\pi}{6}$$



During revolution satellite leaves two spherical patches 1 and 2 on earth surface at north and south poles where no signals can be transmitted due to curvature. The areas of these patches can be obtained by solid angles. The solid angle subtended by a patch on earth's centre is

$$\Omega = 2\pi\,(1 - \cos\theta) = \pi\,(2 - \sqrt{3})\ \text{st.}$$

Area of patch 1 and 2 is

$$A_P = \Omega R_e^2 = \pi(2 - \sqrt{3})R_e^2$$

Thus total area on earth's surface to which communication can be made is

$$A_C = 4\pi R_e^2 - 2A_P$$

$$= 4\pi R_e^2 - 2\pi(2-\sqrt{3})R_e^2$$

$$= 2\pi R_e^2(2-2+\sqrt{3}) = 2\sqrt{3}\ \pi R_e^2$$

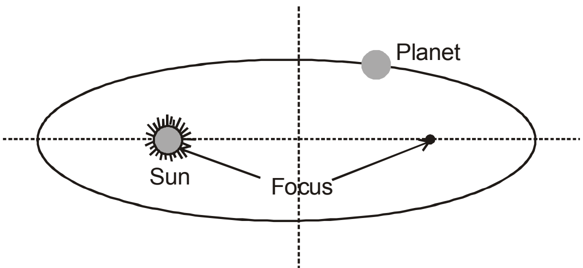
9. KEPLER'S LAWS OF PLANETARY MOTION

The motions of planet in universe have always been a puzzle. In 17th century Johannes Kepler, after a life time of study worded out some empirical laws based on the analysis of astronomical measurements of Tycho Brahe. Kepler formulated his laws, which are kinematical description of planetary motion. Now we discuss these laws step by step.

9.1 Kepler's First Law [The Law of Orbits]

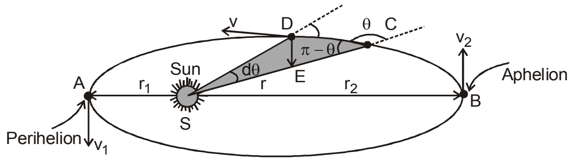
Kepler's first law is illustrated in the image shown in figure. It states that "All the planets move around the sun in ellipitcal orbits with sun at one of the focus not at centre of orbit."

It is observed that the orbits of planets around sun are very less ecentric or approximately circular

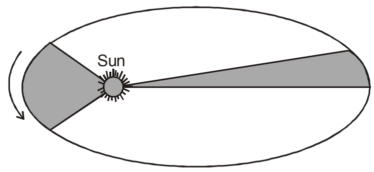


9.2 Kepler's Second Law [The Law of Areas]

Kepler's second Law is basically an alternative statement of law of conservation of momentum. It is illustrated in the image shown in figure(a). We know from angular momentum conservation, in elliptical orbit plane will move faster when it is nearer to the sun. Thus when a planet executes elliptical orbit its angular speed changes continuously as it moves in the orbit. The point of nearest approach of the planet to the sun is termed perihelion. The point of greatest seperation is termed aphelion. Hence by angular momentum conservation we can state that the planet moves with maximum speed when it is near perihelion and moves with slowest speed when it is near aphelion.



(a)



(b)

Kepler's second law states that "The line joining the sun and planet sweeps out equal areas in equal time or the rate of sweeping area by the position vector of the planet with respect to sun remains constant. "This is shown in figure (b).

The above statement of Kepler's second law can be verified by the law of conservation of angular momentum. To verify this consider the moving planet around the sun at a general point C in the orbit at speed v. Let at this instant the distance of planet from sun is r. If θ be the angle between position vector $\vec{r}$ of planet and its velocity vector then the angular momentum of planet at this instant is

$$L = mvr \sin\theta \qquad ...(1)$$

In an elemental time the planet will cover a small distance CD = dl and will travel to another adjacent point D as shown in figure (a), thus the distance CD = vdt. In this duration dt, the position vector $\vec{r}$ sweeps out an area equal to that of triangle SCD, which is calculated as

Area of triangle SCD is $dA = \frac{1}{2} \times r \times vdt \sin(\pi - \theta)$

$$= \frac{1}{2} \, r \, v \sin\theta . \, dt$$

Thus the rate of sweeping area by the position vector $\vec{r}$ is

$$\frac{dA}{dt} = \frac{1}{2} rv \sin\theta$$

Now from equation (1)

$$\frac{dA}{dt} = \frac{L}{2m} = \text{constant} \qquad ...(2)$$

The expression in equation (2) verifies the statement of Kepler' II law of planetary motion.

9.3 Kepler's Third law [The Law of Periods]

Kepler's Third Law is concerned with the time period of revolution of planets. It states that "The time period of revolution of a planet in its orbit around the sun is directly proportional to the cube of semi-major axis of the elliptical path around the sun"

If 'T' is the period of revolution and 'a' be the semi-major axis of the path of planet then according to Kepler's III law, we have

$$T^2 \propto a^3$$

For circular orbits, it is a special case of ellipse when its major and minor axis are equal. If a planet is in a circular orbit of radius r around the sun then its revolution speed must be given as

$$v = \sqrt{\frac{GM_s}{r}}$$

Where M_s is the mass of sun. Here you can recall that this speed is independent from the mass of planet. Here the time period of revolution can be given as

$$T = \frac{2\pi r}{v} \qquad \text{or} \qquad T = \frac{2\pi r}{\sqrt{\frac{GM_s}{r}}}$$

Squaring equation written above, we get

$$T^2 = \frac{4\pi^2}{GM_s} r^3 \qquad ...(1)$$

Equation (1) verifies the statement of Kepler's third law for circular orbits. Similarly we can also verify it for elliptical orbits. For this we start from the relation we've derived earlier for rate of sweeping area by the position vector of planet with respect to sun which is given as

$$\frac{dA}{dt} = \frac{L}{2m}$$

EXAMPLE 22

The moon revolves around the earth 13 times per year. If the ratio of the distance of the earth from the sun to the distance of the moon from the earth is 392, find the ratio of mass of the sun to the mass of the earth.

Sol. The time period T_e of earth around sun of mass M_s is given by

$$T_e^2 = \frac{4\pi^2}{GM_s} \times r_e^3 \qquad ...(1)$$

Where r_e is the radius of the earth.

Similarly, time period T_m of moon around earth is given by

$$T_m^2 = \frac{4\pi^2}{GM_e} \times r_m^3 \qquad ...(2)$$

Dividing equation(1) by equation (2), we get

$$\left(\frac{T_e}{T_m}\right)^2 = \left(\frac{M_e}{M_s}\right)\left(\frac{r_e}{r_m}\right)^3$$

or $$\left(\frac{M_s}{M_e}\right) = \left(\frac{T_m}{T_e}\right)^2 \times \left(\frac{r_e}{r_m}\right)^3 \qquad ...(3)$$

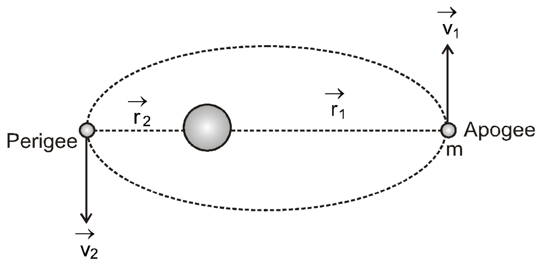
Substituting the given values, we get

$$\left(\frac{M_s}{M_e}\right) = \left\{\frac{(13)}{1}\right\}^2 \times (392)^3 = 3.56 \times 10^5$$

EXAMPLE 23

A satellite revolves around a planet in an elliptical orbit. Its maximum and minimum distances from the planet are 1.5×10^7 m and 0.5×10^7 m respectively. If the speed of the satellite at the farthest point be 5×10^3 m/s, calculate the speed at the nearest point.

Sol.



In case of elliptical orbit, the speed of satellite varies constantly as shown in figure. Thus according to the law of conservation of angular momentum, the satellite must move faster at a point of closest approach (Perigee) than at a farthest point (Appogee).

We know that, $\vec{L} = \vec{r} \times m\,\vec{v}$

Hence, at the two points,

$L = m\,v_1\,r_1 = m\,v_2\,r_2$

or $$\frac{v_1}{v_2} = \frac{r_2}{r_1}$$

Substituting the given values, we get

$$\frac{5\times10^3}{v_2} = \frac{0.5\times10^7}{1.5\times10^7}$$

$\Rightarrow$ $v_2 = 1.5 \times 10^4$ m/s

EXAMPLE 24

Imagine a light planet revolving around a very massive star in a circular orbit of radius r with a period of revolution T. On what power of r, will the square of time period depend if the gravitational force of attraction between the planet and the star is proportional to $r^{-5/2}$.

Sol. As gravitation provides centripetal force

$$\frac{mv^2}{r} = \frac{K}{r^{5/2}},$$

i.e., $$v^2 = \frac{K}{mr^{3/2}}$$

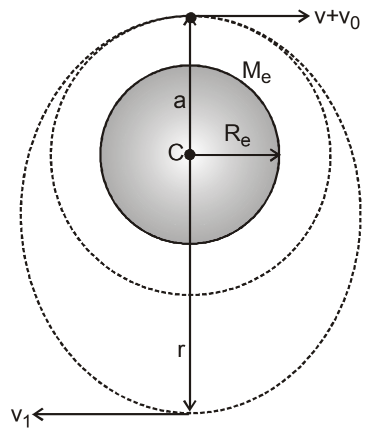
So that $$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{mr^{3/2}}{K}}$$

or $$T^2 = \frac{4\pi^2 m}{K} r^{7/2}; \quad \text{so } T^2 \propto r^{7/2}$$

EXAMPLE 25

A satellite is revolving round the earth in a circular orbit of radius a with velocity v_0. A particle is projected from the satellite in forward direction with relative velocity $v = (\sqrt{5/4} - 1)v_0$. Calculate, during subsequent motion of the particle its minimum and maximum distances from earth's centre. The corresponding situation is shown in figure.

Initial velocity of satellite $v_0 = \sqrt{\left(\frac{GM}{a}\right)}$



When particle is thrown with the velocity v relative to satellite, the resultant velocity of particle will become

$$v_R = v_0 + v$$

$$= \sqrt{\left(\frac{5}{4}\right)} v_0 = \sqrt{\left(\frac{5}{4}\frac{GM}{a}\right)}$$

As the particle velocity is greater than the velocity required for circular orbit, hence the particle path deviates from circular path to elliptical path. At position of minimum and maximum distance velocity vectors are perpendicular to instantaneous radius vector. In this elliptical path the minimum distance of particle from earth's centre is a and maximum speed in the path is v_R and let the maximum distance and minimum speed in the path is r and v_1 respectively.

Now as angular momentum and total energy remain conserved. Applying the law of conservation of angular momentum, we have

$$m v_1 r = m(v_0 + v) a$$

[m = mass of particle]

or $$v_1 = \frac{(v_0 + v)a}{r}$$

$$= \frac{a}{r}\left[\sqrt{\left(\frac{5}{4}\frac{GM}{a}\right)}\right]$$

$$= \frac{1}{r}\left[\sqrt{\left(\frac{5}{4} \times GMa\right)}\right]$$

Applying the law of conservation of energy

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r} = \frac{1}{2}m(v_0 + v)^2 - \frac{GMm}{a}$$

or $$\frac{1}{2}m\left(\frac{5}{4}\frac{GMa}{r^2}\right) - \frac{GMm}{r}$$

$$= \frac{1}{2}m\left(\frac{5}{4}\frac{GM}{a}\right) - \frac{GMm}{a}$$

$$\frac{5}{8} \times \frac{a}{r^2} - \frac{1}{r} = \frac{5}{8} \times \frac{1}{a} - \frac{1}{a} = -\frac{3}{8a}$$

or $$3r^2 - 8\,ar + 5\,a^2 = 0$$

or $$r = a \text{ or } \frac{5a}{3}$$

Thus minimum distance of the particle = a

And maximum distance of the particle = $\frac{5a}{3}$

EXAMPLE 26

A sky lab of mass 2×10^3 kg is first launched from the surface of earth in a circular orbit of radius 2 R (from the centre of earth) and then it is shifted from this circular orbit to another circular orbit of radius 3 R. Calculate the minimum energy required (a) to place the lab in the first orbit (b) to shift the lab from first orbit to the second orbit. Given, R = 6400 km and g = 10 m/s^2.

Sol. (a) The energy of the sky lab on the surface of earth

$$E_S = KE + PE = 0 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{R}$$

And the total energy of the sky lab in an orbit of radius 2 R is

$$E_1 = -\frac{GMm}{4R}$$

So the energy required to placed the lab from the surface of earth to the orbit of radius 2R is given as

$$E_1 - E_s = -\frac{GMm}{4R} - \left[-\frac{GMm}{R}\right] = \frac{3}{4}\frac{GMm}{R}$$

or $$\Delta E = \frac{3}{4}\frac{m}{R} \times gR^2 = \frac{3}{4}mgR$$

$$\left[\text{As } g = \frac{GM}{R^2}\right]$$

or $$\Delta E = \frac{3}{4}(2 \times 10^3 \times 10 \times 6.4 \times 10^6)$$

$$= \frac{3}{4}(12.8 \times 10^{10}) = 9.6 \times 10^{10}\ \text{J}$$

(b) As for II orbit of radius 3R the total energy of sky lab is

$$E_2 = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$$

or $$E_2 - E_1 = -\frac{GMm}{6R} - \left(-\frac{GMm}{4R}\right) = \frac{1}{12}\frac{GMm}{R}$$

or $$\Delta E = \frac{1}{12} mgR$$

$$= \frac{1}{12}(12.8 \times 10^{10})$$

$$= 1.1 \times 10^{10} \text{ J}$$

EXAMPLE 27

A satellite is revolving around a planet of mass M in an elliptic orbit of semimajor axis a. Show that the orbital speed of the satellite when it is at a distance r from the focus will be given by :

$$v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$

Sol. As in case of elliptic orbit with semi major axes a, of a satellite total mechanical energy remains constant, at any position of satellite in the orbit, given as

$$E = -\frac{GMm}{2a}$$

or $$KE + PE = -\frac{GMm}{2a} \quad ...(1)$$

Now, if at position r, v is the orbital speed of satellite, we have

$$KE = \frac{1}{2}mv^2 \text{ and } PE = -\frac{GMm}{r} \quad ...(2)$$

So from equation (1) and (2), we have

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a},$$

i.e., $$v^2 = GM\left[\frac{2}{r} - \frac{1}{a}\right]$$

Note

The student can now attempt section C from exercise.

Exercise - 1   Objective Problems | JEE Main

Section A - Newton's law of Gravitation & Gravitational Field, Potential & Potential energy

1. On doubling the distance between two masses the gravitational force between them will -

(A) remain unchanged (B) become one-fourth
(C) become half (D) become double

2. A hollow spherical shell is compressed to half its radius. The gravitational potential at the centre

(A) increases
(B) decreases
(C) remains same
(D) during the compression increases then returns at the previous value

3. Two different masses are dropped from same heights, then just before these strike the ground, the following is same :

(A) kinetic energy (B) potential energy
(C) linear momentum (D) Acceleration

4. A body of mass m rises to height h = R/5 from the earth's surface, where R is earth's radius. If g is acceleration due to gravity at earth's surface, the increase in potential energy is

(A) mg/h (B) $\frac{5}{6}$ mgh
(C) $\frac{3}{5}$ mgh (D) $\frac{6}{7}$ mgh

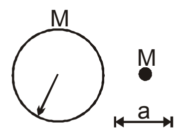
5. A planet has mass 1/10 of that of earth, while radius is 1/3 that of earth. If a person can throw a stone on earth surface to a height of 90m, then he will be able to throw the stone on that planet to a height

(A) 90m (B) 40 m
(C) 100 m (D) 45 m

6. **Statement - I :** Assuming zero potential at infinity, gravitational potential at a point cannot be positive.
Statement - 2 : Magnitude of gravitational force between two particle has inverse square dependence on distance between two particles.

(A) **Statement - 1** is true, **statement-2** is true and **statement-2** is correct explanation for **statement-1**
(B) **Statement -1** is true, **statement-2** is true and **statement - 2** is NOT the correct explanation for statement-1
(C) **Statement - 1** is true, **statement - 2** is false.
(D) **Statement - 1** is false, **statement - 2** is true.

7. A particle of mass M is at a distance a from surface of a thin spherical shell of equal mass and having radius a.



(A) Gravitational field and potential both are zero at centre of the shell
(B) Gravitational field is zero not only inside the shell but at a point outside the shell also
(C) Inside the shell, gravitational field alone is zero
(D) Neither gravitational field nor gravitational potential is zero inside the shell

8. Work done in taking a body of mass m to a height nR above surface of earth will be : (R = radius of earth)

(A) mgnR (B) mgR (n/n + 1)
(C) mgR$\frac{(n+1)}{n}$ (D) $\frac{mgR}{n(n+1)}$

9. If the distance between sun and earth is made 3 times of the present value then gravitational force between them will become :

(A) 9 times (B) $\frac{1}{9}$ times
(C) $\frac{1}{3}$ times (D) 3 times

10. Two point masses of mass 4m and m respectively separated by d distance are revolving under mutual force of attraction. Ratio of their kinetic energies will be

(A) 1 : 4 (B) 1 : 5

(C) 1 : 1 (D) 1 : 2

Section B - Variations in 'g'

11. If R is the radius of the earth and g the acceleration due to gravity on the earth's surface, the mean density of the earth is

(A) $4\pi G/3gR$ (B) $3\pi R/4gG$

(C) $3g/4\pi RG$ (D) $\pi Rg/12G$

12. The height above surface of earth where the value of gravitational acceleration is one fourth of that at surface, will be

(A) $R_e/4$ (B) $R_e/2$

(C) $3R_e/4$ (D) R_e

13. The decrease in the value of g on going to a height R/2 above the earth's surface will be -

(A) g/2 (B) $\frac{5g}{9}$

(C) $\frac{4g}{9}$ (D) $\frac{g}{3}$

14. At what altitude will the acceleration due to gravity be 25% of that at the earth's surface (given radius of earth is R) ?

(A) R/4 (B) R

(C) 3R/8 (D) R/2

15. If the radius of the earth be increased by a factor of 5, by what factor its density be changed to keep the value of g the same ?

(A) 1/25 (B) 1/5

(C) $1/\sqrt{5}$ (D) 5

Section C - Kepler's law, Orbital velocity, Escape velocity, Geo - Stationary Satellites

16. The potential energy of a body of mass 3kg on the surface of a planet is 54 joule. The escape velocity will be -

(A) 18m/s (B) 162 m/s

(C) 36 m/s (D) 6 m/s

17. If the kinetic energy of a satellite orbiting around the earth is doubled then -

(A) the satellite will escape into the space.

(B) the satellite will fall down on the earth

(C) radius of its orbit will be doubled

(D) radius of its orbit will become half.

18. The escape velocity from a planet is v_0. The escape velocity from a planet having twice the radius but same density will be -

(A) 0.5 v_0 (B) v_0

(C) $2v_0$ (D) $4v_0$

19. Two planets A and B have the same material density. If the radius of A is twice that of B, then the ratio of the escape velocity $\frac{v_A}{v_B}$ is

(A) 2 (B) $\sqrt{2}$

(C) $1/\sqrt{2}$ (D) 1/2

20. Select the correct choice(s) :

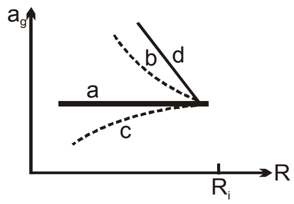
(A) The gravitational field inside a spherical cavity, within a spherical planet must be non zero and uniform.

(B) When a body is projected horizontally at an appreciable large height above the earth, with a velocity less than for a circular orbit, it will fall to the earth along a parabolic path

(C) A body of zero total mechanical energy placed in a gravitational field will escape the field

(D) Earth's satellite must be in equatorial plane.

21. A (nonrotating) star collapses onto itself from an initial radius R_i with its mass remaining unchanged. Which curve in figure best gives the gravitational acceleration a_g on the surface of the star as a function of the radius of the star during the collapse ?



(A) a (B) b

(C) c (D) d

22. A satellite of the earth is revolving in circular orbit with a uniform velocity V. If the gravitational force suddenly disappears, the statellite will

(A) continue to move with the same velocity in the same orbit

(B) move tangentially to the original orbit with velocity V

(C) fall down with increasing velocity

(D) come to a stop somewhere in its original orbit

23. A satellite revolves in the geostationary orbit but in a direction east to west. The time interval between its successive passing about a point on the equator is

(A) 48 hrs (B) 24 hrs

(C) 12 hrs (D) never

24. A satellite can be in a geostationary orbit around earth at a distance r from the centre. If the angular velocity of earth about its axis doubles, a satellite can now be in a geostationary orbit around earth if its distance from the center is

(A) $\frac{r}{2}$ (B) $\frac{r}{2\sqrt{2}}$

(C) $\frac{r}{(4)^{1/3}}$ (D) $\frac{r}{(2)^{1/3}}$

25. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is υ. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is –

(A) $\frac{3}{2}\upsilon$ (B) $\sqrt{\frac{3}{2}}\,\upsilon$

(C) $\upsilon\sqrt{\frac{2}{3}}$ (D) $\upsilon\,\frac{2}{3}$

Section A - Newton's law of Gravitation & Gravitational Field, Potential & Potential energy

1. Two masses m_1 & m_2 are initially at rest and are separated by a very large distance. If the masses approach each other subsequently, due to gravitational attraction between them, their relative velocity of approach at a separation distance of d is

(A) $\frac{2Gd}{(m_1+m_2)}$ (B) $\frac{(m_1+m_2)G}{2d}$

(C) $\left[(m_1+m_2)\frac{2G}{d}\right]^{1/2}$ (D) $(m_1+m_2)^{1/2}\,2Gd$

2. A man of mass m starts falling towards a planet of mass M and radius R. As he reaches near to the surface, he realizes that he will pass through a small hole in the planet. As he enters the hole, he sees that the planet is really made of two pieces a spherical shell of negligible thickness of mass 2M/3 and a point mass M/3 at the centre. Change in the force of gravity experienced by the man is

(A) $\frac{2}{3}\frac{GMm}{R^2}$ (B) 0

(C) $\frac{1}{3}\frac{GMm}{R^2}$ (D) $\frac{4}{3}\frac{GMm}{R^2}$

Paragraph Q. 3 & Q. 4

Two uniform spherical stars made of same material have radii R and 2R. Mass of the smaller planet is m. They start moving from rest towards each other from a large distance under mutual force of gravity. The collision between the stars is inelastic with coefficient of restitution 1/2.

3. Kinetic energy of the system just after the collision is

(A) $\frac{8Gm^2}{3R}$ (B) $\frac{2Gm^2}{3R}$

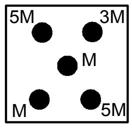
(C) $\frac{4Gm^2}{3R}$ (D) cannot be determined

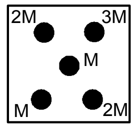
4. The maximum separation between their centres after their first collision

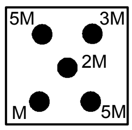
(A) 4R (B) 6R

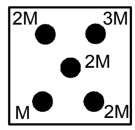
(C) 8R (D) 12R

5. A mass is at the center of a square, with four masses at the corners as shown.

(A) 

(B) 

(C) 

(D) 

Rank the choices according to the magnitude of the gravitational force on the center mass.

(A) $F_A = F_B < F_C = F_D$ (B) $F_A > F_B < F_D < F_C$

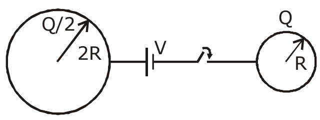
(C) $F_A = F_B > F_C = F_D$ (D) none

6. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of earth. Its radius in terms of earth R will be

(A) R/4 (B) R/2

(C) R/3 (D) R/8

7. There are two spheres of radii R and 2R having charges Q and $\frac{Q}{2}$ respectively. These two spheres are connected with a cell of emf V volts as shown in figure. When switch is closed, the final charge on sphere of radius 2R is $Q+\frac{8\pi\varepsilon_0 RV}{n}$. Then find the value of n.



(A) 4

(B) 11

(C) 6

(D) 3

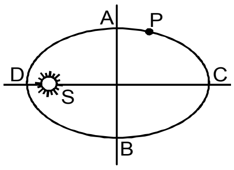
8. On the surface of earth acceleration due to gravity is g and gravitational potential is V.R. is the radius of earth. Match the following :

Column I	Column II
(A) At height h = R, magnitude of acceleration due at surface to gravity	**(P)** Decreases by a factor 1/4 as compared to
(B) At depth h = R/2, magnitude of acceleration due to gravity	**(Q)** Decreases by a factor 1/2 as compared to at surface
(C) At height h = R,magnitude of potential due to gravity	**(R)** Increases by a factor 2 as compared to at surface
(D) Magnitude of potential V at the centre of earth	**(S)** Increase by a factor 3/2 as compared to at surface

(A) $A \rightarrow P$; $B \rightarrow Q$; $C \rightarrow R$; $D \rightarrow S$
(B) $A \rightarrow P$; $B \rightarrow Q$; $C \rightarrow S$; $D \rightarrow R$
(C) $A \rightarrow P$; $B \rightarrow Q$; $C \rightarrow Q$; $D \rightarrow S$
(R) $A \rightarrow R$; $B \rightarrow P$; $C \rightarrow Q$; $D \rightarrow S$

Section C - Kepler's law, Orbital velocity, Escape velocity, Geo - Stationary Satellites

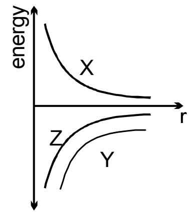
9. Figure shows the orbit of a planet P round the sun S. AB and CD are the minor and major axes of the ellipse.



If U is the potential energy and K kinetic energy then |U| > |K| at

(A) Only D (B) Only C
(C) both D & C (D) neither D nor C

10. Figure shows the variation of energy with the orbit radius r of a satellite in a circular motion. Select the correct statement.



(A) Z is total energy, Y is kinetic energy and X is potential energy
(B) X is kinetic energy, Y is total energy and Z is potential energy
(C) X is kinetic energy, Y is potential energy and Z is total energy
(D) Z is kinetic energy, X is potential energy and Y is total energy

11. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth
(A) the acceleration of S is always directed towards the centre of the earth
(B) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant
(C) the total mechanical energy of S varies periodically with time
(D) the linear momentum of S remains constant in magnitude

12. An earth satellite is moved from one stable circular orbit to another larger and stable circular orbit. The following quantities increase for the satellite as a result of this change
(A) gravitational potential energy
(B) angular velocity
(C) linear orbital velocity
(D) centripetal acceleration

13. A spherical uniform planet is rotating about its axis. The velocity of a point on its equator is V. Due to the rotation of planet about its axis the acceleration due to gravity g at equator is 1/2 of g at poles. The escape velocity of a particle on the pole of planet in terms of V.

(A) $V_e = 2V$ (B) $V_e = V$
(C) $V_e = V/2$ (D) $V_e = \sqrt{3}\,V$

Exercise - 2 (Level-II) Multiple Correct | JEE Advanced

Section A - Newton's law of Gravitation & Gravitational Field, Potential & Potential energy

1. Two masses m_1 and m_2 ($m_1 < m_2$) are released from rest from a finite distance. They start under their mutual gravitational attraction

(A) acceleration of m_1 is more than that of m_2

(B) acceleration of m_2 is more than that of m_1

(C) centre of mass of system will remain at rest in all the references frame

(D) total energy of system remains constant

2. In side a hollow spherical shell

(A) everywhere gravitational potential is zero

(B) everywhere gravitational field is zero

(C) everywhere gravitational potential is same

(D) everywhere gravitational field is same

Section B - Variations in 'g'

3. The magnitudes of the gravitational field at distance r_1 and r_2 from the centre of a uniform sphere of radius R and mass M are F_1 and F_2 respectively. Then :

(A) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

(B) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ If $r_1 > R$ and $r_2 > R$

(C) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ If $r_1 > R$ and $r_2 > R$

(D) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ If $r_1 < R$ and $r_2 < R$

4. Three mass 'm' each are kept at corner of a equilateral triangle and are rotating under effect of mutual gravitational force -

(A) Radius of circular path followed by mass is a/2

(B) Velocity of mass is $\sqrt{\frac{Gm}{a}}$

(C) Binding energy of system is $\frac{1.5Gm^2}{a}$

(D) Time period of mass is $\sqrt{\frac{\pi a^3}{2Gm}}$

5. The gravitational potential changes uniformly from –20 j/kg to –40 J/kg as one moves along x-axis from x = –1m to x = +1m. Then gravitational field at origin

(A) must be equal to 10 N/kg

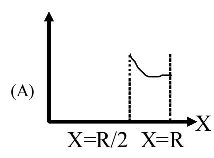
(B) may be equal to 10 N/kg

(C) may be greater than 10 N/kg

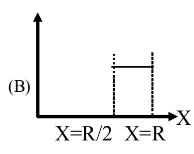
(D) may be less than 10 N/kg

6. A tunnel is dug along a chord of the earth at a perpendicular distance R/2 from the earth's centre. The wall of the funnel may be assumed to be frictionless. A particle is released from one end of the tunnel. The pressing force by the particle on the wall and the acceleration of the particle varies with x (distance of the particle from the centre) according to:

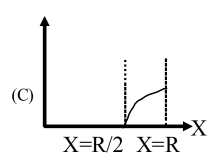
Pressing force



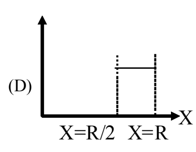
Pressing force



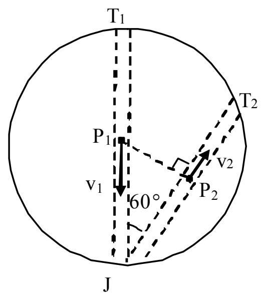
acceleration



acceleration



7. Two tunnels are dug across the earth as shown in figure. Two particles P_1 and P_2 are oscillating from one end to the other of tunnel T_1 and T_2 respectively. At some instant particles are at position shown in figure. Then -



(A) Phase difference between the particle P_1 and P_2 is 180°

(B) Phase difference between the particle P_1 and P_2 is 120°

(C) Ratio of maximum velocity of particle P_1 is to P_2 is 2 : 1

(D) Particle P_1 and P_2 may meet at the junction of the tunnels after some instant

Section C - Kepler's law, Orbital velocity, Escape velocity, Geo - Stationary Satellites

8. If a satellite orbits as close to the earth's surface as possible,

(A) its speed is maximum

(B) time period of its rotation is minimum

(C) the total energy of the 'earth plus satellite' system is minimum

(D) the total energy of the 'earth plus satellite' system is maximum

9. When a satellite in a circular orbit around the earth enters the atmospheric region, it encounters small air resistance to its motion. Then

(A) its kinetic energy increases

(B) its kinetic energy decreases

(C) its angular momentum about the earth decreases

(D) its period of revolution around the earth increases

10. A communications Earth satellite

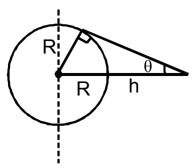
(A) goes round the earth from east to west

(B) can be in the equatorial plane only

(C) can be vertically above any place on the earth

(D) goes round the earth from west to east

11. A geostationary satellite is at a height h above the surface of earth. If earth radius is R



(A) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1}$ (R/R + h)

(B) The maximum colatitudes on earth upto which the satellite can be used for communication is $\sin^{-1}$ (R/R + h)

(C) The area on earth escaped from this satellite is given as $2\pi R^2(1 + \sin\theta)$

(D) The area on earth escaped from this satellite is given as $2\pi R^2(1 + \cos\theta)$

12. For a satellite to orbit around the earth, which of the following must be true ?

(A) It must be above the equator at some time

(B) It cannot pass over the poles at any time

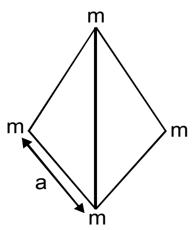
(C) Its height above the surface cannot exceed 36,000 km

(D) Its period of rotation must be $> 2\pi\sqrt{R/g}$ where R is radius of earth

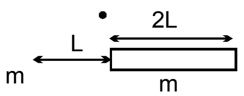
Exercise - 3 | Level-I   Subjective | JEE Advanced

Section A - Newton's law of Gravitation & Gravitational Field, Potential & Potential energy

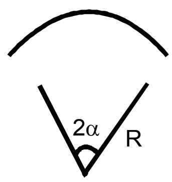
1. Four masses (each of m) are placed at the vertices of a regular pyramid (triangular base) of side 'a'. Find the work done by the system while taking them apart so that they form the pyramid of side '2a'.



2. A small mass and a thin uniform rod each of mass 'm' are positioned along the same straight line as shown. Find the force of gravitational attraction exerted by the rod on the small mass.



3. Find the gravitational field strength and potential at the centre of arc of linear mass density λ subtending an angle 2α at the centre.



4. Find the potential energy of a system of eight particles placed at the vertices of a cube of side L. Neglect the self energy of the particles.

5. Calculate the distance from the surface of the earth at which above and below the surface acceleration due to gravity is the same.

Section B - Variations in 'g'

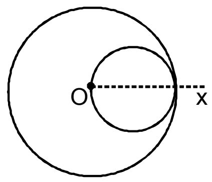
6. An object is projected vertically upward from the surface of the earth of mass M with a velocity such that the maximum height reached is eight times the radius R of the earth. Calculate:

(i) the initial speed of projection

(ii) the speed at half the maximum height.

7. A sphere of radius R has its centre at the origin. It has a uniform mass density ρ_0 except that there is a spherical hole of radius $r = R/2$ whose centre is at $x = R/2$ as in fig. (a) Find gravitational field at points on the axis for $x > R$

(b) Show that the gravitational field inside the hole is uniform, find its magnitude and direction.

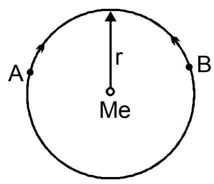


Section C - Kepler's law, Orbital velocity, Escape velocity, Geo - Stationary Satellites

8. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time ___________.

9. A satellite is moving in a circular orbit around the earth. The total energy of the satellite is $E = -2 \times 10^5$ J. The amount of energy to be imparted to the satellite to transfer it to a circular orbit where its potential energy is $U = -2\times10^5$J is equal to ___________.

10. Consider two satellites A and B of equal mass m, moving in the same circular orbit of radius r around the earth E but in opposite sense of rotation and therefore on a collision course (see figure).



(a) In terms of G, M_e, m and r find the total mechanical energy $E_A + E_B$ of the two satellite plus earth system before collision.

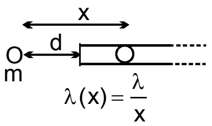
(b) If the collision is completely inelastic so that wreckage remains as one piece of tangle d material (mass = 2m), find the total mechanical energy immediately after collision.

11. A satellite of mass m is orbiting the earth in a circular orbit of radius r. It starts losing energy due to small air resistance at the rate of C J/s. Then the time taken for the satellite to reach the earth is ________.

12. A remote sensing satellite is revolving in an orbit of radius x the equator of earth. Find the area on earth surface in which satellite can not send message.

Section A - Newton's law of Gravitation & Gravitational Field, Potential & Potential energy

1. Find the gravitational force of interaction between the mass m and an infinite rod of varying mass density λ such that $\lambda(x) = \lambda/x$, where x is the distance from mass m. Given that mass m is placed at a distance d from the end of the rod on its axis as shown in figure.



2. Calculate the ratio of the mean densities of the earth and the sun from the following approximate data. θ = angular diameter of the sun seen from the earth $= \frac{1}{2}^{\circ}$. ℓ = length of 1° of latitude on the earth's surface = 100 km.
T = one year = 3×10^7s.
g = 10 ms^{-2}.

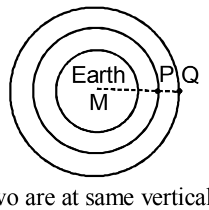
3. Find the gravitational force between a point like mass M and an infinitely long, thin rod, of mass density ρ, which is at a distance L from the mass M.

Section B - Variations in 'g'

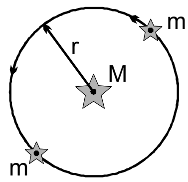
4. A ring of radius R is made from a thin wire of radius r. If ρ is the density of the material of wire then what will be the gravitational force exerted by the ring on the material particle of mass m placed on the axis of ring at a distance x from its centre. Show that the force will be maximum when $x = R/\sqrt{2}$ and the maximum value of force will be given as $F_{max} = \frac{4\pi^2 G r^2 \rho m}{(3)^{3/2} R}$

Section C - Kepler's law, Orbital velocity, Escape velocity, Geo - Stationary Satellites

5. A satellite P is revolving around the earth at a height h = radius of earth (R) above equator. Another satellite Q is at a height 2h revolving in opposite direction. At an instant the two are at same vertical line passing through centre of sphere. Find the least time of after which again they are in this is situation.



6. A certain triple-star system consists of two stars, each of mass m, revolving about a central star, mass M, in the same circular orbit. The two stars stay at opposite ends of a diameter of the circular orbit, see figure. Derive an expression for the period of revolution of the stars; the radius of the orbit is r.



7. A man can jump over b = 4m wide trench on earth. If mean density of an imaginary planet is twice that of the earth, calculate its maximum possible radius so that he may escape from it by jumping. Given radius of earth = 6400 km.

8. A launching pad with a spaceship is moving along a circular orbit of the moon, whose radius R is triple that of moon Rm. The ship leaves the launching pad with a relative velocity equal to the launching pad's initial orbital velocity $\vec{v}_0$ and the launching pad then falls to the moon. Determine the angle θ with the horizontal at which the launching pad crashes into the surface if its mass is twice that of the spaceship m.

9. A satellite of mass m is in an elliptical orbit around the earth of mass M(M >>m). The speed of the satellite at its nearest point to the earth (perigee) is $\sqrt{\frac{6GM}{5R}}$ where R = its closest distance to the earth. It is desired to transfer this satellite into a circular orbit around the earth of radius equal its largest distance from the earth. Find the increase in its speed to be imparted at the apogee (farthest point on the elliptical orbit).

10. Assume that a tunnel is dug across the earth (radius = R) passing through its centre. Find the time a particle takes to reach centre of earth if it is projected into the tunnel from surface of earth with speed needed for it to escape the gravitational field of earth.

1. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere, (you may take $G = 6.67 \times 10^{-11}\ Nm^2/kg^{-2}$) **[AIEEE 2005]**

(A) 13.34×10^{-10} J (B) 3.33×10^{-10} J

(C) 6.67×10^{-9}J (D) 6.67×10^{-10} J

2. The change in the value of g at a height **h** above the surface of the earth is the same as at a depth **d** below the surface of earth. When both **d** and **h** are much smaller than the radius of earth, then which one of the following is correct ? **[AIEEE 2005]**

(A) $d = \frac{h}{2}$ (B) $d = \frac{3h}{2}$

(C) d = 2h (D) d = h

3. Average density of the earth **[AIEEE 2005]**

(A) does not depend on g

(B) is a complex function of g

(C) is directly proportional to g

(D) is inversely proportional to g

4. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be **[AIEEE 2007]**

(A) 1 (B) zero

(C) $\frac{g_E}{g_M}$ (D) $\frac{g_M}{g_E}$

Directions Question number 6 is Assertion-Reason type question. This question contains two statements : Statement I (Assertion) and Statement II (Reason). The question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

(A) Statement I is true; Statement II is true; Statement II is not a correct explanation for Statement I.

(B) Statement I is true; Statement II is false.

(C) Statement I is false; Statement II is true.

(D) Statement I is true; Statement II is true; Statement II is a correct explanation for Statement I.

5. **Statement I :** For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is 4π GM.

and

Statement II : If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $\frac{1}{r^2}$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface.

[AIEEE 2008]

6. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1}, the escape velocity from the surface of the planet would be **[AIEEE 2008]**

(A) 1.1 kms^{-1} (B) 11 kms^{-1}

(C) 110 kms^{-1} (D) 0.11 kms^{-1}

7. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where g = the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is **[AIEEE 2009]**

(A) 2R (B) $\frac{R}{\sqrt{3}}$

(C) $\frac{R}{2}$ (D) $\sqrt{2}R$

8. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero, is **[AIEEE 2011]**

(A) $-\frac{4Gm}{r}$ (B) $-\frac{6Gm}{r}$

(C) $-\frac{9Gm}{r}$ (D) zero

9. Two particles of equal mass m go around a circle of radius R under action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **[AIEEE 2011]**

(A) $\sqrt{\frac{Gm}{R}}$ (B) $\sqrt{\frac{Gm}{4R}}$

(C) $\sqrt{\frac{Gm}{3R}}$ (D) $\sqrt{\frac{Gm}{2R}}$

10. The mass of a spaceship is 1000kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 Km respectively. The required energy for this work will be **[AIEEE 2012]**

(A) 6.4×10^{11} J (B) 6.4×10^{8} J

(C) 6.4×10^{9}J (D) 6.4×10^{10}J

11. What is the minimum energy required to launch a statellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[JEE MAIN 2013]**

(A) $\frac{GmM}{2R}$ (B) $\frac{GmM}{3R}$

(C) $\frac{5GmM}{6R}$ (D) $\frac{2GmM}{3R}$

12. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitiational attraction. The speed of each particle is : **[JEE MAIN 2014]**

(A) $\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$ (B) $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

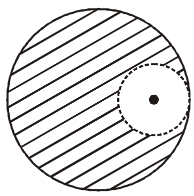
(C) $\sqrt{\frac{GM}{R}}$ (D) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

13. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is

(A) $m_1r_1 : m_2 r_1$ (B) $m_1 : m_2$

(C) $r_1 : r_2$ (D) 1 : 1

14. From a solid sphere of mass M and radius R, a spherical portion of radius $\left(\frac{R}{2}\right)$ is removed as shown in the figure. Taking gravitational potential V = 0 at r = ∞, the potential at the centre of the cavity thus formed is (G = gravitational constant) **[JEE Main 2015]**



(A) $\frac{-GM}{R}$ (B) $\frac{-GM}{2R}$

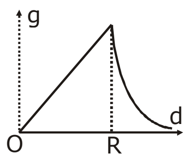
(C) $\frac{-2GM}{3R}$ (D) $\frac{-2GM}{R}$

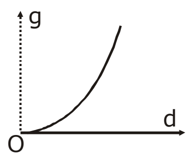
15. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R ; h<<R). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to :(Neglect the effect of atmosphere.) **[AIEEE-2016]**

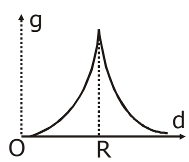
(A) $\sqrt{gR}$ (B) $\sqrt{gR/2}$

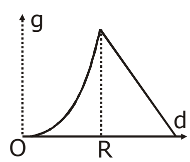
(C) $\sqrt{gR}(\sqrt{2}-1)$ (D) $\sqrt{2gR}$

16. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) : **[AIEEE-2017]**

(A)


(B)


(C)


(D)


1. A system of binary stars of masses m_A and m_B are moving in circular orbits of radii r_A and r_B respectively. If T_A and T_B are the time periods of masses m_A and m_B respectively, then **[JEE 2006]**

(A) $T_A > T_B$ (if $r_A > r_B$) (B) $T_A > T_B$ (if $m_A > m_B$)

(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$ (D) $T_A = T_B$

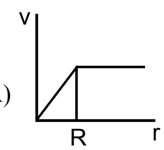
2. Column-I describes some situations in which a small object moves. Column-II describes some characteristics of these motions. Match the situations in Column-I with the characteristics in Column-II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[JEE-2007]**

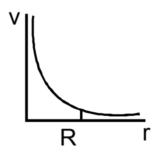
Column-I		**Column-II**
(A) The object moves on the x-axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p)	The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its isplacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q)	The object does not change its direction.
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion during the period it maintains this acceleration.	(r)	The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where, M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s)	The object can change its direction only once.

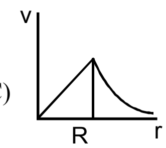
3. A spherically symmetric gravitational system of particles has a mass density **[JEE 2008]**

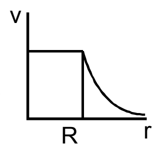
$$\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by

(A) 

(B) 

(C) 

(D) 

4. **STATEMENT-1**

An astronaut in an orbiting space station above the Earth experiences weightlessness. **[JEE 2008]**

and

STATEMENT-2

An object moving around the Earth under the influence of Earth's gravitaitonal force is in a state of 'free-fall'.

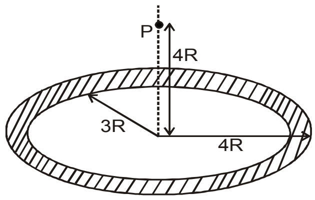
(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

5. A thin uniform annular disc (see figure) of mass M has outer radius 4 R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is **[JEE 2010]**



(A) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

(C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}(\sqrt{2}-1)$

6. A binary star consists of two stars A (mass 2.2 M_s) and B (mass 11 M_s), where M_s is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is. **[JEE 2010]**

7. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}$g. where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **[JEE 2010]**

8. A satellite is moving with a constant speed 'V' in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is **[JEE 2011]**

(A) $\frac{1}{2}mV^2$ (B) mV^2

(C) $\frac{3}{2}mV^2$ (D) $2mV^2$

9. Two spherical planets P and Q have the same uniform density ρ, masses M_p and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_p, V_Q and V_R, respectively. Then **[JEE 2012]**

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$

(C) $V_R / V_P = 3$ (D) $V_P / V_Q = \frac{1}{2}$

10. Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) **[JEE 2013]**

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$.

(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$.

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$.

(D) The energy of the mass m remains constant.

11. A planet of radius $R = \frac{1}{10} \times$(radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius oif Earth=6×10^6m and the acceleration dur to gravity on Earth is 10 ms^{-2})

[JEE Advance 2014]

(A) 96 N (B) 108 N

(C) 120 N (D) 150 N

12. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length l and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance r = 3l from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is - **[JEE-2015]**

13. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $1/4^{th}$ of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[JEE-2015]**

14. A planet of mass M, has two natural satellites with masses m1 and m2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

[JEE Advance 2018]

List–I		**List–II**	
P.	$\frac{v_1}{v_2}$	1.	$\frac{1}{8}$
Q.	$\frac{L_1}{L_2}$	2.	1
R.	$\frac{K_1}{K_2}$	3.	2
S.	$\frac{T_1}{T_2}$	4.	8

(A) P → 4 ; Q → 2 ; R → 1 ; S → 3
(B) P → 3 ; Q → 2 ; R → 4 ; S → 1
(C) P → 2 ; Q → 3 ; R → 1 ; S → 4
(D) P → 2 ; Q → 3 ; R → 4 ; S → 1

ANSWER KEYS

Exercise - 1 Objective Problems | JEE Main

1.	B	2.	B	3.	D	4.	B	5.	C
6.	B	7.	D	8.	B	9.	B	10.	A
11.	C	12.	D	13.	B	14.	B	15.	B
16.	D	17.	A	18.	C	19.	A	20.	C
21.	B	22.	B	23.	C	24.	C	25.	C

Exercise - 2 (Leve-I) Objective Problems | JEE Main

1.	C	2.	A	3.	B	4.	A	5.	A
6.	B	7.	D	8.	C	9.	C	10.	C
11.	A	12.	A	13.	A				

Exercise - 2 (Level-II) Multiple Correct | JEE Advanced

1.	A,D	2.	B,C,D	3.	A,B	4.	B,C	5.	B,C
6.	B,C	7.	A,C	8.	A,B,C	9.	A,C	10.	B,D
11.	A,C	12.	A,D						

Exercise - 3 | Level-I Subjective | JEE Advanced

1. $-\dfrac{3Gm^2}{a}$

2. $\dfrac{Gm^2}{3L^2}$

3. $\dfrac{2G\lambda}{R}$ **(sin α), (–Gλ 2α)**

4. $\dfrac{-4GM^2}{L}\left[3+\dfrac{3}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}\right]$

5. $h=\dfrac{\sqrt{5}-1}{2}R$

6. **(i)** $\dfrac{4}{3}\sqrt{\dfrac{Gm}{R}}$, **(ii)** $\dfrac{2}{3}\sqrt{\dfrac{2Gm}{5R}}$

7. $\vec{g}=+\dfrac{\pi G\rho_0 R^3}{6}\left[\dfrac{1}{\left(x-\dfrac{R}{2}\right)^2}-\dfrac{8}{x^2}\right]\hat{i}$, $\vec{g}=-\dfrac{2\pi G\rho_0 R}{3}\hat{i}$

8. **1.6 hours if is rotating from west to east, 24/17 hours if it is rotating from west to east.**

9. **1×10^5 J**

10. **(a) $-GmM_e/r$, (b) $-2GmM_e/r$**

11. $t=\dfrac{GMm}{2C}\left(\dfrac{1}{R_e}-\dfrac{1}{r}\right)$

12. $\left(1-\dfrac{\sqrt{x^2-R^2}}{x}\right)4\pi R^2$

Exercise - 3 | Level-II — Subjective | JEE Advanced

1.	$\dfrac{Gm\lambda}{2d^2}$	**2.**	$\dfrac{\rho_e}{\rho_s} = 3.31$	**3.**	$\dfrac{2MG\rho}{L}$	**4.**	$\dfrac{da}{dx} = 0$ for $a_{max} \Rightarrow x = \dfrac{R}{\sqrt{2}}$
5.	$\dfrac{2\pi R^{3/2}(6\sqrt{6})}{\sqrt{GM}(2\sqrt{2}+3\sqrt{3})}$	**6.**	$\dfrac{4\pi r^{3/2}}{\sqrt{G(4M+m)}}$	**7.**	$\sqrt{6.4}\,\text{km}$		
8.	$\cos\theta = \dfrac{3}{\sqrt{10}}$	**9.**	$\sqrt{\dfrac{GM}{R}}\left[\sqrt{\dfrac{2}{3}} - \sqrt{\dfrac{8}{15}}\right]$	**10.**	$T = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)\sqrt{\dfrac{R_e}{g}}$		

Exercise - 4 | Level-I — Previous Year | JEE Main

1.	**D**	**2.**	**C**	**3.**	**C**	**4.**	**A**	**5.**	**A**
6.	**C**	**7.**	**A**	**8.**	**C**	**9.**	**B**	**10.**	**D**
11.	**C**	**12.**	**B**	**13.**	**C**	**14.**	**A**	**15.**	**C**
16.	**A**								

Exercise - 4 | Level-II — Previous Year | JEE Advanced

1.	**D**	**2.**	**A-P; B-Q,R; C-P; D-Q,R**			**3.**	**C**	**4.**	**A**
5.	**A**	**6.**	**6**	**7.**	**3**	**8.**	**B**	**9.**	**B, D**
10.	**B**	**11.**	**B**	**12.**	**7**	**13.**	**2**	**14.**	**B**