# SEMICONDUCTOR

(CHAPTER-14)

| CONDUCTOR | SEMI-CONDUCTOR | INSULATOR |
|---|---|---|
| 1 It conducts easily | It conducts moderately | It doesn't conduct easily |
| 2 It has positive temp. coefficient of resistivity | It has negative temp. coefficient of resistivity. | It has negative temp. coefficient of resistivity |

## CLASSIFICATION OF SEMICONDUCTORS ON THE BASIS OF THEIR CHEMICAL COMPOSITION:-

(A) ELEMENTAL SEMICONDUCTORS:- Si and Ge

(B) COMPOUND SEMICONDUCTORS:-

(i) INORGANIC - CdS, GaAs, InP etc.

(ii) ORGANIC - Polypyrrole, polyaniline, polythiophene etc.

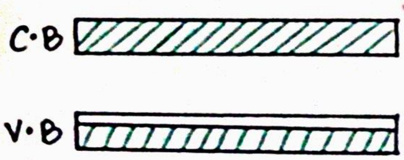
**VALENCE BAND:-** It is the energy band, which include the energy levels of valence electrons.

**CONDUCTION BAND:-** It contains free $e^-$ of solid. It is the energy band above valence band.

**ENERGY GAP ($E_g$):-** The difference in energy gap between the upper level of valence band and lower level of conduction band.

Classification of conductors, semi-conductors and Insulators on the basis of energy gap:-

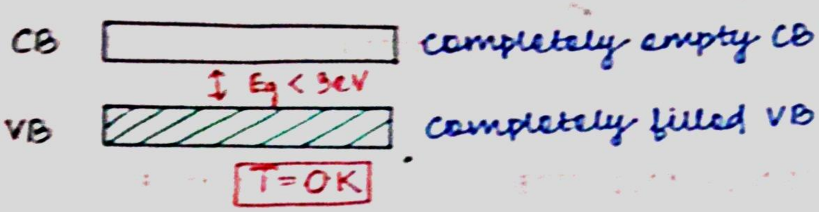
1 CONDUCTORS



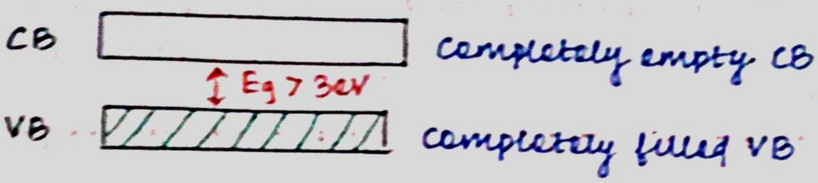

Completely filled conduction band.

partially filled valence band.

## ② SEMICONDUCTORS




## ③ INSULATORS:-




On the basis of purity, semiconductors are of two types:-

① Intrinsic semiconductors

② Extrinsic semiconductors

It is basically two types:-

(a) n-type semiconductors (b) p-type semiconductors.

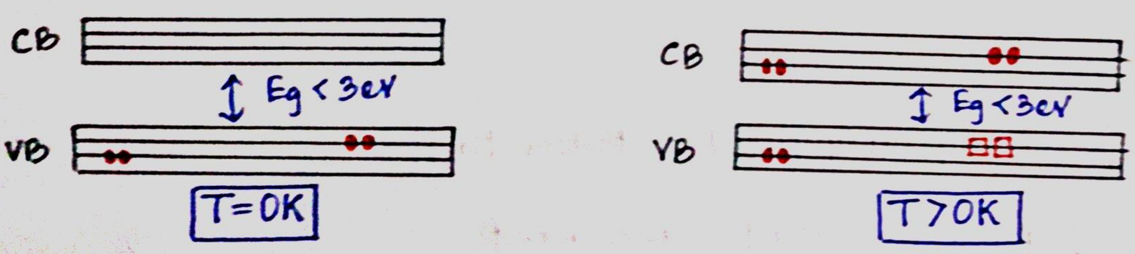
## ① INTRINSIC/ PURE SEMICONDUCTORS:-

* In this type, $n_e = n_h$ (no. of $e^-$ = no. of holes)
* So, intrinsic carrier concentration:-

$$n_i = n_e = n_h$$

* At equilibrium in any semiconductor:-

$$n_i^2 = n_e \cdot n_h$$

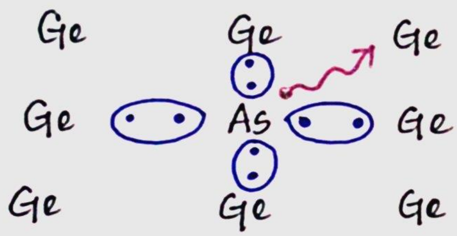


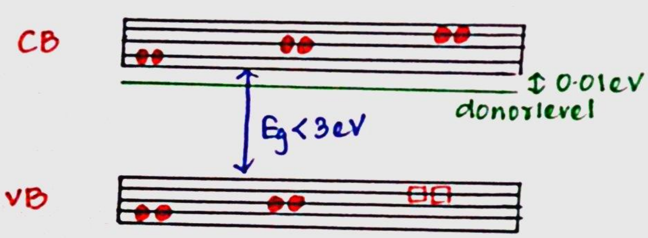
**DOPING:-** The process of deliberate addition of a desirable impurity to a pure semiconducter so as to increase its condutivity is called doping. The impurity atoms are called dopants.

**EXTRINSIC OR DOPED SEMICONDUCTORS:-** The semicondutors doped with impurity atoms are called extrinsic semiconductors.

(a) n-type:- This semiconductor is obtained by doping the tetravalent semiconducter Si or Ge with pentavalent impurities such as As, P or Sb of group V of the periodic table. When pentavalent impurity atom (As) is added to pure semiconductor, then $4e^-$s of As participate in band formation. $1e^-$ remains extra on it. Addition of large no. of As atom large no. of such $e^-$ are obtained and they lie in a level called as Donor level which is very close to conduction band.

So, majority charge carriers are free electrons and minority charge carriers are holes.



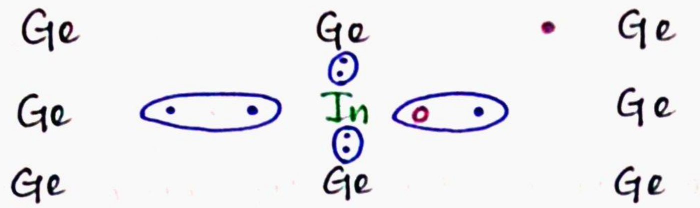

$(n_e >>> n_h)$

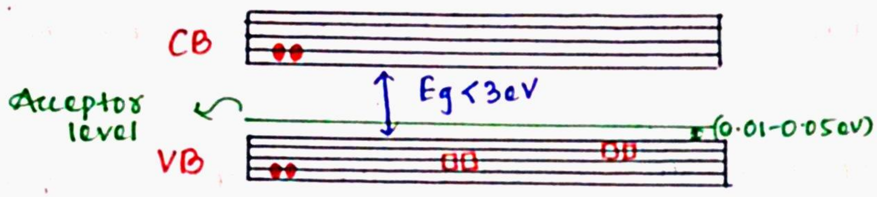
(b) p-type:- This semiconductor is obtained by doping the tetravalent semiconducter Si or Ge with trivalent impurities such as In, B, Al or Ga. When trivalent impurity atom (In) is added to pure semiconducter (Ge), $3e^-$s of In participate in band formation. The lack of $1e^-$ on it is called hole.

→ Addition of large no. of In atom produces large no. of holes in V.B.

→ One level is created just above the V.B called as acceptor level

→ So, majority charge carriers are holes and minority charge carriers are electrons.



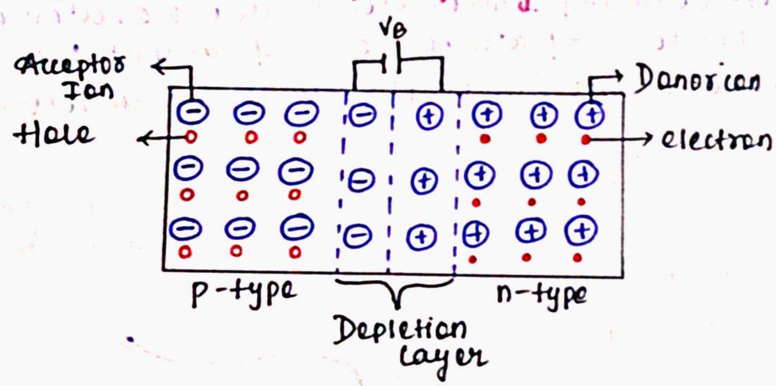

## P-N JUNCTION DIODE:-

When p-type semiconductor comes in n-type semiconduter, two processes happen:-

① DIFFUSION:- Due to concentration difference, holes from p-side and e⁻ from n-side move towards each other. The current constituted is called as diffusion current. A potential difference is built at the junction.

② DRIFTING:- Due to a potential difference, minor charge carriers move and the current constituted is called drift current. Diffusing current and drift current are in opposite direction. Equilibrium is reached when diffusion current is equal to drift current. The layer formed at the junction is called depletion layer and the Potential difference is called as barrier potential

* The device formed is called as p-n junction diode.



## WORKING OF A p-n JUNCTION :-

### ① FORWARD BIASING :-

P side is connected to +ve and n side is connected to −ve. Applied potential difference is in opposite direction to barrier potential. So effective barrier potential ($V_f$). $V_f = V_b - V$. By increasing applied potential, $V_f$ gradually decreases, majority charge carrier moves and diode conducts. So diode behaves like a low resistive device.

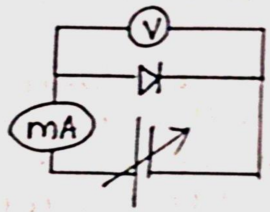
### ② REVERSE BIASING :-

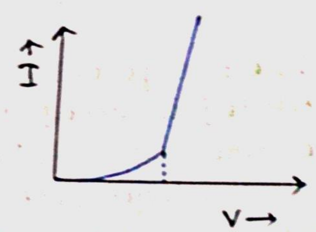
P side is connected to −ve and n side is connected to +ve. Applied potential difference is in same direction to barrier potential. So effective barrier potential ($V_f$). $V_f = V_b + V$. By increasing applied potential, $V_f$ gradually increases. So diode behaves like a high resistive device.

### CHARACTERSTICS CURVE :- It is of 2 types :-

**(1) Forward characterstic :-**

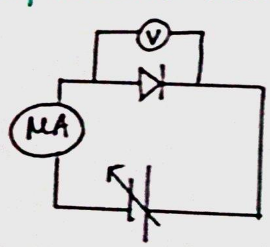
The graphical representation of variation of forward current and forward voltage is called forward characterstics

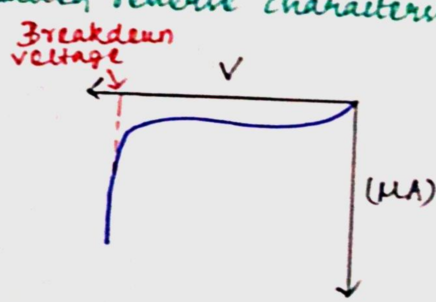



When forward voltages ↑ses, forward I ↑ses slowly due to existence of barrier potential. After a particular forward voltage, forward current ↑ses rapidly, that voltage is known as knee voltage / threshold voltage / cut-in voltage

**(2) Reverse characterstic :-** The graphical representation of variation of reverse current and reverse voltage is called reverse characterstic



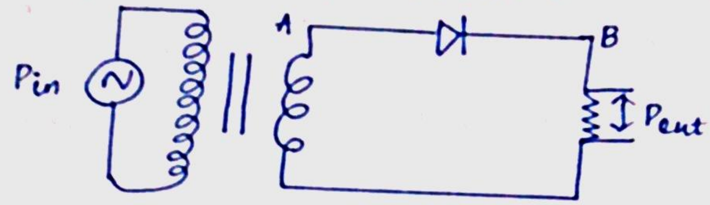
When reverse voltage ↑ses, there is almost no change in reverse current because it is due to minority charge carrier. At a particular reverse voltage, reverse current ↑ses suddenly, that voltage is called as breakdown voltage.

**RECTIFIER:-** It is an electronic device which converts AC to DC.

**PRINCIPLE:-** When diode is forward bias it conducts, when diode is reverse bias, it doesnot conduct.

It is of two types:-

**① HALF-WAVE RECTIFIER:-** It consist of single diode connected to step down transformer and a load resistance. Input is given to 1° transformer and DC is taken from the load resistance.




**WORKING:-**

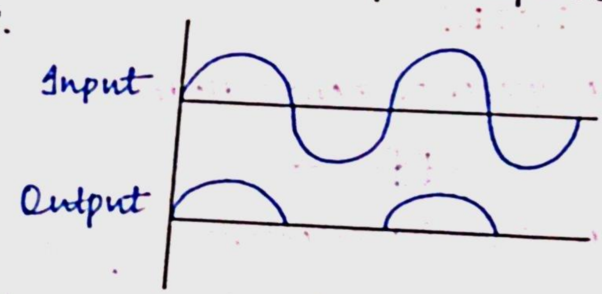
For +ve half cycle of input AC, A is +ve, D is -ve, diode is forward biased and it conducts. This half cycle appears in the output.

For -ve half cycle of input AC, A is -ve, D is +ve, diode is reverse biased and it doesn't conducts. So, this half cycle doesn't appear in the output.

In this way, half of AC converts to DC. So this is called half wave rectifier.

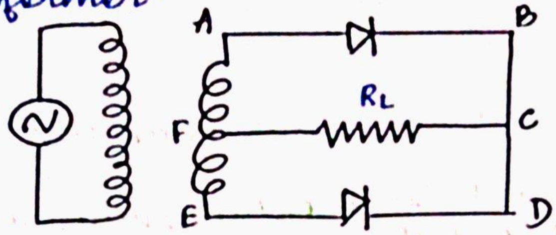
Efficiency:- It is the ratio of output DC power to input AC power.

$$\eta = \frac{P_{out}}{P_{in}} = 40.6\%$$




**② FULL-WAVE RECTIFIER:-** It consist of 2 diodes, $D_1$ and $D_2$ connected to a centre taped step down transformer and load resistance.

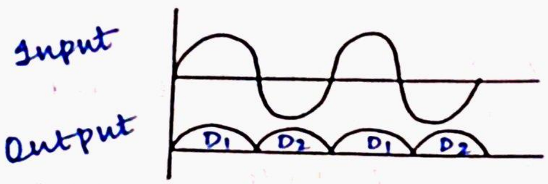
Load resistance is connected to the middle of the 2° coil of the transformer.




**WORKING**:- For the +ve half cycle of input AC, A is +ve, E is −ve. $D_1$ is forward biased and $D_2$ is reverse biased. $D_1$ conducts and current passes in the cycle ABCFA.

For −ve half cycle of input AC, A is −ve, E is +ve. $D_2$ is forward biased and $D_1$ is reverse biased. $D_2$ conducts and current passes in the cycle EDCFE.

In this way, full cycle of AC converts to DC. So it is called full wave rectifier.



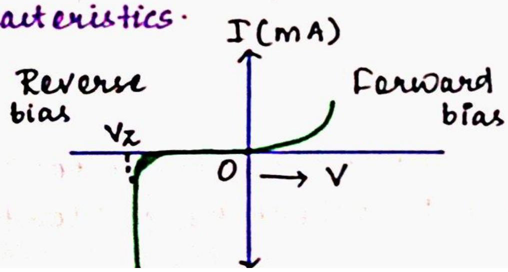

Efficiency:- It is the ratio of output DC power to input AC power.

$$\eta = \frac{P_{out}}{P_{in}} = 81.2\%$$

## ZENER DIODE:-

Draw the symbol of zener diode.

Draw the V−I characteristics.



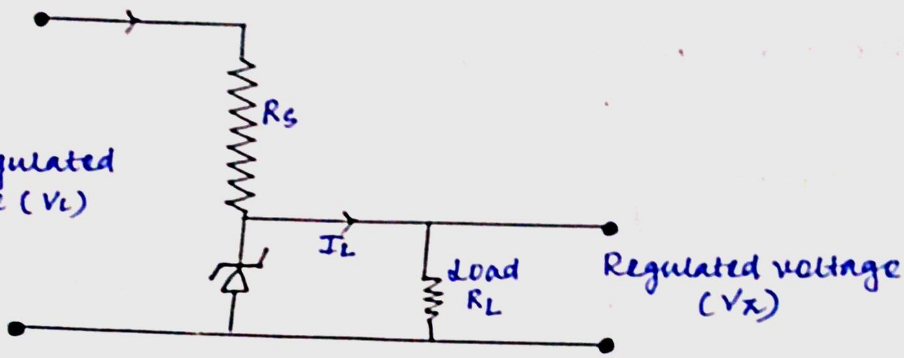
How zener diode is fabricated?

Zener diode is fabricated by heavily doping p and n sides of the junction.

What is the advantage of heavily doping?

Due to heavily doping, depletion region formed is very thin and the electric field to the junction is extremely high even for a small reverse bias voltage.

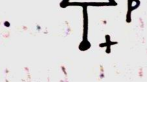
How zener diode is used as voltage regulator?




The unregulated dc voltage is connected to a zener diode through a series resistance $R_S$ such that zener diode is reverse biased. If the input voltage increases, the current through $R_S$ and zener diode also increases. This increases the voltage drop across $R_S$ without any change in the voltage across the zener diode. This is because in the breakdown region, zener voltage remains constant even though the current through the zener diode changes. Similarly, if the input voltage decreases, the current through $R_S$ and zener diode also decreases. The voltage drop across $R_S$ decreases without any change in the voltage across the zener diode. Thus, any increase or decrease in the input voltage results in increase/decrease of the voltage drop across $R_S$ without any change in voltage across the zener diode. Thus, the zener diode act as a voltage regulator.

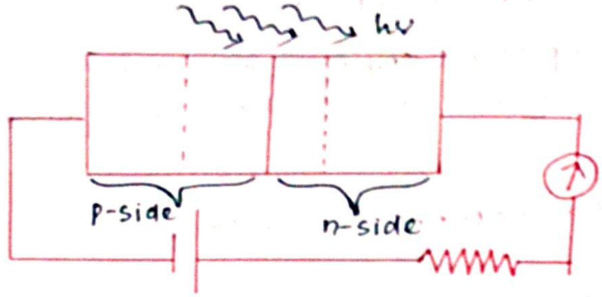
## PHOTODIODE :-

(Q):- Draw its symbol.



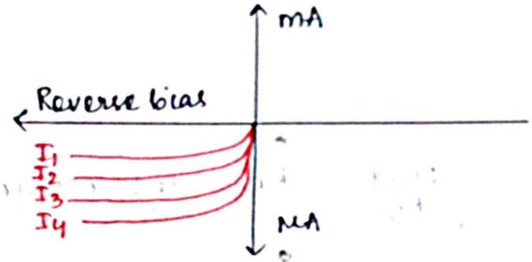
(Q):- How is photodiode fabricated?

It is fabricated with a transparent window to allow light to fall on the diode.




(Please refer NCERT Pg no 487)

(Q):- Draw the characteristics curve.




The magnitude of the photocurrent depends on the intensity of the incident light.

WORKING:-

When the photodiode is illuminated with light (photons), with energy greater than the energy gap of the semiconductor, the electron-hole pairs are generated due to absorption of photons. These charge carriers contribute to the reverse current.

(Q) Why photodiode is always reverse biased?

In case of an n-type semiconductor, the majority carrier density (n) is considerably larger than the minority hole density.

$n \rightarrow$ majority carrier density
$\Delta n \rightarrow$ excess $e^-$ generated
$p \rightarrow$ minority hole density
$\Delta p \rightarrow$ excess holes generated

$n' = n + \Delta n$
$p' = p + \Delta p$ , where $\Delta n = \Delta p$, $n >> p$

$$\Rightarrow \frac{\Delta n}{n} < \frac{\Delta p}{p}$$

The fractional charge due to the photo effects on the minority carrier dominated reverse bias current is more easily measureable than the fractional change in the forward bias current. Hence, photodiodes are preferably used in the reverse bias condition for measuring light intensity.

## LIGHT EMITTING DIODE:-

(Q):- Draw the circuit symbol of LED.


(Q):- How LED is fabricated?

The diode is fabricated with a transparent cover so that the emitted light can come out.

(Q):- Write the advantages of LED.

1. Low operational voltage and less power.
2. Fast action and no warm-up time required.
3. The bandwidth of emitted light is 100 Å to 500 Å
4. Long life and ruggedness.
5. Fast on-off switching capability.

(Q):- For visible LED band gap should be minimum 1.8 eV. Why?

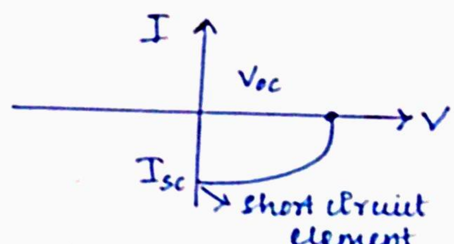
Because the minimum energy of photon of the visible range is 1.8 eV

(Q):- Why elemental semiconductors is not used in LED?

Because number of charge carriers is very less.

## SOLAR CELL:-

(Q):- Draw the circuit symbol of solar cell.


(Q):- Draw the characterstic curve of solarcell.

(Q):- Write the criteria to choose a material for solar cell.

1. band gap (~1 to 1.8 eV) 2. cost 3. electrical conductivity
4. availability of raw material 5. high optical absorption.

(Q):- Write the 3 processes of solar cell.

1. Generation of e-h pairs due to light ($h\nu > E_g$) close to the junction.
2. Separation of electrons and holes due to electric field of the depletion region.
3. The electrons reaching the n-side are collected by the front contact & holes reaching the p-side are collected by back contact. Thus, p side becomes +ve & n side becomes -ve giving rise to photovoltage.

# NUCLEI

(CHAPTER-13)

* Nucleus is made up of neutron and proton.

$\boxed{\text{NEUTRON + PROTON = NUCLEONS}}$

**PROTON :-** CHARGE- $+e = 1.6 \times 10^{-19} C$

MASS- $1.67 \times 10^{-27} kg = 1$ amu

SYMBOL- ${}^{1}_{1}H$

**NEUTRON :-** CHARGE :- 0

MASS :- $1.674 \times 10^{-27} kg \approx 1$ amu

SYMBOL :- ${}^{1}_{0}n$

**ATOMIC NUMBER :- (Z)**

It is the number of protons present inside nucleus.

Z = Number of protons
= Number of electrons (in neutral atom)

**MASS NUMBER :- (A)**

It is the total number of protons and neutrons inside the atomic nucleus of the element.

A = Number of protons + Number of neutrons

Eg :- ${}^{16}_{8}O$ = 8p, 8n

**ISOTOPES :-** Atoms of the same element whose nuclei have same number of protons but different number of neutrons.

Atomic number → same
Mass number → Different

Eg:- Protium , Deuterium , Tritium
$^{1}_{1}H$ $^{2}_{1}H$ $^{3}_{1}H$

## ISOBAR:-

Atoms of different element whose nuclei have same number of nucleons but different number of protons and neutrons.

$$\boxed{\begin{array}{l}\text{Atomic number — Different}\\ \text{Mass number — Same}\end{array}}$$

Eg:- $^{14}_{6}C$ $^{14}_{7}N$
(6p, 8n) (7p, 7n)

## ISOTONES:-

Atoms of different element whose nuclei have same number of neutrons but different number of protons.

$$\boxed{\begin{array}{l}\text{Atomic number — Different}\\ \text{Mass number — Different}\end{array}}$$

Eg:- $^{3}_{1}H$ $^{4}_{2}He$
(1p, 2n) (2p, 2n)

## SIZE OF NUCLEUS:-

Volume of nucleus $\propto$ Mass number

$\Rightarrow \frac{4}{3}\pi R^3 \propto A$

$\Rightarrow R^3 \propto A$

$\Rightarrow R \propto (A)^{1/3}$

$\Rightarrow \boxed{R = R_0 (A)^{1/3}}$ It is also known as nuclear unit radius

where, $R$ = radius of nucleus.
$A$ = mass number
$R_0 = 1.2 \times 10^{-15} m = 1.2\ fm$

## NUCLEAR DENSITY :- ($\rho$)

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{A \times amu}{V} = \frac{A \times 1 amu}{\frac{4}{3}\pi R^3}$$

$$= \frac{A \times 1 amu}{\frac{4}{3}\pi [R_0 (A)^{1/3}]^3} = \frac{A \times 1 amu}{\frac{4}{3}\pi R_0^3 A} = \frac{3 \times 1 amu}{4\pi R_0^3}$$

$$\boxed{\rho = \frac{3 \times 1 amu}{4\pi R_0^3}}$$

$$\rho = \frac{3 \times 1.6 \times 10^{-27}}{4 \times 3.14 \times (1.25 \times 10^{-15})^3} \approx 2 \times 10^{17} kg/m^3$$

## MASS-ENERGY RELATION :-

According to Einstein relation of mass and energy

$$\boxed{E = mc^2 \text{ —— (I)}}$$

$$E = E_0 + K \cdot E$$

$$\Rightarrow E = m_0 c^2 + \frac{1}{2} m v^2 \text{ —— (II)}$$

where, $E$ = total energy
$E_0$ = rest mass energy
$m_0$ = normal mass (rest)
$m$ = mass (light speed)

From equation (I) and (II),

$$mc^2 = m_0 c^2 + K \cdot E$$

$$\Rightarrow K \cdot E = mc^2 - m_0 c^2$$

$$\Rightarrow K \cdot E = c^2 (m - m_0)$$

$$\Rightarrow \boxed{K \cdot E = \Delta m c^2}$$

## MASS DEFECT :- ($\Delta m$)

The mass of nucleus is less than the sum of mass of all nucleons making it. The mass that dissapered is termed as 'Mass Defect'.

$$\boxed{\Delta m = [Z m_p + (A - Z) m_n] - M}$$ , $M$ = mass of nucleus.

- Mass defect is taken in amu.

## NUCLEAR BINDING ENERGY:-



Binding energy :- ($\Delta E_b$)

It is the amount of energy required to separate all nucleons from the nucleus.

$$\boxed{\Delta E_b = [Zm_p + (A-Z)m_n - M]C^2}$$

$$\boxed{\text{Energy in 1 amu} = 931 \text{ MeV}}$$

$$\Delta E_b = \Delta mc^2 \quad \text{—— (I)}$$

If mass defect is taken in amu,

$$\Delta m = [Zm_p + (A-Z)m_n - M] \text{ amu} \quad \text{—(II)}$$

Substituting eqn (II) in eqn (I)

$$\Delta E_b = [Zm_p + (A-Z)m_n - M] 931 \text{ MeV}$$

$$\Rightarrow \boxed{\Delta E_b = \Delta m \times 931 \text{ MeV}}$$

- More binding energy means more stable nucleus.

Nuclear binding energy per nucleons:-

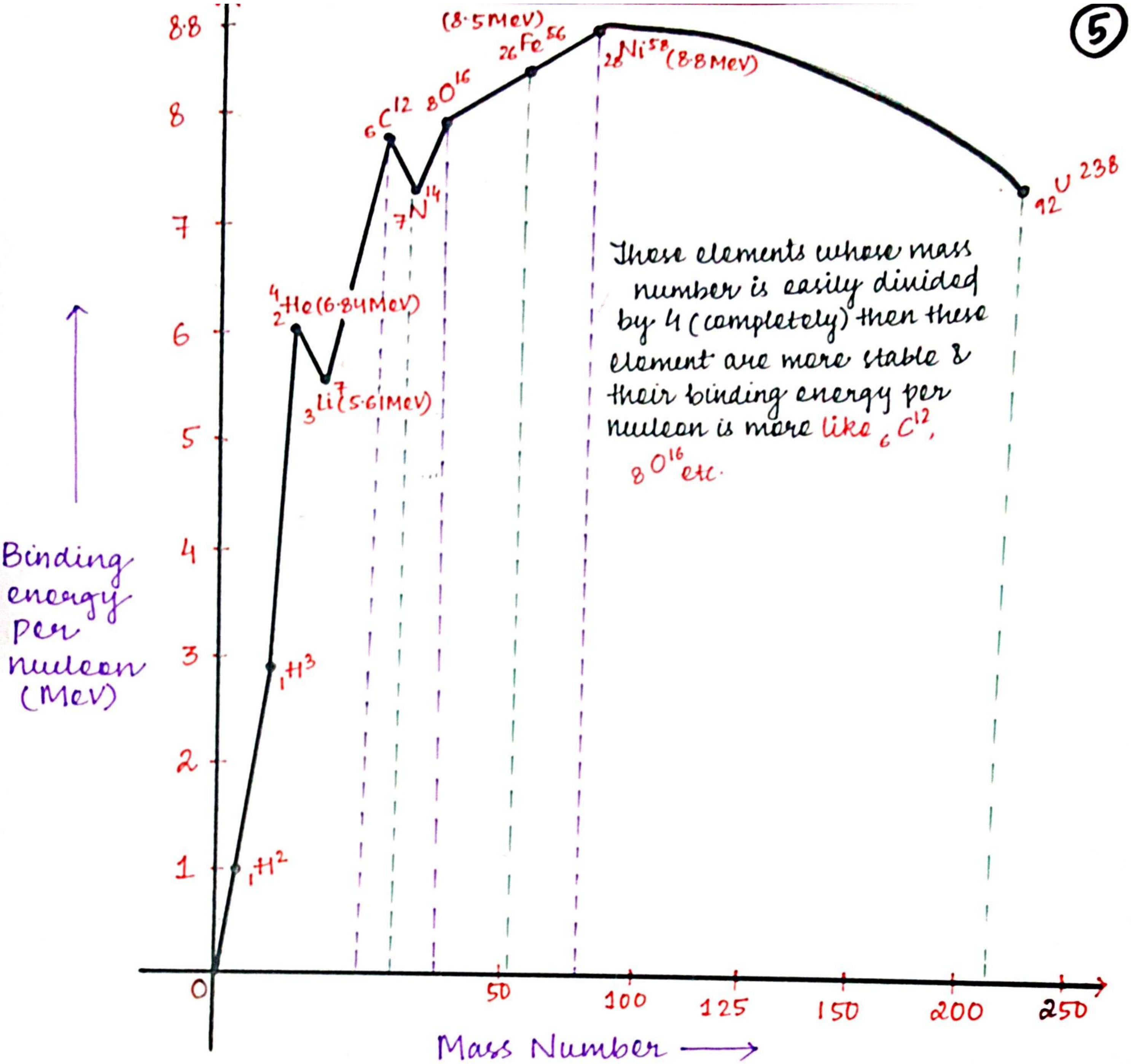
The ratio of total binding energy of nucleus to total number of nucleons is defined as nuclear binding energy per nucleons.

$$\bar{E}_b = \frac{\text{Total binding energy}}{\text{Number of nucleons}(A)}$$

$$\Rightarrow \bar{E}_b = \frac{E_b}{A}$$

- The average energy required to release nucleons from a nucleus is called binding energy per nucleons.

## BINDING ENERGY CURVE:-




The following are the features of the plot:-

① Average binding energy per nucleon for mass number less than 3 is very small. (hydrogen).

② Some nuclei with mass number (3 to 20) have large binding energy per nucleon than their neighbouring nuclei. For eg:- $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$.

③ For (30-56) binding energy per nucleons increases gradually till it attains a max. value 8.8 MeV. Thus, Iron, nickel are stable element.

(4) For nuclei whose mass number is greater than 56, their binding energy per nucleon decreases. For uranium, one of the heaviest natural element, the binding energy per nucleon drops to 7.5 MeV

## CONCLUSION:-

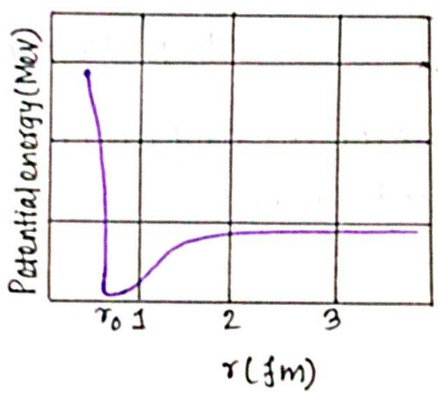
### (1) NUCLEAR FISSION :-

When a heavy nucleus splits up into lighter nuclei (eg. uranium) then binding energy per nucleons of lighter nuclei is more than that of the original heavy nucleus. This process is called nuclear fission.

### (2) NUCLEAR FUSION :-

When two very light nuclei (eg hydrogen) combine to form a heavy nucleus then binding energy per nucleons of heavy nucleus becomes more than the lighter nuclei. In other words, the nucleons of the fused heavy nucleus are tightly bound. i.e energy is released. This process is called as nuclear fusion

## NUCLEAR FORCE:-




Some of the important characteristics are:-

1. Nuclear forces are independent of charge.
2. Nuclear forces are very short range forces.
3. They are (dependent) on spin or angular momentum of nuclei
4. The nuclear force is much stronger than the coulomb force acting between charges or gravitational forces between masses.

## RADIOACTIVITY:-

The unstable nuclei gains stability by emitting $\alpha$-particles or $\beta$-particles and $\gamma$- EM waves. This phenomenon is called radioactivity

## LAW OF RADIOACTIVE DECAY:-

- It is also known as Rutherford and soddy law)
- Radioactivity is a random process.

### STATISTICAL LAW:-

When there is a large number of nuclei, rate of decay or disintegratio is directly proportional to the number of nuclei in the sample.

$$\text{Rate of decay} = \frac{\text{no. of nuclei decays}}{\text{time}}$$

$$= \frac{-dN}{dt}$$

$$\Rightarrow \frac{-dN}{dt} \propto N$$

$$\Rightarrow \boxed{\frac{-dN}{dt} = \lambda N}$$

$\lambda$ = decay constant or disintegration constant.

$\lambda \rightarrow$ depends on choice of element and isotope.

$N \rightarrow$ number of undecayed nuclei

$$\int_{N_0}^{N} \frac{dN}{N} = \int_{0}^{t} -\lambda dt$$

$$\Rightarrow [\log N]_{N_0}^{N} = -\lambda (t)_0^t$$

$$\Rightarrow \log N - \log N_0 = -\lambda(t-0)$$

$$\Rightarrow \boxed{\log \frac{N}{N_0} = -\lambda t}$$ Logarithmic form

$$\frac{N}{N_0} = e^{-\lambda t}$$

$N_0 \rightarrow$ initial number of nuclei

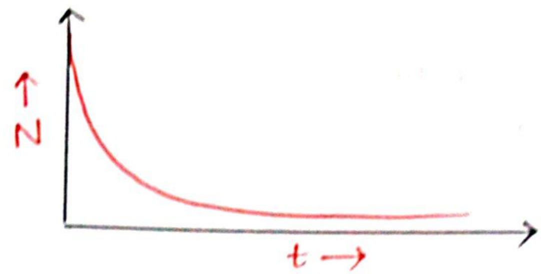
$$\Rightarrow \boxed{N = N_0 e^{-\lambda t}}$$ Exponential form.

### DECAY CONSTANT ($\lambda$):-

when $t = 1/\lambda$, $N = N_0 e^{-1} = N_0(1/e) = 0.368 N_0$

Decay constant is the reciprocal of time in which no. of nuclei left undecayed at 36.8% of initial number of nuclei.

## GRAPH OF N VS t :-




## HALF LIFE OF DECAY ($t_{1/2}$) :-

when $t = t_{1/2}$ , $N = N_0/2$

$$N = N_0 e^{-\lambda t}$$

$$\log\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\Rightarrow \log\left(\frac{N_0}{2N_0}\right) = -\lambda t_{1/2}$$

$$\Rightarrow \log(2)^{-1} = -\lambda t_{1/2}$$

$$\Rightarrow -\log 2 = -\lambda t_{1/2}$$

$$\Rightarrow t_{1/2} = \frac{\log 2}{\lambda}$$

$$\Rightarrow \boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

## NUMBER OF NUCLEI LEFT AFTER 'n' HALF LIVES :-

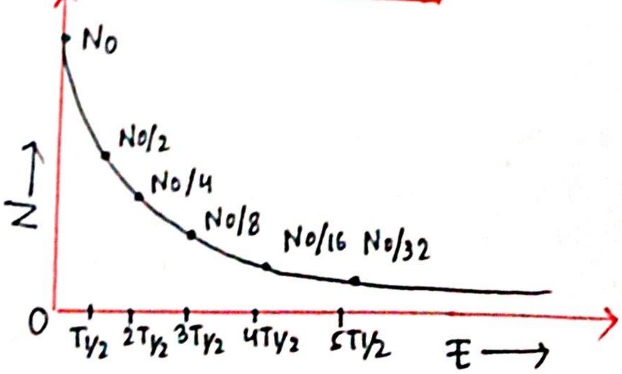
After 1 half life , $N = \frac{N_0}{2} = \frac{N_0}{2}$

After 2 half life , $N = \frac{N_0}{4} = \frac{N_0}{2^2}$

After 3 half life , $N = \frac{N_0}{8} = \frac{N_0}{2^3}$

⋮

After n half life, $\boxed{N = \frac{N_0}{2^n}}$



## ACTIVITY OF A RADIOACTIVE SAMPLE:-

$$\text{Activity} = \frac{\text{Rate of decay}}{\text{Disintegration}}$$

$$R = -\frac{dN}{dt}$$

$$\Rightarrow R = \lambda N$$

$$\Rightarrow R = \lambda N_0 e^{-\lambda t}$$

$$\boxed{R = R_0 e^{-\lambda t}}$$

### S.I units of activity:-

1. Becquerel (Bq)
2. Curie (Ci)
3. Rutherford (rd)

## α-DECAY:-

1. α-particle is helium nucleus
   (2p, 2n)

2. Mass = 4 amu
   Charge = +2e

3. $${}^{A}_{Z}X \longrightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y$$
   Parent nucleus — Daughter nucleus

eg: $${}^{238}_{92}U \rightarrow {}^{4}_{2}He + {}^{234}_{90}Th + Q$$

4. $$Q = [m_X - m_Y - m_{He}]\,c^2$$

## β-decay:-

(a) β⁻ decay ( ${}^{0}_{-1}e$ or ${}^{0}_{-1}\beta$ )

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$$
(Antineutrino)

eg:- $^{14}_{6}C \rightarrow ^{0}_{-1}e + ^{14}_{7}N + \bar{\nu}$

* Actually $\beta^{\ominus}$ decay is conversion of a neutron to a proton inside nucleus.

(b) $\beta^{\oplus}$ decay:-

(1) $^{A}_{Z}X \longrightarrow ^{A}_{Z-1}Y + ^{0}_{+1}e + \nu$

eg:- $^{22}_{11}Na \longrightarrow ^{22}_{10}Ne + ^{0}_{+1}e + \nu$

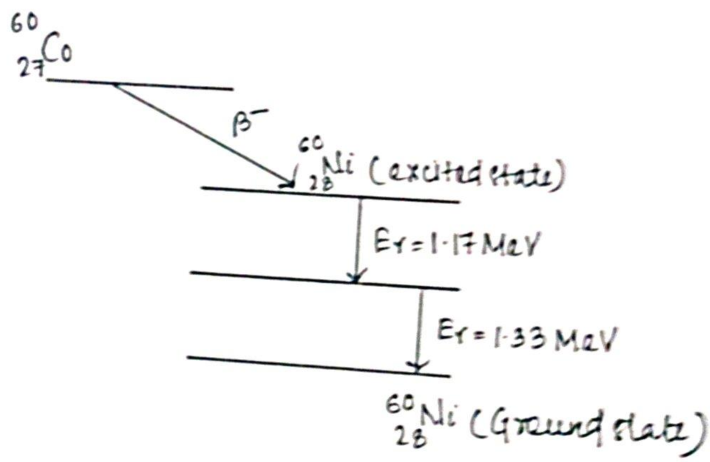
## GAMMA DECAY:-

The process of emission of γ-ray photon during the radioactive disintegration of nucleus is called gamma decay.

$$^{A}_{Z}X \longrightarrow ^{A}_{Z}X + \gamma$$

(Excited state) (Ground state)

Eg:-




## NUCLEAR FISSION:-

In this nuclear reaction, a heavy nucleus splits into lighter nuclei and large amount of energy is produced.

eg:- $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n + Q$

When a slow moving neutron strikes with uranium splits into barium and krypton.

## NUCLEAR FUSION :-

The process in which two very light nuclei combine to form a nucleus with a large mass number along with release of large amount of energy is called fusion.

Eg:- $${}_{1}^{3}H + {}_{1}H^{2} \rightarrow {}_{2}He^{4} + {}_{0}n^{1} + Q\ (17.6\,MeV)$$

$${}_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{1}H^{3} + {}_{1}H^{1} + Q\ (4.0\,MeV)$$

- Nuclear fusion is known as thermo nuclear reaction because it cannot take place so easily.
- A temperature of the order of $10^{8}$ kelvin is required to start nuclear fusion.

# ATOMS :-

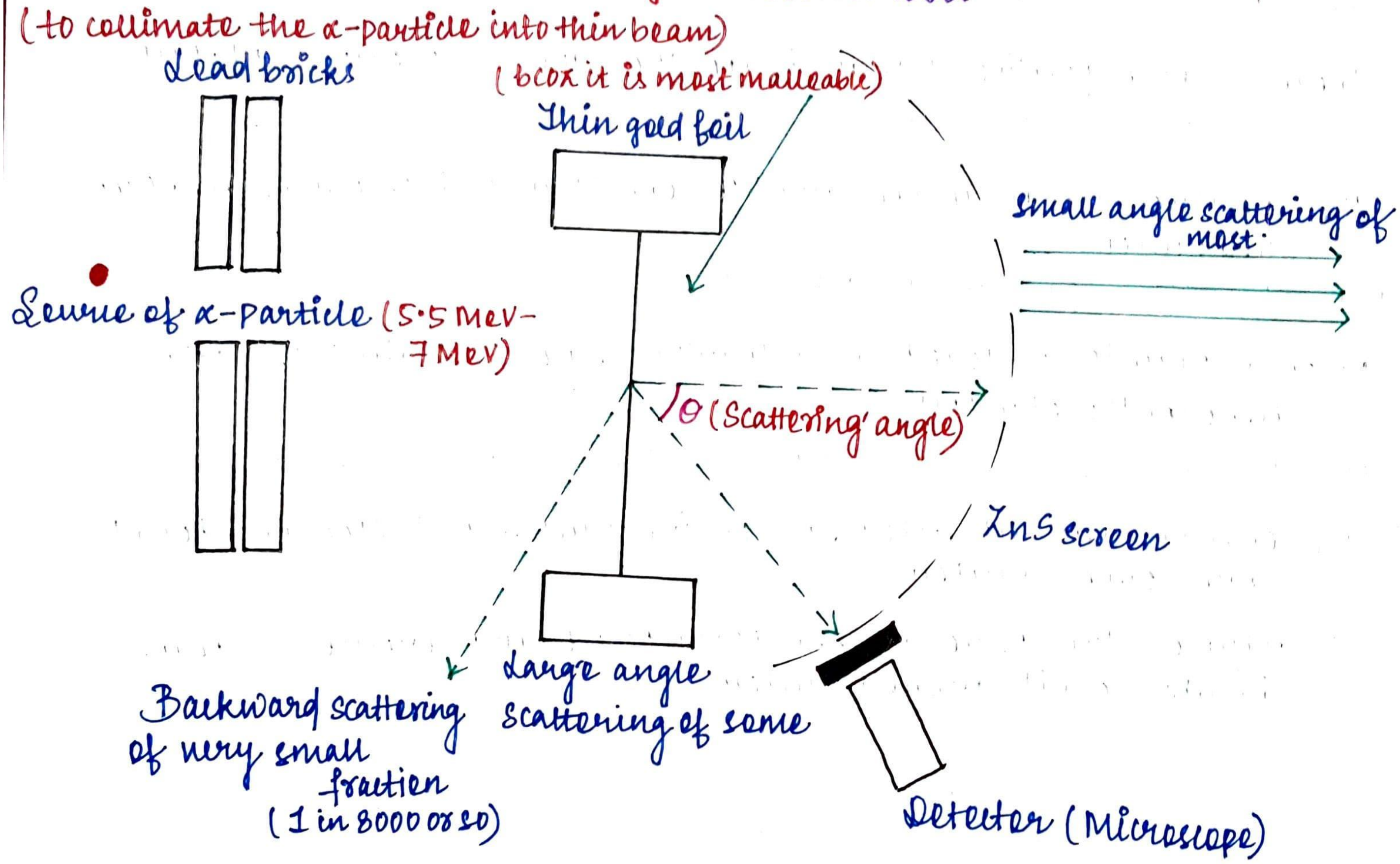


* All elements consist of very small invisible particles are called atoms.
* The first model of atom was propesed by JJ Thomson in 1898 called plum pudding model of the atom.

## α-particle scattering expt :-

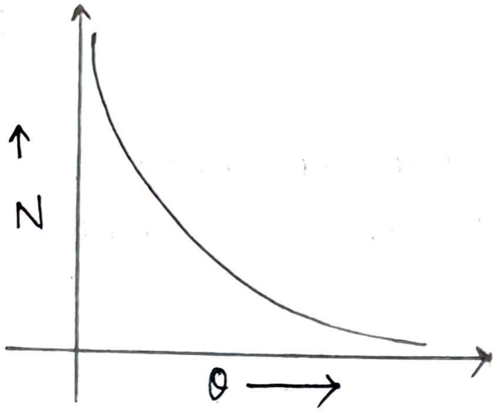
* In 1911, Geiger and Marsden performed α-particle scattering expt. on the suggestion of Rutherford.

## Schematic arrangement of Geiger-Marsden expt :-




## OBSERVATIONS :-

1. Most of the α-particle (99·86%) went undeviated. i.e they didnot suffer any collision. i.e they went straight.
2. About 0·14% of α-particle deviate more than 0°
3. 1 in 8000 of the α-particle deviate more than 90°
4. 1 in $10^6$ of the α-particle return back i.e (deviate through an angle of 180°)




N = no. of α-particles deviated
θ = Scattering angle

No of α-particle, deviated at an angle θ,

$$N = \frac{KZ^2}{\sin^4(\theta/2)}$$

Z = atomic no. of foil

## RESULTS/CONCLUSIONS:-

① Most part of the atom is empty space (hollow)
Reason:- 99.86% α-particles passes undeviated.

② There is some positive charge inside the atom in a very small space.
Reason:- 0.14% α-particle deflect more than 1°.

③ The positive charge inside atom is conc. to an extremely small space called nucleus.
Reason:- 1 in $10^6$ α-particle deviate through 180°.

④ No. of α-particle scattered at particular angle 'θ' is different for different metal foils.
Reason:- Different metals have different +ve charge on their nucleus.

⑤ The electron are attracted by positive nucleus but electron do not move toward nucleus.

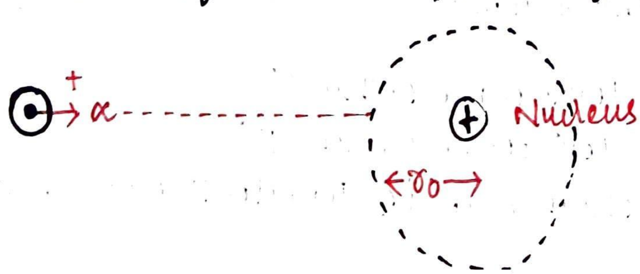
$e^-$ utilises this force as centripetal force and revolves in any circular orbit around nucleus.

$$F_e = F_c$$

⇒ electrostatic force = centripetal force.

## DISTANCE OF CLOSEST APPROACH:- ($r_0$)

It is the minimum distance between which the α-particle passes through the central line of nucleus and centre of atom.



∴ If $r_0$ is the distance of closest approach,
then KE = PE

$$\Rightarrow KE = \frac{1}{4\pi\epsilon_0}\frac{2e \cdot Ze}{r_0}$$

$$\Rightarrow KE = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{r_0}$$

$$\Rightarrow KE \cdot 4\pi\epsilon_0 \cdot r_0 = 2Ze^2$$

$$\Rightarrow r_0 = \frac{2Ze^2}{4\pi\epsilon_0 \cdot KE}$$

$$\Rightarrow r_0 = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{KE}$$

$$\Rightarrow \boxed{r_0 = \frac{2kZe^2}{KE}}$$ (If K.E is max then $r_0$ is min)

Max value of KE = 7.7 MeV, Z = 79

$$r_0 = \frac{2 \times 9 \times 10^9 \times 79 \times (1.6 \times 10^{-19})^2}{7.7 \times 10^6 \times 1.6 \times 10^{-19}}$$

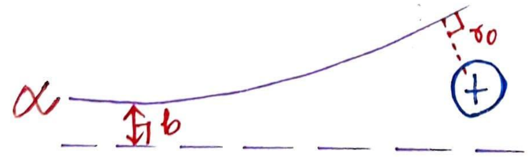
$$= 30 \times 10^{-15} m$$

$$r_0 = 30 fm$$

* Actual gold size of nucleus is 6 fm.

## IMPACT PARAMETER (b)

It is the perpendicular distance of the initial velocity vector of the α-particle from the central line of nucleus.




$$\boxed{b = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2 \cot \theta/2}{KE}}$$

Where, b = impact parameter
θ = scattering angle
KE = kinetic energy of α-particle

when $b \to 0$, $\theta = \pi$
$b \to \infty$, $\theta = 0$ ($b >> r_0$)

## ELECTRON ORBITS :-



Centripetal force required by the electron is provided by the electrostatic force of attraction between the nucleus and electron.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e \cdot e}{r^2}$$

$$\Rightarrow \frac{mv^2}{r} = \frac{Ke^2}{r^2}$$

$$\Rightarrow mv^2 r = Ke^2$$

$$\Rightarrow \boxed{r = \frac{Ke^2}{mv^2} = \frac{e^2}{4\pi\varepsilon_0 mv^2}}$$

This is the expression for radius.

**GOLDEN KEY POINTS :-**

- Size of Nucleus – $10^{-15}$ m
- Size of Atom – $10^{-10}$ m
- α-particle :- $He^{++}$ = Helium nucleus (2P 2n)
  Charge = +2e
  Mass = 4 amu.

Kinetic energy,

$$K.E = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times \frac{Ke^2}{r}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}$$

$$\boxed{K.E = \frac{e^2}{8\pi\varepsilon_0 r}}$$

Potential energy,

$$PE = \frac{Kq_1 q_2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{-e \cdot e}{r} = \frac{-2e^2}{8\pi\varepsilon_0 r}$$

Total energy,

$$T.E = K.E + PE = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{2e^2}{8\pi\varepsilon_0 r}$$

$$\Rightarrow \boxed{T.E = \frac{-e^2}{8\pi\varepsilon_0 r}}$$

–ve sign indicates that electron is bound to the nucleus.
i.e energy is required to free the electron from nucleus.

## DRAWBACK'S OF RUTHERFORD'S MODEL :-

Rutherford's model suffers two major drawbacks:-

① He cannot explain stability of atom.

According to electromagnetic theory, an accelerated charged particle emit energy in the form of electromagnetic radiation. So, electron comes closer to the nucleus and the entire model is collapsed.

② He cannot explain discrete spectrum of atoms.

## BOHR'S MODEL OF HYDROGEN ATOM:-

These three postulates are as follows:-

(i) Bohr's first postulate:- electron can revolve only in certain stable orbits. These orbits have fixed energy and these are called as energy levels or stationary state. These were named K L M N.
1 2 3 4

* electron have same energy as that of the orbit in which it is revolving.
* while revolving in a particular energy level (orbit) electron donot emit any radiation.
* If an electron absorb or emit energy, it must move to different energy level.

ii) Bohr's second postulate :- electron can revolve only in those circular orbits for which the orbital angular momentum of electron is an integral multiple of $h/2\pi$ (Defining stable orbits)

$$L = \frac{nh}{2\pi} \quad , n = \text{any +ve integer } 1, 2, 3, \ldots$$

$$\Rightarrow \boxed{mvr = \frac{nh}{2\pi}} \text{ (Bohr's quantisation cond}^n\text{)}.$$

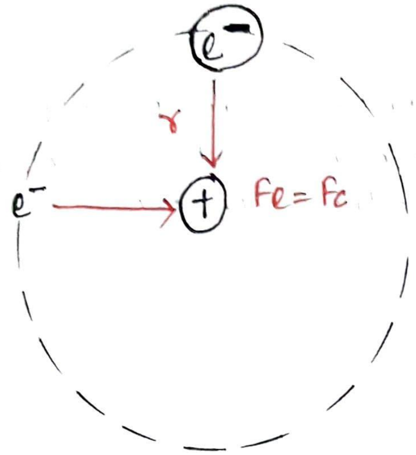
iii) Bohr's third postulate :- While revolving in a higher energy level an electron may emit energy radiation (photon) of a specific wavelength and falls (de-excites) to a lower energy level. The energy of such photon is always equal to the difference in two energy levels.

$$\boxed{E = E_{n_2} - E_{n_1}}$$

$$\Rightarrow \frac{hc}{\lambda} = E_{n_2} - E_{n_1}$$

# BOHR'S THEORY OF HYDROGEN LIKE ATOMS :-






So, the necessary centripetal force of electron is provided by the electrostatic force of attraction.

$$F_e = F_c$$

$$\Rightarrow \frac{K q_1 q_2}{r^2} = \frac{m v^2}{r}$$

$$\Rightarrow \frac{K e \cdot Ze}{r^2} = \frac{m v^2}{r}$$

$$\Rightarrow \frac{K \cdot e \cdot Ze}{r} = m v^2 \text{ —— (I)}$$

According to Bohr's postulate,

$$m v r = \frac{nh}{2\pi} \text{ —— (II)}$$

$eq^n (II)^2 \div eq^n (I)$

$$\frac{m^2 v^2 r^2}{m v^2} = \frac{n^2 h^2}{4\pi^2} \times \frac{r}{K Z e^2}$$

$$\Rightarrow m r = \frac{n^2 h^2}{4\pi^2} \times \frac{4\pi \epsilon_0}{Z e^2}$$

$$\Rightarrow m r = \frac{n^2 h^2}{\pi} \times \frac{\epsilon_0}{Z e^2}$$

$$\Rightarrow \boxed{r = \frac{n^2 h^2 \epsilon_0}{m \pi Z e^2}} \text{ —— (III)}$$

Radius for $n^{th}$ orbit,

$$\boxed{r = \frac{n^2 h^2 \epsilon_0}{\pi m Z e^2}}$$

if $Z = 1, n = 1$

$$r = \frac{h^2 \epsilon_0}{\pi m e^2} = 0.53 \times 10^{-10} m = \boxed{0.53 \mathring{A}}$$

(It is called as Bohr's radius)

N.B :- 1) $r \propto \frac{n^2}{Z}$

$$\Rightarrow \frac{r_1}{r_2} = \frac{n_1^2}{n_2^2} \times \frac{Z_2}{Z_1}$$

2) $\boxed{r_n = 0.53 \frac{n^2}{Z} A^\circ}$

## Velocity of electron in stationary orbits :-

Putting the value of r in eqn (II)

$$mvr = \frac{nh}{2\pi}$$

$$\Rightarrow m \times v \times \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2} = \frac{nh}{2\pi}$$

$$\Rightarrow \frac{nh\varepsilon_0 v}{Ze^2} = \frac{1}{2}$$

$$\Rightarrow 2nh\varepsilon_0 v = Ze^2$$

$$\Rightarrow \boxed{v = \frac{Ze^2}{2nh\varepsilon_0}}$$

velocity of electron in the $n^{th}$ energy level :-

$$\boxed{v = \frac{Ze^2}{2nh\varepsilon_0}}$$

for 1st orbit of hydrogen atom,

$$Z=1,\ n=1$$

$$v = \frac{e^2}{2h\varepsilon_0} = \boxed{2.2 \times 10^6 \, m/s}$$

N.B :- 1) $v \propto \frac{Z}{n}$

$$\Rightarrow \frac{v_1}{v_2} = \frac{Z_1}{Z_2} \times \frac{n_2}{n_1}$$

2) $\boxed{v_n = 2.2 \times 10^6 \frac{Z}{n} \, m/s}$

Energy of electron in stationary orbits:-

Electron has two type of energy:-

① $K.E = \frac{1}{2}mv^2$

$= \frac{1}{2} \times m \times \left(\frac{Ze^2}{2nh\varepsilon_0}\right)^2$

$= \frac{1}{2} \times m \times \frac{Z^2e^4}{4n^2h^2\varepsilon_0^2}$

$$\boxed{K.E = \frac{mZ^2e^4}{8n^2h^2\varepsilon_0^2}}$$

② $PE = \frac{Kq_1q_2}{r} = \frac{K\cdot(-e)\cdot Ze}{\frac{n^2h^2\varepsilon_0}{\pi mZe^2}}$

$= \frac{(-e)\cdot Ze\cdot Ze^2\pi m}{4\pi\varepsilon_0 n^2h^2\varepsilon_0}$

$= \frac{-Z^2e^4m}{4\varepsilon_0 n^2h^2\varepsilon_0}$

$$\boxed{P.E = \frac{-2Z^2e^4m}{8n^2h^2\varepsilon_0^2}}$$

Total energy $= K.E + P.E$

$= \frac{mZ^2e^4}{8n^2h^2\varepsilon_0^2} - \frac{2Z^2e^4m}{8n^2h^2\varepsilon_0^2}$

$$\boxed{T.E = \frac{-mZ^2e^4}{8n^2h^2\varepsilon_0^2}}$$

Total energy is -ve. i.e electron is bound to the nucleus.

N.B:- ① $PE = 2TE = -2KE$

② $T.E = -13.6\frac{Z^2}{n^2}\,eV$ = energy of $n^{th}$ orbit.

$T.E = -13.6\frac{Z^2}{n^2}\,eV$

for H atom, $Z=1$, $n=1$

$T.E = \boxed{-13.6\,eV}$ (ground state) (1st orbit)

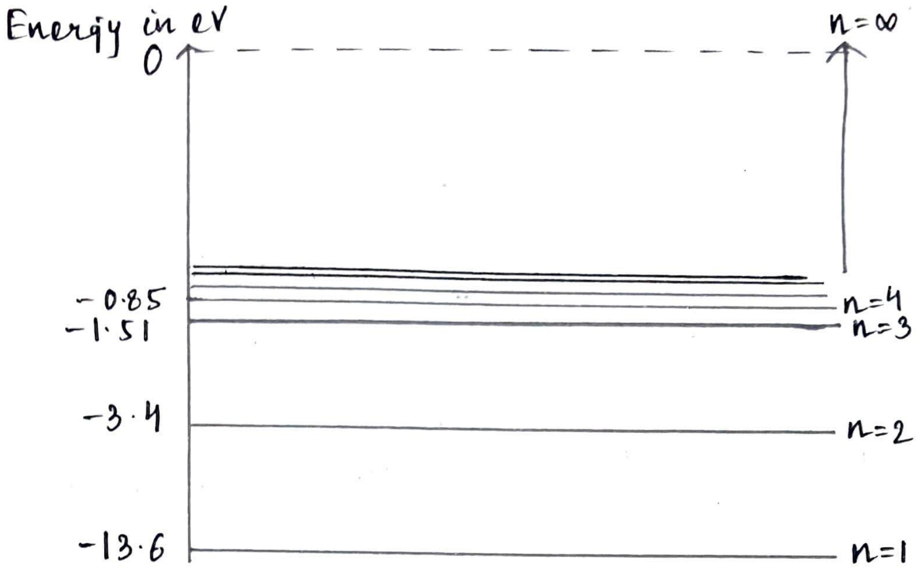
$T.E = -13.6 \times \frac{1}{4} = \boxed{-3.4\,eV}$ (2nd orbit) (1st excited state)

$T.E = -13.6 \times \frac{1}{9} = \boxed{-1.51\,eV}$ (3rd orbit) (2nd excited state)

$T.E = -13.6 \times \frac{1}{16} = \boxed{-0.85\,eV}$ (4th orbit) (3rd excited state)

$T.E = -13.6 \times \frac{1}{25} = \boxed{-0.54\,eV}$ (5th orbit) (4th excited state)

## ENERGY LEVEL DIAGRAM:-




* While remaining in a lower energy level an $e^-$ may absorb a radiation (photon) of a specific wavelength and jumps (excites) to a higher energy level.

The energy of such photon is always equal to the diff in 2 energy level.

$$E = E_{n_2} - E_{n_1}$$

$$\Rightarrow \boxed{\frac{hc}{\lambda} = E_{n_2} - E_{n_1}}$$

## SHORTCUT FORMULA FOR FINDING λ OF EMITTED PHOTON:-

$$\boxed{\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}$$

where, R = Rydberg constant

$R = 1.097 \times 10^7\ m^{-1}$

$\frac{1}{R} \approx 911\ A^\circ$.

N.B:- $\boxed{\underset{(\text{in } A^\circ)}{\lambda} = \frac{12375}{E(eV)}}$

Longest wavelength ⟶ Small transition

Shortest wavelength ⟶ larger transition.

(i) For Lyman series

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \; n = 2, 3, 4,}$$ (UV region)

(ii) For Balmer series

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \; n = 3, 4, 5, \ldots}$$ (Visible region)

(iii) For Paschen series

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \; n = 4, 5, 6, \ldots}$$ (Low infrared region)

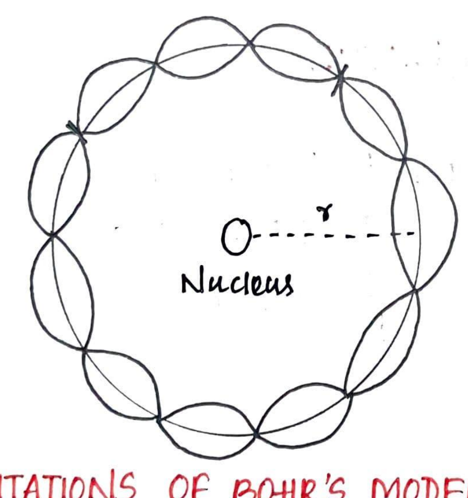
(iv) For Brackett series

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \; n = 5, 6, 7, \ldots}$$ (Low infrared region)

(v) For Pfund series:-

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \; n = 6, 7, 8, \ldots}$$ (High infrared region)

## DE-BROGLIE'S COMMENT ON BOHR'S SECOND POSTULATE:-




According to de-Broglie wavelength,

$$\boxed{\lambda = \frac{h}{mv}}$$

$$\therefore \; 2\pi r = \frac{nh}{mv}$$

$$\Rightarrow mvr = \frac{nh}{2\pi} = n(h/2\pi)$$

## LIMITATIONS OF BOHR'S MODEL:-

① It stands true for only H-atom and H like atom (single $e^-$ species)
② It cannot explain spectrum of multi $e^-$ species.
③ It doesnot take into account the wave nature of $e^-$ violating de-broglie hypothesis.
④ It violates heisenberg's uncertainity principle.
⑤ He cannot explain splitting of spectra lines in electricfield (Stark effect) and magnetic field (Zeeman effect)

# DUAL NATURE OF RADIATION AND MATTER

(CHAPTER-11)

## ELECTRON EMISSION :-

- The process of emission of electron from a metal surface is called electron emission.

* In metal large number of free electrons are present which can move everywhere in a metal. But these electron cannot leave the surface of the metal.

## WORK FUNCTION ($\phi_0$):-

The minimum energy required by an electron to escape from the metal surface is called work function of the metal.

- It is measured in eV.

$$\boxed{1\,eV = 1.602 \times 10^{-19}\,J}$$

- It depends on the properties of the metal and nature of its surface.

The mininum energy required for the electron emission from the metal surface can be supplied to the free electrons by any one of the following physical processes :-

### (1) THERMIONIC EMISSION :-

- The process of emission of an electron when a metal is heated is known as thermionic emission.
- The free electrons in the metal absorb the heat energy and can overcome the surface barrier. As a result, the free electrons are emitted from the metal surface.
- The electrons emitted are known as Thermions because they are emitted due to thermal energy.

### (2) FIELD EMISSION :-

- The process of emission of free electrons when a strong electric field of the order $10^8\,V/m$ is applied across the metal surface is known as field emission.
- It is also known as cold cathode emission.

## (3) PHOTO-ELECTRIC EMISSION :-

- The process of emission of electrons when light of suitable frequency is incident on a metal surface is known as photo electric emission.
- When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface.
- The electrons emitted are known as photoelectrons.

## PHOTOELECTRIC EFFECT :-

- The emissions of electrons from the surface of the metals due to the incidence of light of suitable frequency is called photoelectric effect.
- The ejected electrons are called as photoelectrons and the current constituted is called photocurrent.

## ① HERTZ'S OBSERVATION :-

Hertz observed that when ultraviolet rays are incident on negative plate of electric discharge tube then conduction takes place easily in the tube.

## ② HALLWACHS' AND LENARD'S OBSERVATIONS :-

### Hallwach observation :-

Hallwach observed that if negatively charged Zn plate is illuminated by U.V light, its negative charge decreases and it becomes neutral and after some time it gains positive charge. It means, in the effect of light, some negative charged particles are emitted from the metal.

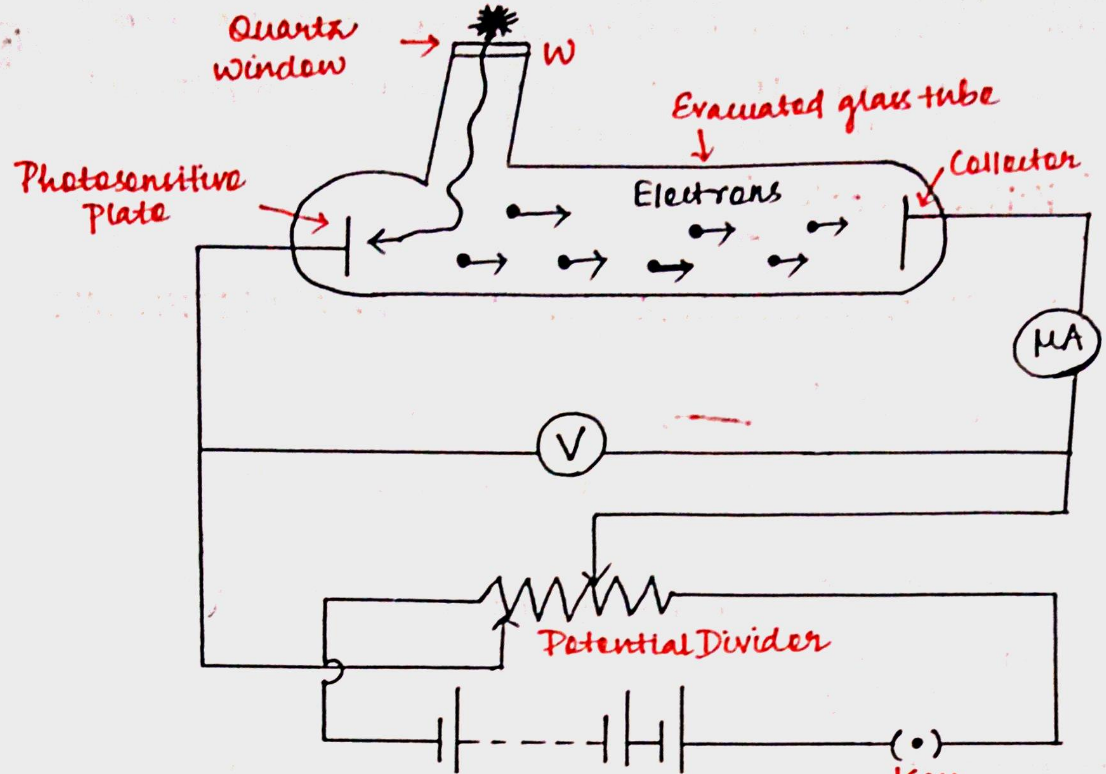
### Lenard observation :-

He told that when U.V rays are incident on cathode, electrons are ejected. These electrons are attracted by anode and circuit is completed due to flow of electrons and ~~current~~ (current) flows. When U.V rays are incident on anode, electrons are ejected but current doesnot flow.

For the photoelectric effect the light of short wavelength (or high frequency) is more effective than the light of long wavelength (low frequency).

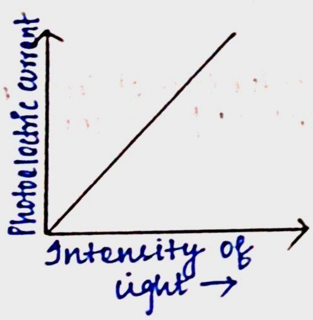
## EXPERIMENTAL STUDY OF PHOTOELECTRIC CURRENT :-

When light of frequency $\nu$ and intensity I falls on the cathode, electrons are emitted from it. The electrons are collected by the anode and a current flows in the circuit. This current is called photoelectric current. This experiment is used to study the variation of photoelectric current with different factors like intensity, frequency and the potential difference betn the anode & cathode.



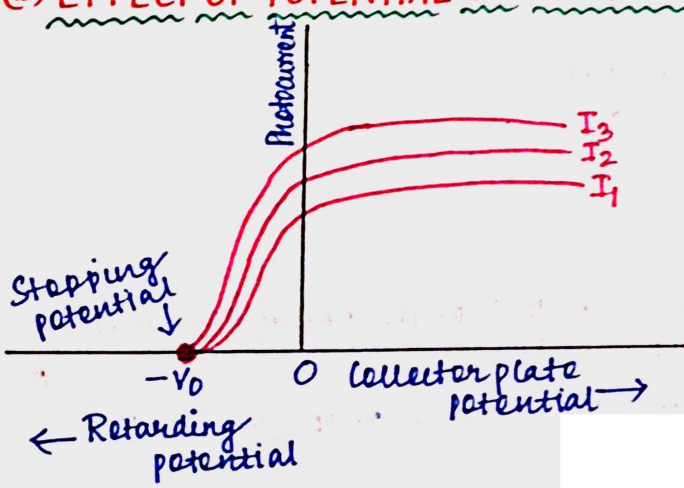

Experimental arrangement for study of photoelectric effect.

## (a) EFFECT OF INTENSITY OF LIGHT ON PHOTOCURRENT:-




- The photocurrent is directly proportional to the number of photoelectrons emitted per second.
- This implies that no. of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.

## (b) EFFECT OF POTENTIAL ON PHOTOELECTRIC CURRENT:-



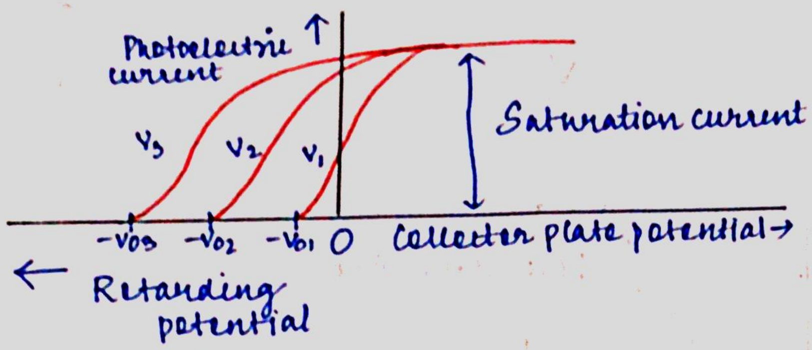

Zero potential- When anode is at zero potential, the photocurrent is not zero.

Positive potential- When anode is at +ve potential, it attracts the ejected $e^{\ominus}$. When it is made more +ve, gradually photocurrent increases and becomes constant called as Saturation current.

Negative potential :- When anode is made $-ve$, the ejected $e^-$ are repelled, so photocurrent decreases. For a particular value of $-ve$ potential, photocurrent is zero, which is called stopping potential.

$$\boxed{K.E_{max} = eV_0}$$

## (c) EFFECT OF FREQUENCY OF INCIDENT RADIATION ON STOPPING POTENTIAL:-

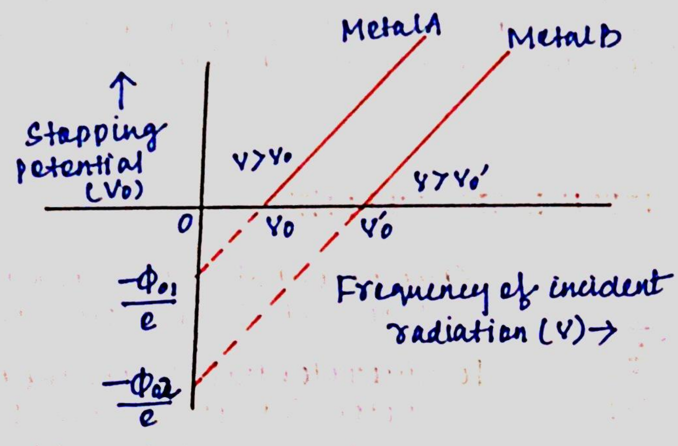



From the graph, we observe that :-

(i) The value of stopping potential is different for radiation of different frequencies but same value of saturation current. (for given intensity).

(ii) Greater the frequency of incident radiation, greater is the max K.E of photoelectrons, consequently greater retarding potential or stopping potential is required to stop them completely.

(iii) The value of the saturation current depends on the intensity of incident radiation but is independent of frequency of the incident radiation.




The graph shows that :-

(i) The stopping potential $V_0$ varies linearly with the frequency of incident radiation for a given photosensitive material.

(ii) There exists a certain minimum cut-off frequency $\nu_0$ for which the stopping potential is zero.

**THRESHOLD FREQUENCY** ($\nu_0$):- For a given metal surface, there exists certain or minimum frequency below which no photoelectric emission takes place.

## LAWS OF PHOTOELECTRIC EFFECT:-

(1) It is an instantaneous process.

(2) For a given metal, there exists a certain/minimum frequency of incident radiation below which no photoelectric emission take place. This frequency is called threshold frequency.

(3) The photoelectric current is directly proportional to intensity of incident radiation but is independent of frequency of light.

(4) The maximum K.E of ejected $e^-$ depends on the frequency of incident radiation and is independent of its intensity.

## EINSTEIN'S PHOTOELECTRIC EQUATION:-

Einstein explained photoelectric emission basing on planck's quantum theory. According to Einstein, when light is incident on a metal, each photon interacts with one $e^-$ and transfer its energy. It is utilized in 2 purposes:-

① To just eject the $e^-$ from metal surface which is called work function ($\phi_0 = h\nu_0$)

② rest energy becomes K.E of $e^-$.

If $\nu$ is the frequency of incident light then,

$$h\nu = \phi_0 + K.E$$

$$h\nu = h\nu_0 + \frac{1}{2}mv_{max}^2$$

$$\Rightarrow K_{max} = h\nu - h\nu_0 = h(\nu - \nu_0) = h\nu - \phi_0$$

$$\boxed{K_{max} = h\nu - \phi_0}$$

## WAVE NATURE OF MATTER:-

The wave associated with moving material particle is called matter wave or de-Broglie wave whose wavelength is called de-Broglie wavelength which is given by:- $\lambda = \frac{h}{mv}$

According to planck's quantum theory, the energy of the photon is given by :-

$$E = h\nu = \frac{hc}{\lambda} \quad \text{---} \text{(I)}$$

According to Einstein's theory, the energy of photon is given by

$$E = mc^2 \quad \text{---} \text{(II)}$$

From (I) & (II), we get,

$$\lambda = \frac{h}{mc} = \frac{h}{P}$$ , $p = mc$ is momentum of a photon.

According to de-broglie hypothesis, the wavelength of wave associated with moving material particle becomes.

$$\boxed{\lambda = \frac{h}{P} = \frac{h}{mv}}$$

## DE-BROGLIE WAVELENGTH OF AN ELECTRON:-

$$\lambda = \frac{12.27}{\sqrt{V}} \ \text{Å}$$

or $$\lambda = \frac{1.227}{\sqrt{V}} \ \text{nm}$$

## DAVISSON AND GERMER EXPERIMENT:-

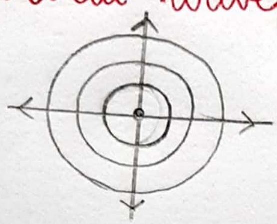
**PURPOSE:-** To prove wave nature of electron.

# WAVE OPTICS

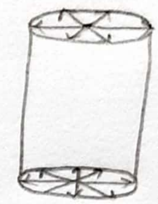
**Wave Front** - A wavefront is the locus of points having the same phase of oscillations. A wavelet is the point of disturbance due to propagation of light

## TYPES OF WAVEFRONT

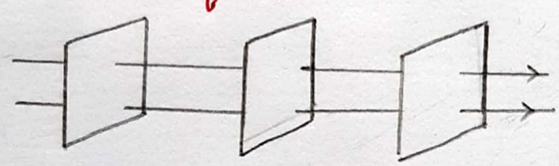
1. Spherical wavefront -



2. Cylindrical wavefront -



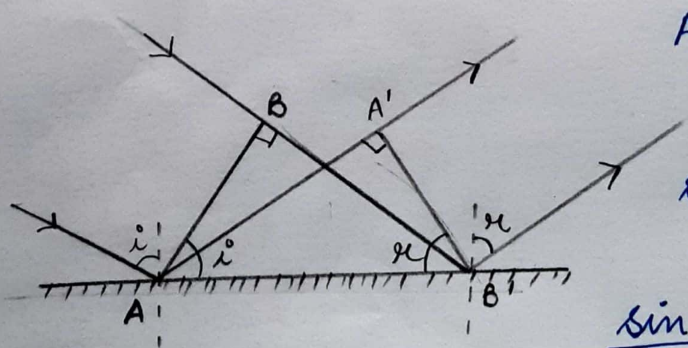
3. Plane wavefront -



## HUYGEN'S PRINCIPLE -

According to Huygen, each point on the wavefront acts as a secondary disturbance and to generate secondary wavelet tangents are drawn from primary wavelet and a common line touching the tangent will form a secondary wavelet.

## LAWS OF REFLECTION -




AB → incident wavefront

A'B' → reflected wavefront

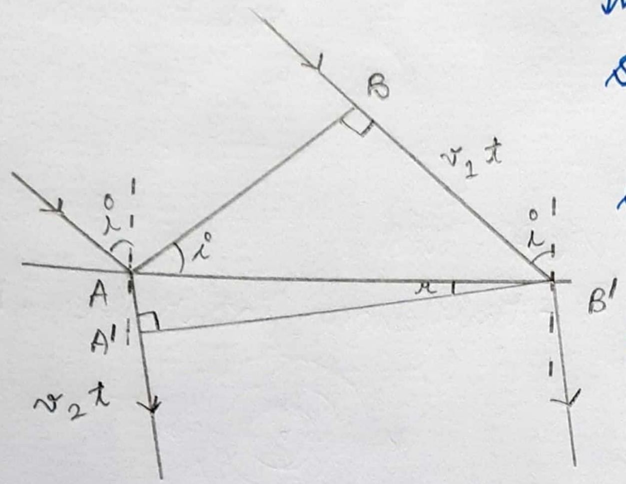
In $\Delta ABB'$ and $\Delta AB'A'$

$\sin i = \frac{BB'}{AB'}$ $\sin r = \frac{AA'}{AB'}$

$\frac{\sin i}{\sin r} = \frac{BB'}{AA'} = \frac{ct}{ct}$ $\sin i = \sin r$

$\boxed{i = r}$

## LAWS OF REFRACTION -




In $\Delta ABB'$ and $\Delta AA'B'$

$$\sin i = \frac{BB'}{AB'} \qquad \sin r = \frac{AA'}{AB'}$$

$$\sin i = \frac{v_1 t}{AB'} \qquad \sin r = \frac{v_2 t}{AB'}$$

$$\frac{\sin i^\circ}{\sin r} = \frac{v_1}{v_2} \rightarrow \text{constant}$$

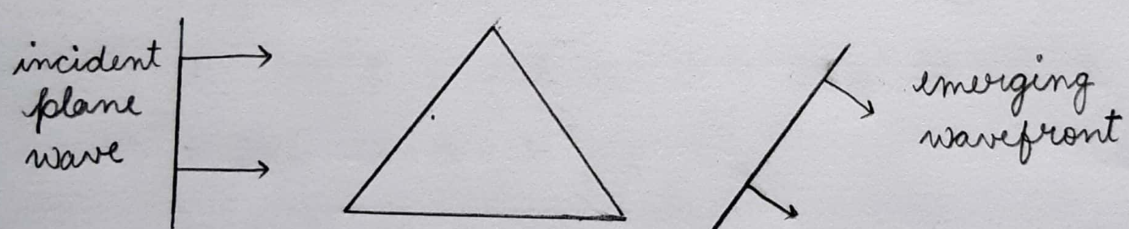
AB → incident wave

A'B' → refracted wave

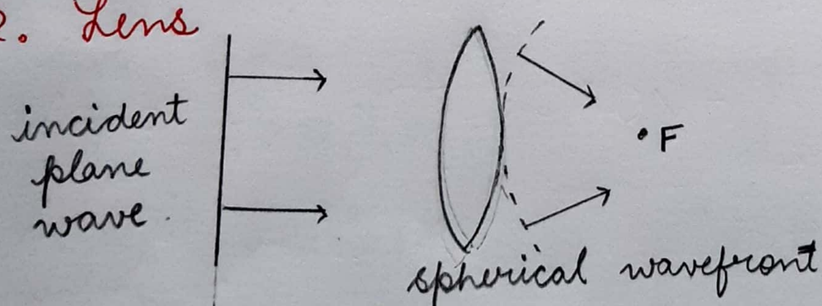
$$\therefore \frac{\sin i^\circ}{\sin r} = {}_1n_2$$

## BEHAVIOUR OF PRISM, LENS AND SPHERICAL MIRROR TOWARDS WAVEFRONT

### 1. Prism




### 2. Lens




### 3. Spherical Mirror

# PRINCIPLE OF SUPERPOSITION

$Y = \bar{y}_1 + \bar{y}_2$

vector sum

$y_1 = a \sin \omega t$

$y_2 = b \sin(\omega t + \phi)$

$Y = \bar{y}_1 + \bar{y}_2$

$= a \sin \omega t + b \sin(\omega t + \phi)$

$Y = a \sin \omega t + b[\sin \omega t \cos \phi + \cos \omega t \sin \phi]$

$Y = \sin \omega t [a + b \cos \phi] + \cos \omega t \sin \phi$

$a + b \cos \phi = R \cos \theta$ —— 1 $\qquad \sin \phi = R \sin \theta$ —— 2

$Y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$

$Y = R \sin(\omega t + \theta)$ $\quad R \rightarrow$ amplitude $\quad \theta \rightarrow$ phase diff.

squaring and adding

$(a + b \cos \phi)^2 + (\sin \phi)^2 = R^2$

$a^2 + b^2 \cos^2 \phi + 2ab \cos \phi + b^2 \sin^2 \phi = R^2$

$a^2 + 2ab \cos \phi + b^2 = R$

$R = \sqrt{a^2 + b^2 + 2ab \cos \phi}$

$R_{max.}$, $\cos \phi = 1$

$\phi = 0, 2\pi, 4\pi \ldots$

$R_{max.} = \sqrt{(a+b)^2}$

$R_{max.} = (a+b)$

$R_{min.}$, $\cos \phi = -1$

$\phi = \pi, 3\pi, 5\pi \ldots$

$R_{min.} = \sqrt{(a-b)^2}$

$R_{min.} = (a-b)$

Intensity $\propto$ amplitude$^2$

$I_1 \propto a^2 \qquad I_2 \propto b^2 \qquad I_R \propto R^2$

$I_1 = K a^2 \qquad I_2 = K b^2 \qquad I_R = K R^2$

$$I_R = K(a^2 + b^2 + 2ab \cos\phi)$$

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\phi$$

$I_{max.} = (\sqrt{I_1} + \sqrt{I_2})^2 \rightarrow$ constructive interference

$I_{min.} = (\sqrt{I_1} - \sqrt{I_2})^2 \rightarrow$ destructive interference

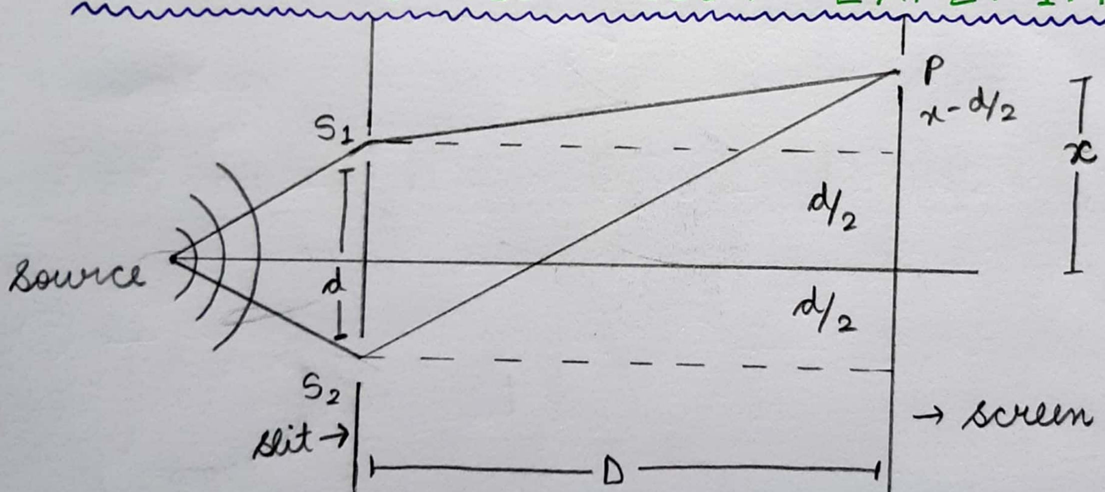
## INTERFERENCE OF LIGHT

Redistribution of light energy from two coherent sources superimposing at a point.

Coherent Sources –

→ Same frequency → zero / constant phase differenc

## YOUNG'S DOUBLE SLIT EXPERIMENT




In young's double slit experiment,

(i) Fringe width of bright and dark fringe

$$\boxed{\beta = \frac{D\lambda}{d}}$$

where, $\lambda$ = wavelength of wave

$D$ = distance between slit and screen

$d$ = distance between two slits

$$\boxed{\text{Angular fringe, width } \theta = \frac{\beta}{D} = \frac{\lambda}{d}}$$

(ii) Separation of nth order bright fringe from central fringe

$$y_n = \frac{Dn\lambda}{d} \quad \text{where , } n = 1, 2, 3 \ldots$$

(iii) Separation of $n^{th}$ order dark fringe from central fringe

$$y_n = (2n-1)\frac{D\lambda}{2d} \quad \text{, where } n = 1, 2, 3 \ldots$$

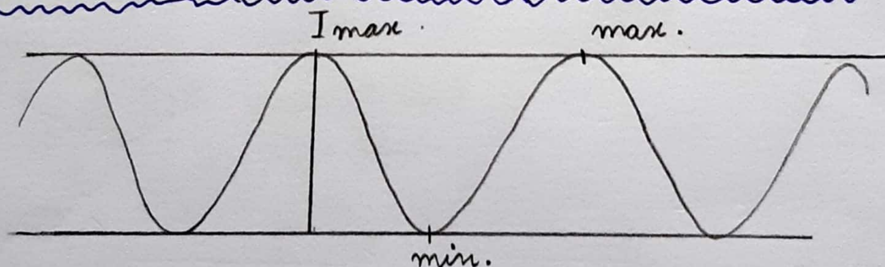
(iv) Angular position of $n^{th}$ order

(a.) Bright fringe $= \frac{y_n}{D} = \frac{n\lambda}{d}$

(b.) Dark fringe $= \frac{y_n}{D} = (2n-1)\frac{\lambda}{d}$ where, $n = 1, 2, 3 \ldots$

(v) Fringe width decreases, when whole apparatus is taken from air to a denser medium, due to the decrease in wavelength of the light.

## DISTRIBUTION OF INTENSITY




- INTENSITY OF LIGHT (I) is proportional to the width (d) of slit and ratio of slit - width (a)

$$\frac{d_1}{d_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

- Ratio of max.$^m$ and min.$^m$ intensity of light

$$\frac{I_{max.}}{I_{min.}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \left(\frac{r+1}{r-1}\right)^2$$

where, $r = \frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}}$

## FRINGE WIDTH

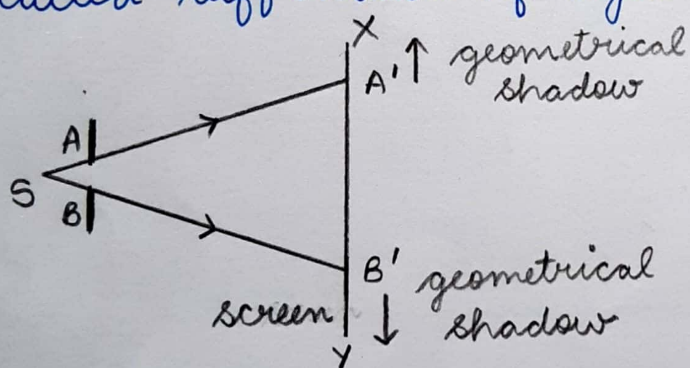
The difference between two consecutive dark and bright fringes.

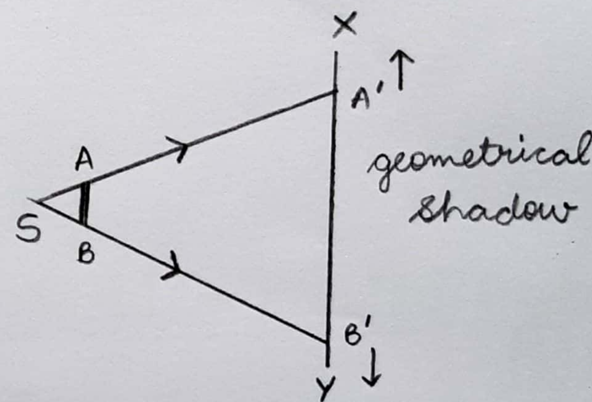
### Conditions for interfernce -

1. Sources must be coherrent.
2. Distance between slit and screen should be large.
3. Distance between slits must be small.

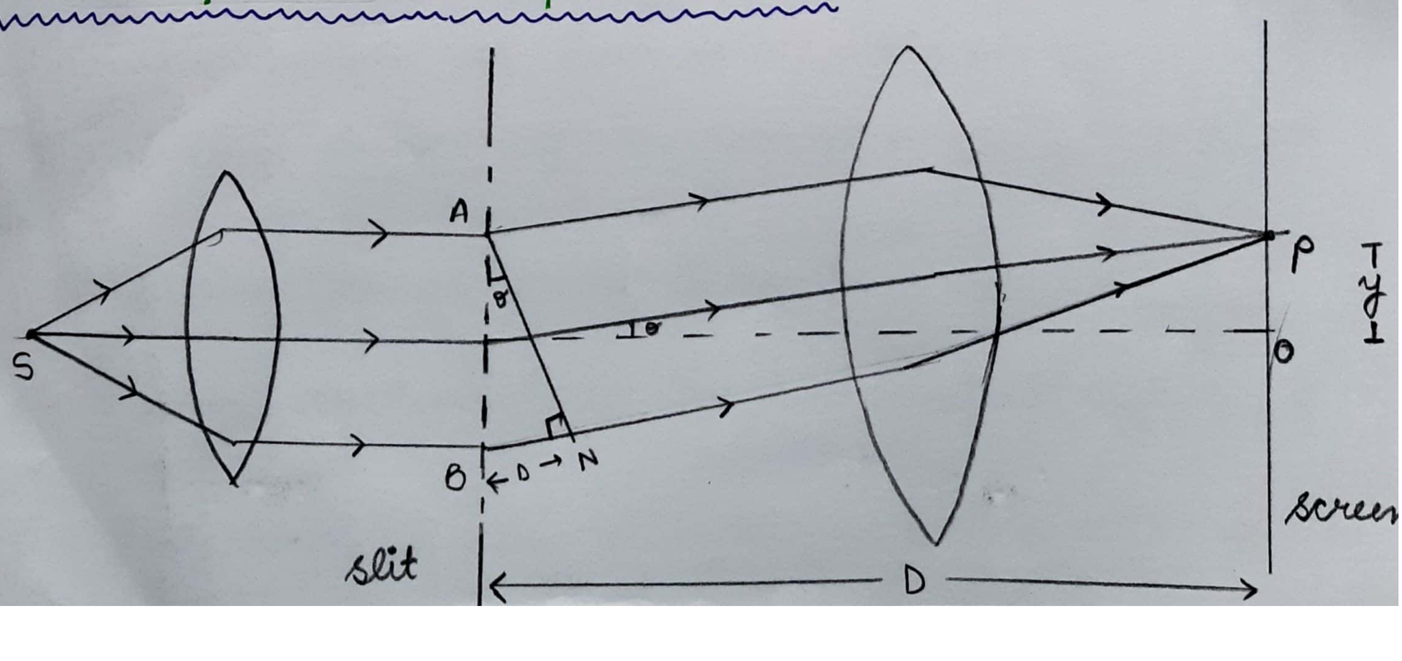
## DIFFRACTION OF LIGHT

The phenomenon of bending of light around the sharp corners and the spreading of light within the geometrical shadow of the opaque obstacles is called diffraction of light.




## YOUNG'S SINGLE SLIT



$$\cos(90-\theta) = \frac{B}{a}$$

$$B = a\sin\theta$$

For minima

$a\sin\theta = n\lambda \quad n = 1, 2, 3\ldots$

$$\sin\theta = \frac{n\lambda}{a}$$

$n = 1 \quad \theta_1 = \frac{\lambda}{a}$

$n = 2 \quad \theta_2 = \frac{2\lambda}{a}$

For maxima

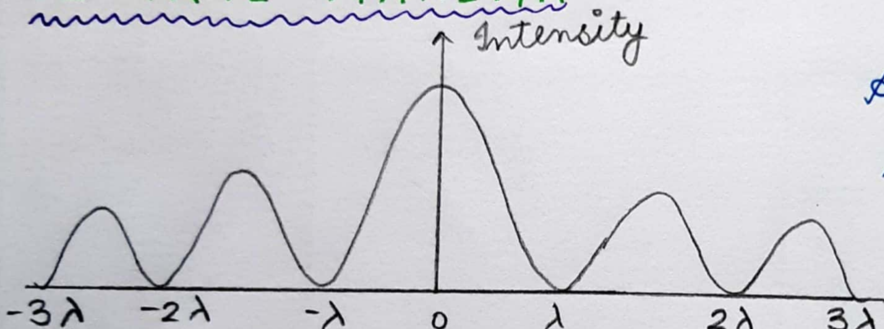
$$a\sin\theta = (2n+1)\frac{\lambda}{2}$$

$$\sin\theta = \frac{(2n+1)\lambda}{2a}$$

$n = 1 \quad \theta_1 = \frac{3\lambda}{2a}$

$n = 2 \quad \theta_2 = \frac{5\lambda}{2a}$

## CENTRAL MAXIMA




Angular width of central maxima

## RESOLVING POWER OF OPIICAL INSTRUMENTS

Resolving power of an optical instrument is the ability of the instrument to produce distinctly separate images of two close objects.

(i) Resolving power of microscope $= \frac{1}{\Delta d} = \frac{2\mu\sin\beta}{\lambda}$

(ii) Resolving power of a telescope $= \frac{1}{d\theta} = \frac{D}{1.22\lambda}$

$d\theta$ = angle subtended by the two distinct objects of objective

$\beta$ = half angle of cone of light

D = diameter of the objective

# DIFFERENCE BETWEEN INTERFERENCE PATTERN AND THE DIFFACTION PATTERN

| CHARACTERISTICS | INTERFERENCE | DIFFRACTION |
|---|---|---|
| FRINGE WIDTH | All bright and dark fringes are of equal width. | The central bright fringe have got double width to that of width of secondary maxima or minima |
| INTENSITY OF BRIGHT FRINGES | All bright fringes are of same intensity. | Central fringe is the brightest and intensity of secondary maxima, decreases with the increase of order of secondary maxima on either side of central maxima. |

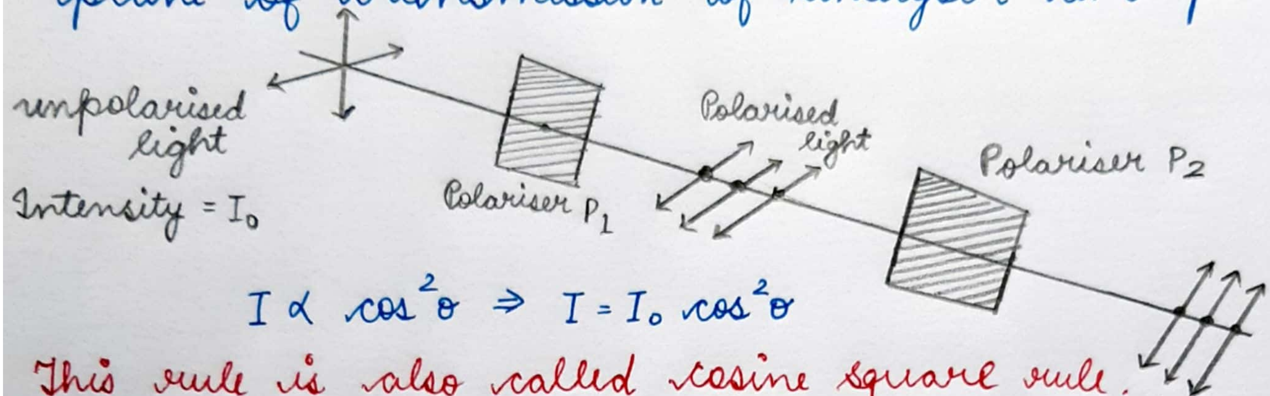
# POLARISATION

The phenomenon of restricting the vibrations of light in a particular direction, perpendicular to the direction of wave motion is called polarisation of light. Polarisation ensures the transverse nature of light.

- Polarisers - A device that plane - polarises the unpolarised light passed through it is called a polariser. Example - Crystal, nicol prism, polaroid. Tourmaline.

# MALUS LAW

According to law of Malus, when a beam of completly plane polarised light is incident on an analyser, the resultant intensity of light (I) transmitted from the analyser varies directly as the square of the cosine of the angle θ between the plane of transmission of analyser and polariser.




$$I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$$

This rule is also called cosine square rule.

where, $I_0$ = intensity of plane polarised light after passing through $P_1$.

# BREWSTER'S ANGLE

(i) The angle of incidence at which the reflected light is completely plane polarised is called polarising angle or Brewster's angle ($i_B$)

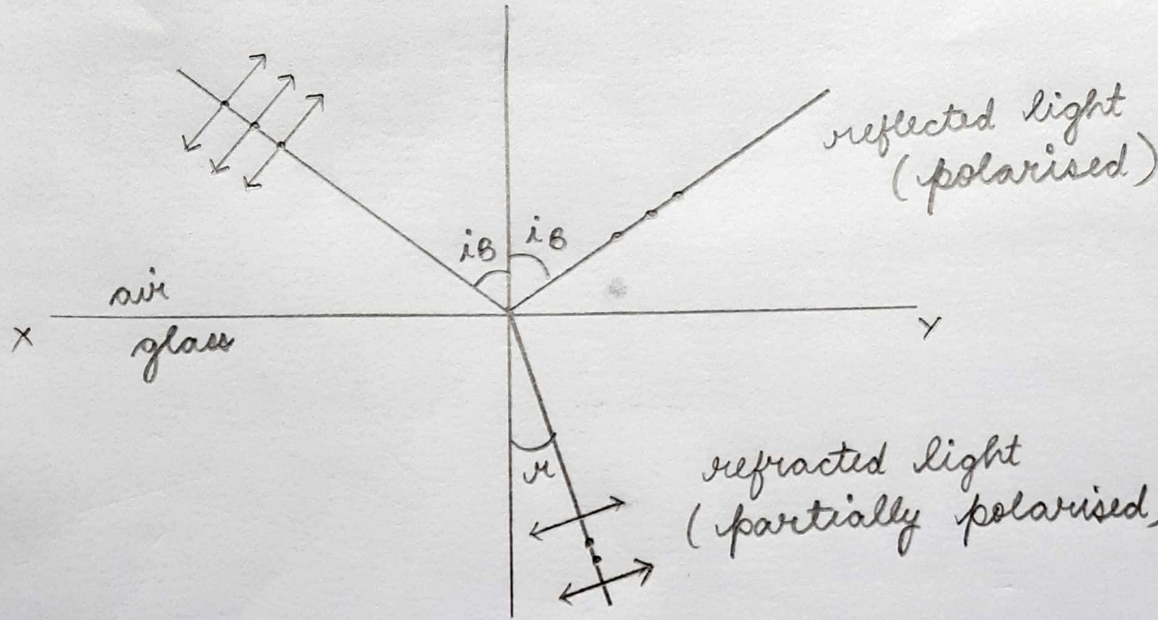
(ii) According to this law, when unpolarised light is incident at polarising angle, $i_B$ on an interface separating air from a medium of refractive index $\mu$, then the reflected light is plane polarised, provided $\mu = \tan i_B$

where, $i_B$ = Brewster's angle  $\mu$ = refractive index

$i_B + r = 90^\circ$

From Snell's law,

$$\boxed{\mu = \frac{\sin i_B}{\sin r_B} = \frac{\sin i_B}{\sin(90 - i_B)} = \frac{\sin i_B}{\cos i_B} = \tan i_B}$$




# POLAROIDS

Polaroids are the commercial devices to produce plane polarised light making use of selective absorption. Polaroids are used in sunglasses, wind screen, window panes of aeroplane and to make images vivid and clear.

Modes of production of plane polarised light -

(i) Reflection (Brewster's law)
(ii) Scattering
(iii) Double refraction (calcite)
(iv) Selective absorption (dichroism)

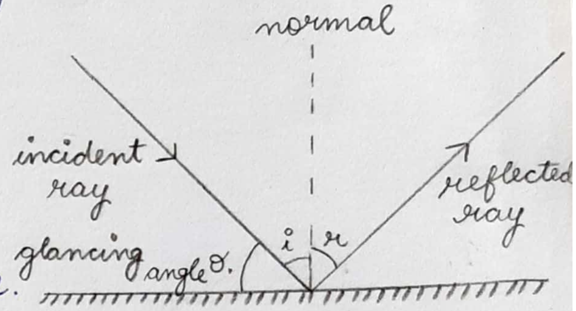
# RAY OPTICS



**Reflection** – The bouncing back of light in the same medium.

**Laws of reflection** –

1. The angle of incidence is equal to the angle of reflection.
2. The normal, incident ray and reflected ray lie in the same plane.




## Relation between f and R

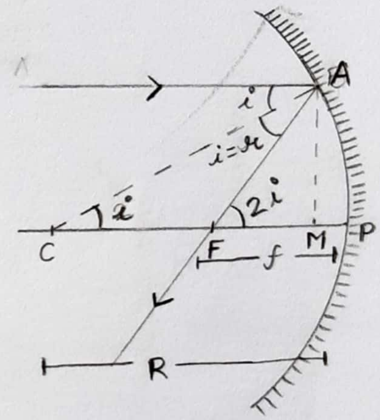
### I] Using concave mirror

In $\Delta AFM$

$$\tan 2i = \frac{AM}{FM} \approx \frac{AM}{PF}$$

$$2i = \frac{AM}{PF} \quad (\because 2i \text{ is small})$$

$$i = \frac{AM}{2PF} \quad \text{—— (1)}$$




concave mirror

In $\Delta ACM$

$$\tan i = \frac{AM}{MC} \approx \frac{AM}{PC}$$

$$i = \frac{AM}{PC} \quad \text{—— (2)}$$

From equations (1) and (2)

$$\frac{AM}{2PF} = \frac{AM}{PC}$$

$$2PF = PC$$

$$\boxed{2f = R}$$

$$\boxed{f = \frac{R}{2}}$$

### II] Using convex mirror

In $\Delta AFM$

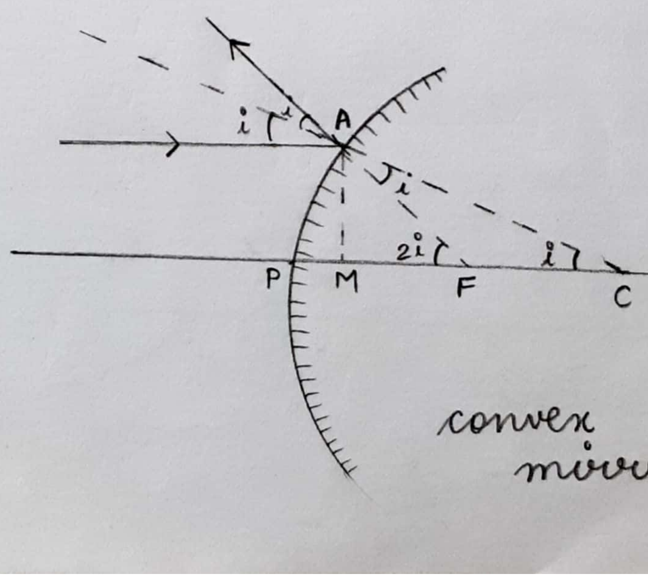
$$\tan 2i = \frac{AM}{MF} \approx \frac{AM}{PF}$$

$$2i = \frac{AM}{PF} \quad \text{—— (1)}$$

In $\Delta AMC$

$$\tan i = \frac{AM}{MC} \approx \frac{AM}{PC}$$

$$i = \frac{AM}{PC} \quad \text{—— (2)}$$




convex mirror

From equations 1 and 2

$$\frac{AM}{PC} = \frac{AM}{2PF}$$

$$PC = 2\,PF$$

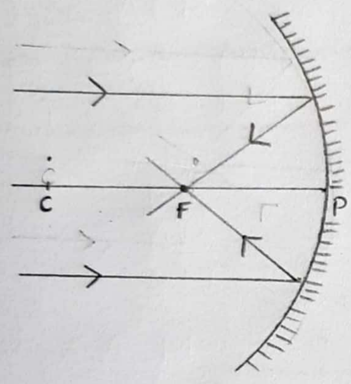
$$\boxed{R = 2f}$$

$$\boxed{f = \frac{R}{2}}$$

- The radius of curvature of a plane mirror is infinity.

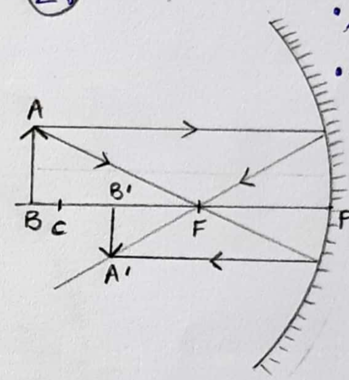
## Ray Diagrams due to concave mirror

1.



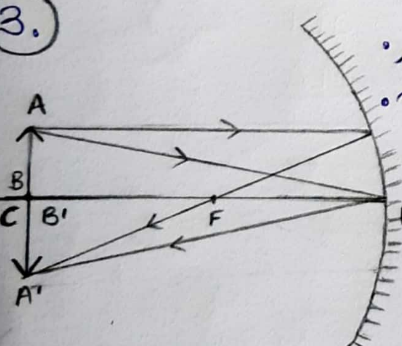

- $u \rightarrow$ infinity
- $v \rightarrow$ focus
- real & inverted
- point sized image

2.



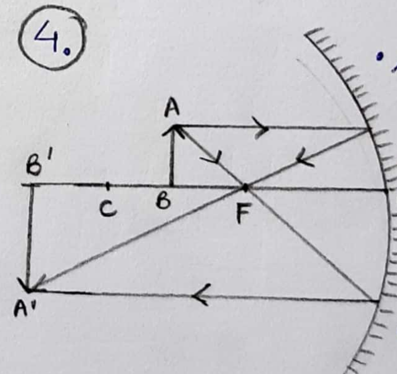

- $u \rightarrow$ beyond C
- $v \rightarrow$ between F & C
- real & inverted
- diminished image

3.



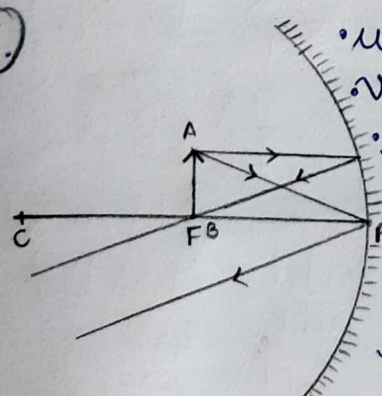

- $u \rightarrow$ curvature
- $v \rightarrow$ curvature
- real & inverted
- same sized image

4.



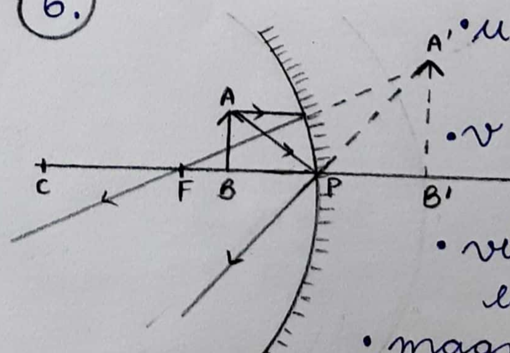

- $u \rightarrow$ between F and C
- $v \rightarrow$ beyond C
- real & inverted
- magnified image

5.




- $u \rightarrow F$
- $v \rightarrow \infty$
- real and inverted
- highly enlarged image

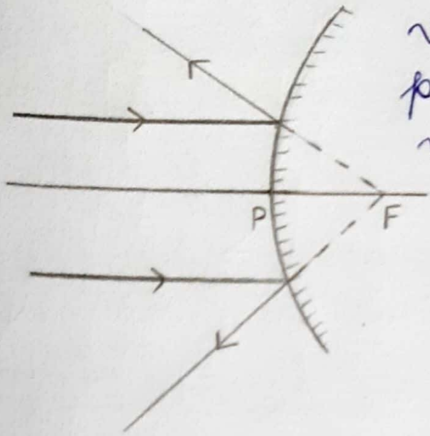
6.




- $u \rightarrow$ between F and P
- $v \rightarrow$ behind the mirror
- virtual and erect
- magnified image

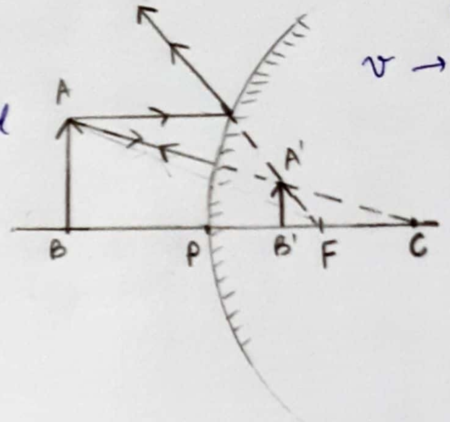
## Ray Diagrams due to convex mirror

1.




$u \to \infty$
$v \to F$
point sized virtual and erect image

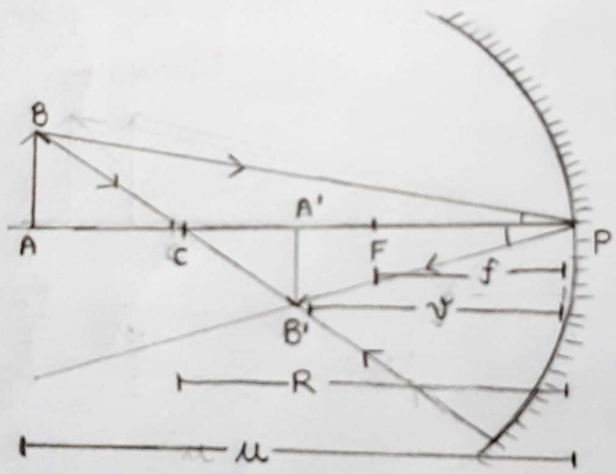
2.




$u \to$ ~~beyond~~ b/w P and F
$v \to$ b/w P and F
diminished virtual and erect image

# MIRROR FORMULA

## I] Due to concave mirror




In $\Delta APB$ and $\Delta A'PB'$

$\angle BAP = \angle PA'B'$ (90°)

$\angle \cancel{ABPB} = \angle A'PB'$

$\because \Delta APB \approx \Delta A'PB'$

$$\frac{AB}{A'B'} = \frac{PA}{PA'} \quad \text{——— (1)}$$

In $\Delta ACB$ and $\Delta A'CB'$

$\angle BAC = \angle B'A'C$

$\angle ACB = \angle A'CB'$

$\Delta ACB \approx \Delta A'CB'$

$$\frac{AB}{A'B'} = \frac{AC}{A'C} \quad \text{——— (2)}$$

from (1) and (2)

$$\frac{PA}{PA'} = \frac{AC}{A'C} \Rightarrow \frac{-u}{-v} = \frac{-u+R}{-R+v}$$

$[PA = -u, \; PA' = -v, \; AC = -u+R, \; A'C = -R+v]$

$$uR - uv = uv - vR$$

$$2uf - uv = uv - 2vf \qquad [\because R = 2f]$$

$$\cancel{2}uf = \cancel{2}uv - \cancel{2}vf$$

$$uf = uv - vf$$

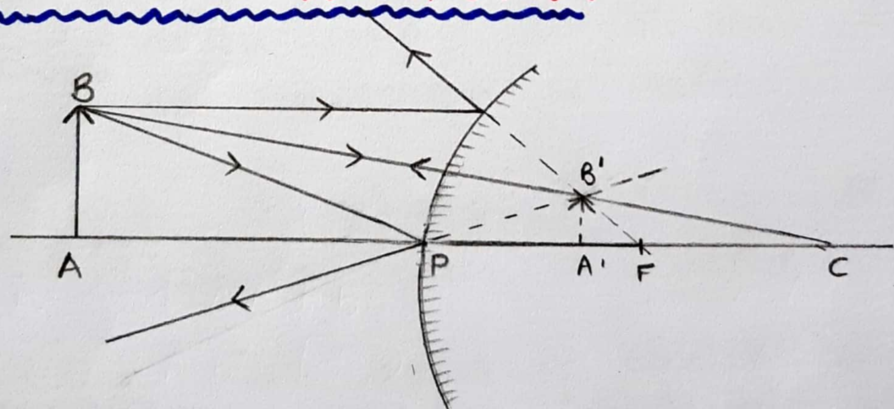
dividing both sides with $uvf$

$$\frac{\cancel{u}\cancel{f}}{\cancel{u}v\cancel{f}} = \frac{\cancel{u}\cancel{v}}{\cancel{u}\cancel{v}f} - \frac{\cancel{v}\cancel{f}}{u\cancel{v}\cancel{f}}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

## II Due to convex mirror




The triangles $\Delta A'B'P$ and $ABP$ are similar

$$\because \frac{A'B'}{AB} = \frac{PA'}{PA} \quad \text{——— (1)}$$

Also the $\Delta A'B'C$ and $ABC$ are similar

$$\because \frac{A'B'}{AB} = \frac{A'C}{AC} = \frac{PC - PA'}{PC + PA} \quad \text{——— (2)}$$

From equations (1) and (2)

$$\frac{PA'}{PA} = \frac{PC - PA'}{PC + PA}$$

$$\frac{+v}{+(-u)} = \frac{+R - (v)}{+R + (-u)}$$

$$-\frac{v}{u} = \frac{R - v}{R - u}$$

$$-vR + uv = Ru - uv$$

Dividing by $uvR$

$$-\frac{1}{u} + \frac{1}{R} = \frac{1}{v} - \frac{1}{R}$$

$$\frac{2}{R} = \frac{1}{v} + \frac{1}{u}$$

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

$R = 2f$

$$\because \frac{2}{R} = \frac{\cancel{2}}{\cancel{2}f}$$

**Real Image** - An image which can be obtained on the screen is called a real image.

**Virtual Image** - An image that cannot be obtained on a screen is called a virtual image.

**Magnification** - The ratio of size of image to the size of object is called magnification.

$$\boxed{M = \frac{h_i}{h_o} = \frac{-v}{u}}$$

- Magnification of real image is positive.
- Magnification of virtual image is negative.

**Refraction** – When light goes from one transparent medium to another, it deviates from its path, this phenomenon is refraction.

**Cause** – Difference in speed of light in different mediums.

**Laws of refraction** –

1. Incident ray, refracted ray and the normal to the interface at the point of incidence, all lie in the same plane.
2. The ratio of sine of angle of incidence to the sine of angle of refraction is constant [Snell's Law]

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \mu_{21}.$$

- $\mu$ depends on –

1. Temperature (inversely)
2. Material (directly)
3. Wavelength (inversly)

**Refraction through a glass slab**

For direct light

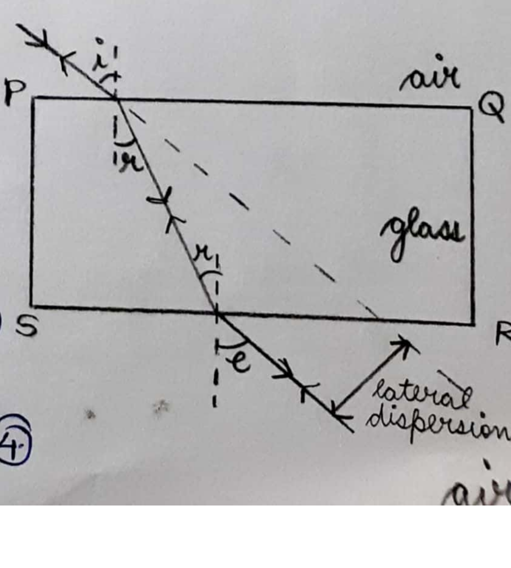
at surface PQ, $\mu_{ga} = \dfrac{\sin i}{\sin r}$ —— (1)

at surface RS, $\mu_{ag} = \dfrac{\sin r}{\sin e}$ —— (2)

For reflected light

at surface RS, $\mu_{ga} = \dfrac{\sin e}{\sin r}$ —— (3)

at surface PQ, $\mu_{ag} = \dfrac{\sin r}{\sin i}$ —— (4)



For equations 1 and 4

$$\boxed{\mu_{ga} = \frac{1}{\mu_{ag}}}$$

For equations 2 and 3

$$\frac{\sin i}{\sin r} = \frac{\sin e}{\sin r} = \boxed{\angle i = \angle e}$$

Provided : 1. Refracting surfaces are parallel to each other.

2. Incident and emergent ray are in same medium.

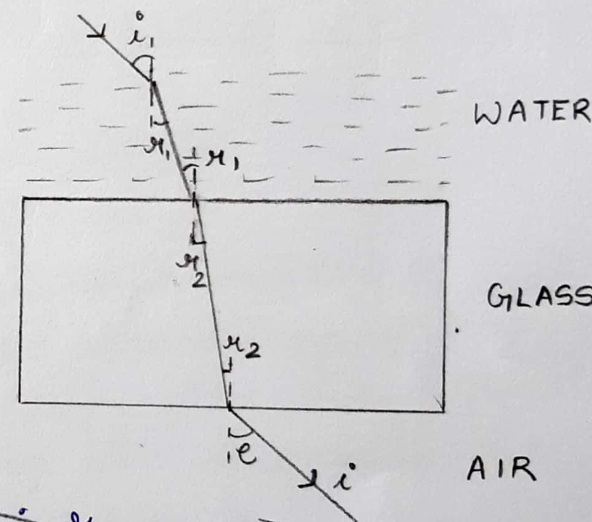
## Refraction through multiple media

$$\mu_{wa} = \frac{\sin i}{\sin r_1} \quad \text{—— 1}$$

$$\mu_{gw} = \frac{\sin r_1}{\sin r_2} \quad \text{—— 2}$$

$$\mu_{ag} = \frac{\sin r_2}{\sin e} = \frac{\sin r_2}{\sin i} \quad \text{—— 3}$$

$$(\because \angle i = \angle e)$$




1 × 2 × 3

$$\mu_{wa} \times \mu_{gw} \times \mu_{ag} = \frac{\cancel{\sin i}}{\cancel{\sin r_1}} \times \frac{\cancel{\sin r_1}}{\cancel{\sin r_2}} \times \frac{\cancel{\sin r_2}}{\cancel{\sin i}} = 1$$

$$\mu_{wa} \times \mu_{gw} = \frac{1}{\mu_{ag}}$$

$$\mu_{wa} \times \mu_{gw} = \mu_{ga}$$

$$\mu_{ba} \times \mu_{cb} \times \mu_{dc} \times \mu_{ed} = \mu_{ea} \quad [\angle i = \angle e]$$

**Optical Density** – Ratio of speed of light in two media. Eg – terpentine and water.

## Real and apparent depth-

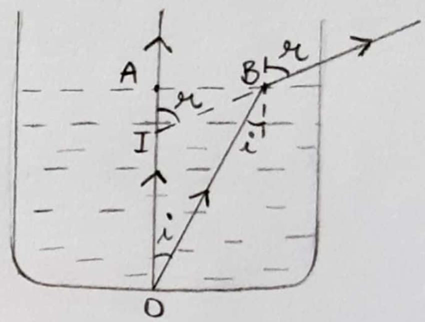
In $\Delta OAB$,

$$\sin i = \frac{AB}{OB} = \frac{AB}{OA} \quad (\because i \text{ is small})$$

In $\Delta IAB$

$$\sin r = \frac{AB}{BI} = \frac{AB}{AI} \quad (\because r \text{ is small})$$

$$\mu_{aw} = \frac{\sin i}{\sin r} = \frac{AB/OA}{AB/AI}$$



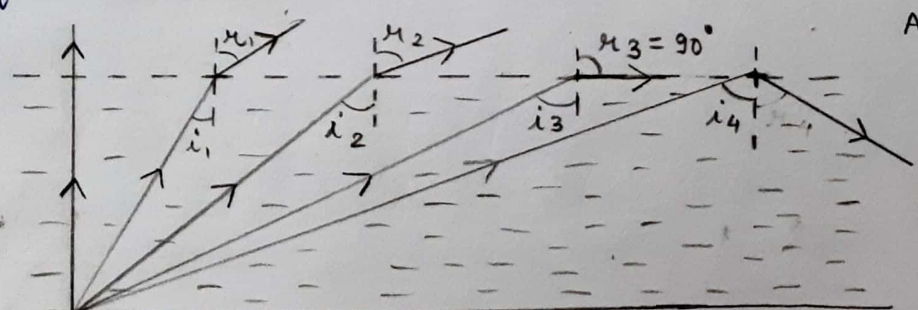

$$_w\mu_a = \frac{AI}{OA} = \frac{\text{Apparent}}{\text{Real}}$$

$$_a\mu_w = \frac{OA}{AI} = \frac{\text{Real}}{\text{Apparent}}$$

$$\boxed{\mu = \frac{\text{real depth}}{\text{apparent depth}}}$$

**Critical Angle-** It is the angle of incidence subtended by a ray of light travelling from denser to rarer for which refracted ray travels along the surface separating the 2 media i.e. for which angle of refraction equals 90°.

**Total Internal Reflection-** When light travels from denser to rarer medium above a certain angle of incidence it will reflect back into the same medium.



$${}_{w}\mu_{a} = \frac{\sin i}{\sin r}$$

when $i = i_c$ then $r = 90°$

$${}_{w}\mu_{a} = \frac{\sin i_c}{\sin 90°}$$

$${}_{w}\mu_{a} = \frac{\sin i_c}{1}$$

$${}_{a}\mu_{w} = \frac{1}{{}_{w}\mu_{a}} = \frac{1}{\sin i_c} \Rightarrow \boxed{\mu = \frac{1}{\sin i_c}}$$

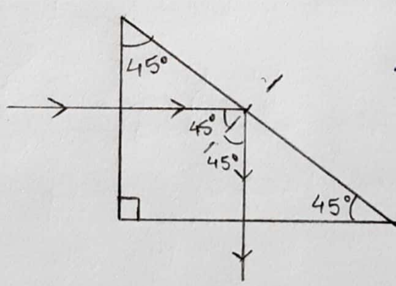
**Conditions -**

1. $\angle i > \angle i_c$
2. Light should travel from denser to rarer.

## Applications of Total Internal Reflection -

**1. Prism**

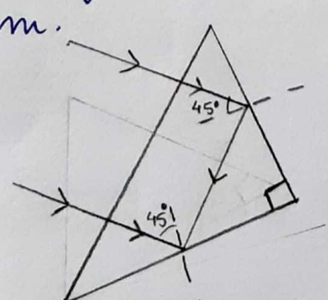
(i) To turn a ray of light by 90° using right angled prism.




glass $\mu = 1.5$

$i_c = 42°$

(ii) To turn a ray of light by 180° using right angled prism.

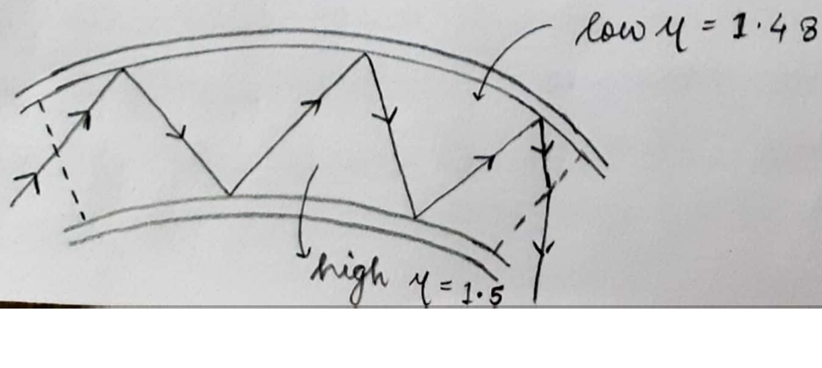



**2. Brilliance of Diamond -**

This brilliance is due to the total internal reflection of light inside them, $i_c = 2.4°$ is very small therefore once light enters a diamond, it is very likely to undergo TIR inside it. By cutting at the diamond suitably, multiple TIR's can be made to occur.

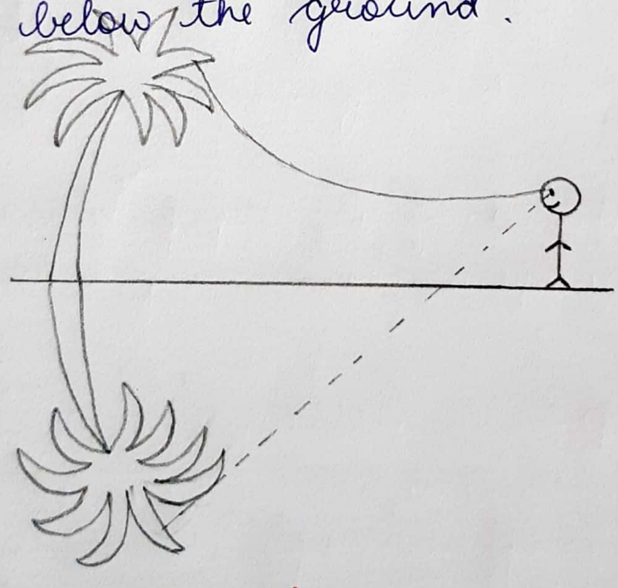
3. Optical Fibre -

- These are fabricated with high quality composite glass/quartz fibres Each fibre consists of a core and cladding. $\mu$ of material of core > $\mu$ of material of cladding. When a signal in the form of light is directed at one end of fibre at a suitable angle, it undergoes repeated TIRs along the fibre's length and comes out at the other end.
- Since light undergoes TIR at each, no appreciable intensity is lost.
- Used for transmitting and receiving electrical signal.
- Used as 'light pipes' to facilitate visual examination of internal organs like oesophagus, stomach and intenstines.
- Requirements : very little absorption of light as it travels long distance can be done by purification and special preparation of materials like quartz.
- Example - Silica glass fibres : 95% light - 1 km



## 4. Mirage -

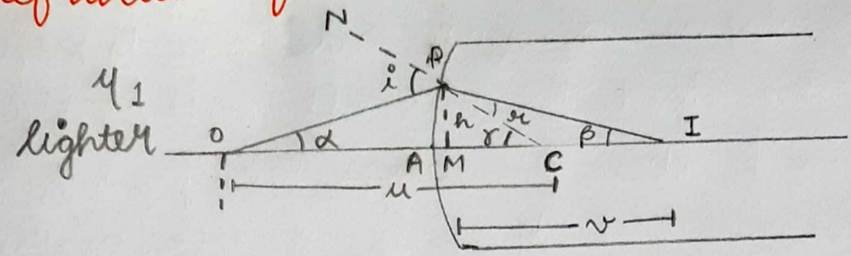
- On hot summer days, air near ground becomes hotter than the air at higher levels. Refractive index of air increases with ↑ in density. Hotter air is less dense and has smaller than cooler air is still, the optical density at different layers of air increases with height.
- As a result, light from tall object passes through a medium whose $\mu$ decreases towards the ground.
- Thus, a ray of light from such an object successively bends away from the normal and undergoes TIR if the $\angle i$ for air near ground exceeds $i_c$.
- To a distant observer, light appears to come from somewhere below the ground.



## Examples of refraction-

1. Twinkling of stars.
2. Early sunrise and sunset.
3. Stars appear higher than they are.
4. Straight rod appears bent in water.
5. Fountain of fire.

# Refraction from convex spherical surface –




$$\tan\alpha = \frac{PM}{MO}$$

$$\tan\beta = \frac{PM}{MI}$$

$$\tan\gamma = \frac{PM}{MC}$$

$$i = \alpha + \gamma \qquad r = \gamma + \beta$$

Snell's Law

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

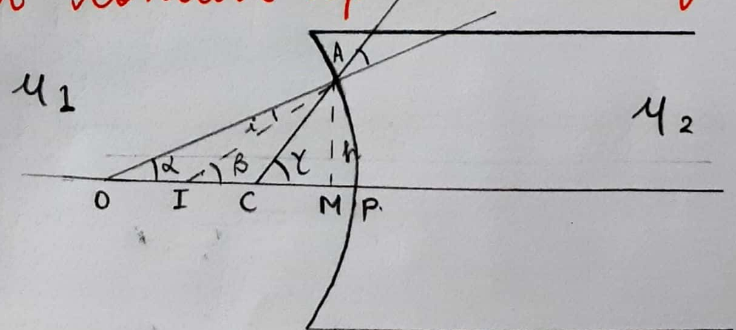
$$\frac{\alpha+\gamma}{\gamma+\beta} = \frac{\mu_2}{\mu_1}$$

$$\left(\frac{PM}{MO} + \frac{PM}{MC}\right)\mu_1 = \left(\frac{PM}{MC} + \frac{PM}{MI}\right)\mu_2$$

$$\left(\frac{1}{-u} + \frac{1}{R}\right)\mu_1 = \left(\frac{1}{R} - \frac{1}{v}\right)\mu_2$$

$$\boxed{\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}}$$

- From concave spherical surface –



$\alpha + i = \gamma$

$\tan\gamma + (-\tan\alpha) = \gamma$ —— ① [$\because \alpha$ & $\gamma$ are small]

$r + \beta = \gamma$

$\tan\gamma - \tan\beta = r$ —— ②

$\mu_1 \sin i = \mu_2 \sin r$

$\mu_1(\tan\gamma - \tan\alpha) = \mu_2(\tan\gamma - \tan\beta)$

$$\mu_1\left(\frac{AM}{MC} - \frac{AM}{MO}\right) = \mu_2\left(\frac{AM}{MC} - \frac{AM}{MI}\right)$$

$$\mu_1\left(\frac{1}{MC} - \frac{1}{MO}\right) = \mu_2\left(\frac{1}{MC} - \frac{1}{MI}\right)$$

$$\mu_1\left(-\frac{1}{R} - \left(-\frac{1}{u}\right)\right) = \mu_2\left(-\frac{1}{R} - \left(-\frac{1}{v}\right)\right)$$

$$-\frac{\mu_1}{R} + \frac{\mu_1}{u} = -\frac{\mu_2}{R} + \frac{\mu_2}{v}$$

$$\frac{\mu_2 - \mu_1}{R} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$$

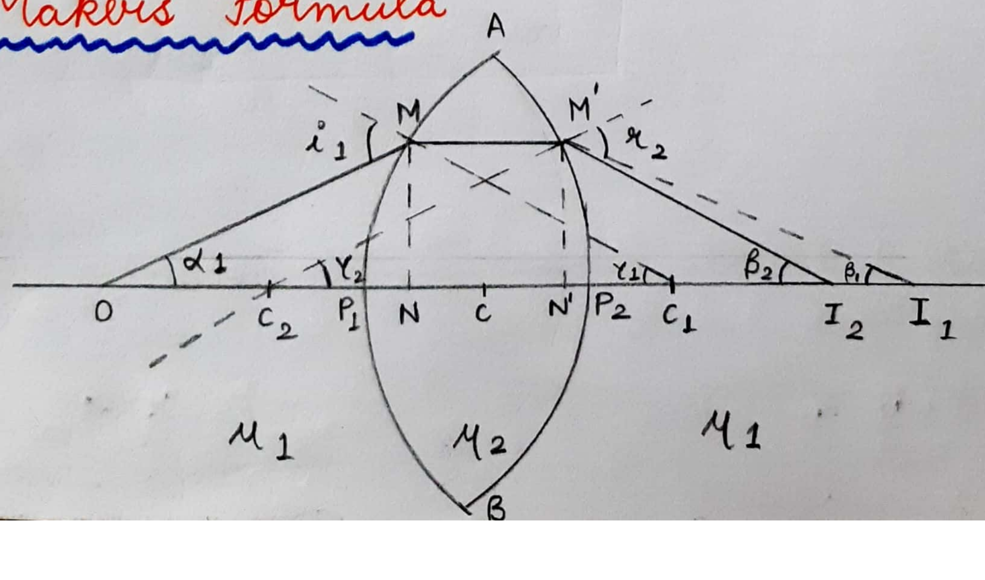
replace $\mu_1$ by $\mu_2$ and $\mu_2$ by $\mu_1$ and we'll get

$$\frac{\mu_1 - \mu_2}{R} = \frac{\mu_1}{v} - \frac{\mu_2}{u}$$

Dividing by $\mu_1$

$$\frac{1-\mu}{R} = \frac{1}{v} - \frac{\mu}{u} \quad \left[\because \frac{\mu_2}{\mu_1} = \mu\right]$$

## Lens Makers Formula



## Case I

First we consider beyond $AP_1B$ surface onto the right of it. $\mu_2$ only is extended i.e. there is no presence of $AP_2B$ surface.

In $\Delta OMC$, $i_1 = \alpha_1 + \gamma_1$

$$i_1 = \tan\alpha_1 + \tan\gamma_1$$

$$= \frac{MN}{ON} + \frac{MN}{NC_1}$$

$$= \frac{MN}{OC} + \frac{MN}{CC_1}$$

In $\Delta I_1MC_1$

$$\gamma_1 = \beta_1 + r_1$$

$$r_1 = -\beta_1 + \gamma_1 = -\tan\beta_1 + \tan\gamma_1$$

As light travelling from rarer to denser medium

$$\mu_1 \sin i_1 = \mu_2 \sin r_1$$

$$\mu_1 i_1 = \mu_2 r_1$$

$$\mu_1\left(\frac{MN}{CO} + \frac{MN}{CC_1}\right) = \mu_2\left(\frac{MN}{CC_1} - \frac{MN}{CI_1}\right)$$

$$\mu_1\left(\frac{1}{CO} + \frac{1}{CC_1}\right) = \mu_2\left(\frac{1}{CC_1} - \frac{1}{CI_1}\right)$$

$$\mu_1\left(-\frac{1}{u} + \frac{1}{R_1}\right) = \mu_2\left(\frac{1}{R_1} - \frac{1}{v_1}\right)$$

$$-\frac{\mu_1}{u} + \frac{\mu_1}{R_1} = \frac{\mu_2}{R_1} - \frac{\mu_2}{v_1}$$

$$\left(\frac{\mu_2 - \mu_1}{R_1}\right) = \frac{\mu_2}{V_1} - \frac{\mu_1}{u} \quad \text{——— (1)}$$

## Case II

Now we consider the presence of $AP_2B$ surface for which $I_1$ behaves as virtual object and ray of light travels from denser to rarer medium forming the final image at $I_2$

In $\Delta I_1 M' C_2$

$$i_2 = \beta_1 + \gamma_2$$
$$= \tan\beta_1 + \tan\gamma_2$$
$$= \frac{M'N'}{N'I_1} + \frac{M'N'}{N'C_2}$$
$$= \frac{M'N'}{CI_1} + \frac{M'N'}{C_1C_2}$$

In $\Delta I_2 M' C_2$

$$r_2 = \beta_2 + \gamma_2$$
$$= \tan\beta_2 + \tan\gamma_2$$
$$= \frac{M'N'}{N'I_2} + \frac{M'N'}{N'C_2}$$
$$= \frac{M'N'}{CI_2} + \frac{M'N'}{CC_2}$$

As light is travelling from denser to rarer medium.

$$\mu_2 \sin i_2 = \mu_1 \sin r_2$$
$$\mu_2(\tan\gamma_2 + \tan\beta_1) = \mu_1(\tan\beta_2 + \tan\gamma_2)$$
$$\mu_2\left(\frac{M'N'}{CC_2} + \frac{M'N'}{CI_1}\right) = \mu_1\left(\frac{M'N'}{CC_2} + \frac{M'N'}{CI_2}\right)$$
$$\mu_2\left(\frac{1}{CC_2} + \frac{1}{CI_2}\right) = \mu_1\left(\frac{1}{CC_2} + \frac{1}{CI_2}\right)$$
$$\mu_2\left(-\frac{1}{R_2} + \frac{1}{V_1}\right) = \mu_1\left(\frac{-1}{R_2} + \frac{1}{V_1}\right)$$
$$-\frac{\mu_2}{R_2} + \frac{\mu_2}{V_1} = -\frac{\mu_1}{R_2} + \frac{\mu_1}{V}$$
$$\boxed{-\left(\frac{\mu_2 - \mu_1}{R_2}\right) = -\frac{\mu_2}{V_1} + \frac{\mu_1}{V}} \quad \text{———} \quad (2)$$

Adding (1) and (2) we get

$$\left(\frac{\mu_2 - \mu_1}{R_1}\right) - \left(\frac{\mu_2 - \mu_1}{R_2}\right) = \cancel{\frac{\mu_2}{V_1}} - \frac{\mu_1}{u} - \cancel{\frac{\mu_2}{V_1}} + \frac{\mu_1}{V}$$
$$\boxed{(\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \mu_1\left(\frac{1}{V} - \frac{1}{u}\right)} \quad \text{—} \quad (3)$$

Dividing by $\mu$, we get

$$\boxed{\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{v}-\frac{1}{u}} \quad \text{——— (4)}$$

when $u=\infty$ ; $v=f$

$$(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f}-\frac{1}{\infty}$$

$$(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f} \quad \text{——— (5)}$$

when $u=-f$ then $v=\infty$

$$(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{\infty}-\left(-\frac{1}{f}\right)$$

$$(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{f} \quad \text{——— (6)}$$

From equations (4) (5) and (6)

$$\boxed{(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{v}-\frac{1}{u}=\frac{1}{f}}$$

## Power of a lens –

It gives the degree or extent of ~~cove~~ convergence or divergence of parallel rays of light incident on the lens.

Mathematically, it is equal to reciprocal of focal length.

$$\boxed{P=\frac{1}{f}}$$

unit : D ( dioptre when $f$ in m)

concave lens : negative

convex lens : positive

## Combination of Lenses –

- When two lenses of focal length $f_1$ and $f_2$ are in contact with each other then,

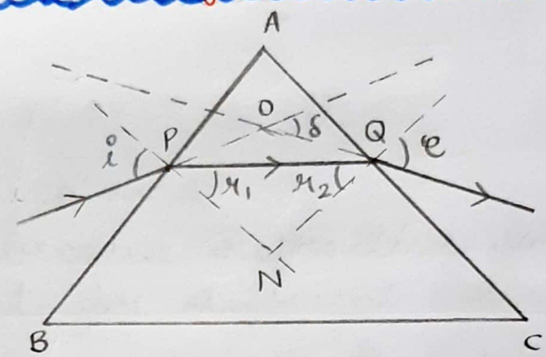
$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$P = P_1 + P_2$$
$$m = m_1 \times m_2$$

- When two lenses are d distance apart,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$
$$P = P_1 + P_2 - d P_1 P_2$$
$$\boxed{m = m_1 \times m_2}$$

## Refraction due to glass prism




Using exterior angle property

$$\delta = i - r_1 + e - r_2$$
$$\delta = i + e - (r_1 + r_2) \quad \text{—— (1)}$$

In $\triangle NPQ$

$$\angle N + r_1 + r_2 = 180°$$

In $\square$ APNQ

$$\angle N + \angle A = 180°$$
$$\angle A = r_1 + r_2 \quad \text{—— (2)}$$
$$\delta = i + e - \angle A \quad \text{—— (3)}$$

$\mu \rightarrow r \cdot i$ using Snell's Law

$$\mu = \frac{\sin i}{\sin r} = \frac{i}{r_1}$$

$i = \mu r_1 \qquad e = \mu r_2$

$$\delta = \mu (r_1 + r_2) - A$$
$$\delta = \mu A - A$$
$$\boxed{\delta = A(\mu - 1)}$$

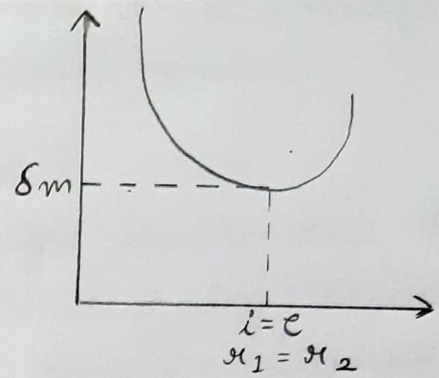
$\delta \rightarrow$ minimum then,

$$i = e \ , \ r_1 = r_2$$

$$A = 2r \qquad \delta_m = 2i - A$$

$$\boxed{r = \frac{A}{2}} \qquad \boxed{i = \frac{\delta_m + A}{2}}$$

$$\boxed{\mu = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$$




**Dispersion** -
Splitting of light into its constituent colours.

- It occurs because refractive index of medium is different for different colours.
- Inversly proportional to wavelength.

**Angular dispersion** - Difference of angle of deviation of two extreme colours.

$$\delta_v - \delta_r = (\mu_v - 1)A - (\mu_r - 1)A$$
$$= (\mu_v - \mu_r)A$$

- depends on A and material of prism.

**Dispersive power** - The ratio of angular deviation and mean deviation.

$$\omega = \frac{(\mu_v - \mu_r)\cancel{A}}{(\mu - 1)\cancel{A}}$$

- depends on material of prism

**Scattering** - spreading of light

**Examples** -
- blue colour of sky
- reddish sun during sunrise & sunse
- white colour of clouds

# Optical Instruments –



## 1. Simple Microscope – Uses convex lens to magnify

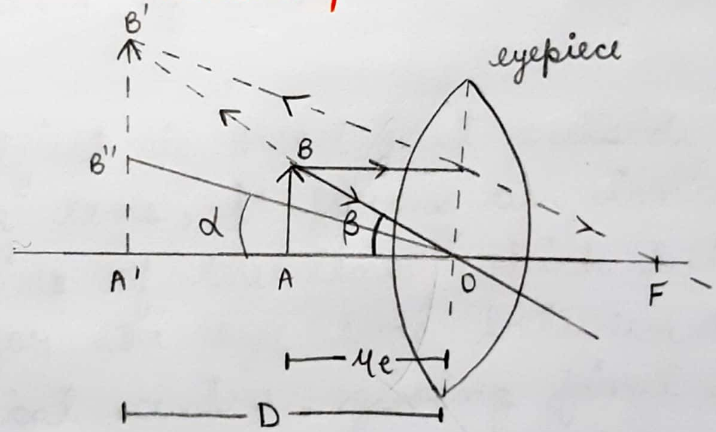
Principle – When an object is placed between the pole and the focus of a convex lens, it forms virtual, magnified and erect image at the least distance of distinct vision.

★ Magnifying Power –

Defined as the ratio of angle subtended by image at eye to angle subtended by object at eye when both of them are considered at the least distance of distinct vision.

$$m = \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha}$$

### I] Image at near point




In $\Delta AOB$

$$\tan\beta = \frac{AB}{-u_e} \quad \text{——— (1)}$$

In $\Delta A'OB''$

$$\tan\alpha = \frac{A'B''}{D} = \frac{AB}{-D} \quad \text{——— (2)}$$

$$m = \frac{\beta}{\alpha} = \frac{AB}{-u_e} \times \frac{-D}{AB} = \frac{D}{u_e}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f_e} = \frac{-1}{D} - \frac{1}{-u_e}$$

$$\frac{1}{f_e} + \frac{1}{D} = \frac{1}{u_e} \times D$$

$$\boxed{\frac{D}{f_e} + 1 = m}$$

II] Image at infinity

$$f_e = u_e$$

$$m = \frac{D}{f_e}$$

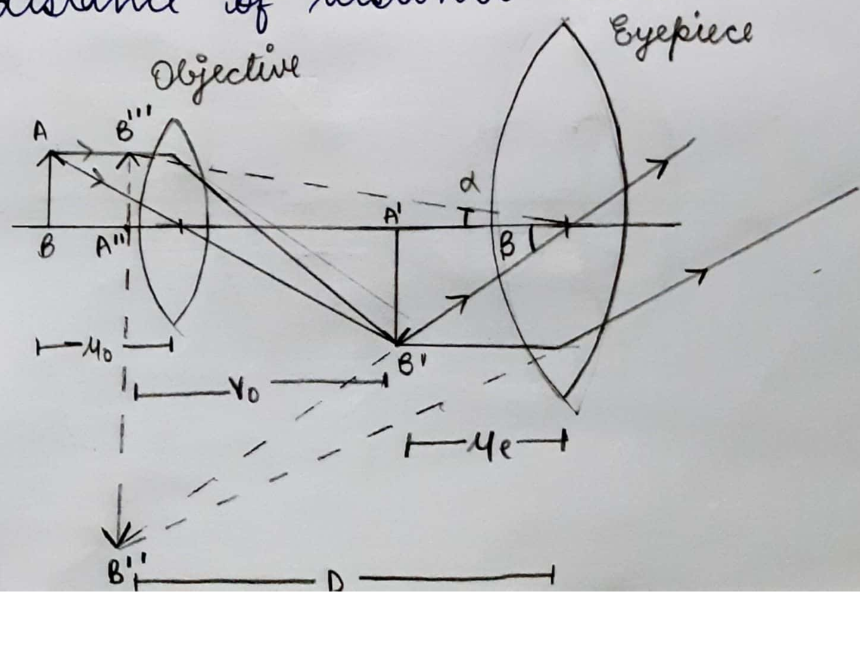
- Simple microscope has limited magnification.

② Compound Microscope -

It consists of two lens → objective and eyepiece

Principle -

When object is held just outside the focus of objective lens, it forms an image on the other side of the lens which behaves as an object for the eye lens between its focus and the optical centre, giving final image at the least distance of distinct vision.



In $\Delta A''OB''$

$$\tan\beta = \frac{A''B''}{-D}$$

In $\Delta A''OB'''$

$$\tan\alpha = \frac{-AB}{-D}$$

$$m = \frac{\tan\alpha}{\tan\beta}$$

$$m = \frac{-A''B''}{AB} \times \frac{A'B'}{A'B'} = m_e \cdot m_o$$

$$m = \frac{v_e}{u_e} \times \frac{v_o}{u_o}$$

$$m = \frac{-v_o}{u_o} \times \frac{D}{u_e} \quad \text{------ (1)}$$

- when at LDDV

$$\frac{1}{f_e} = \frac{1}{-D} - \frac{1}{-u_e}$$

$$\frac{1}{f_e} + \frac{1}{D} = \frac{1}{u_e}$$

$$\frac{D}{f_e} + 1 = \frac{D}{u_e} \quad [\text{multiple by } D]$$

put in (1)

$$\boxed{m = \frac{-v_o}{u_o}\left(1 + \frac{D}{f_e}\right)}$$

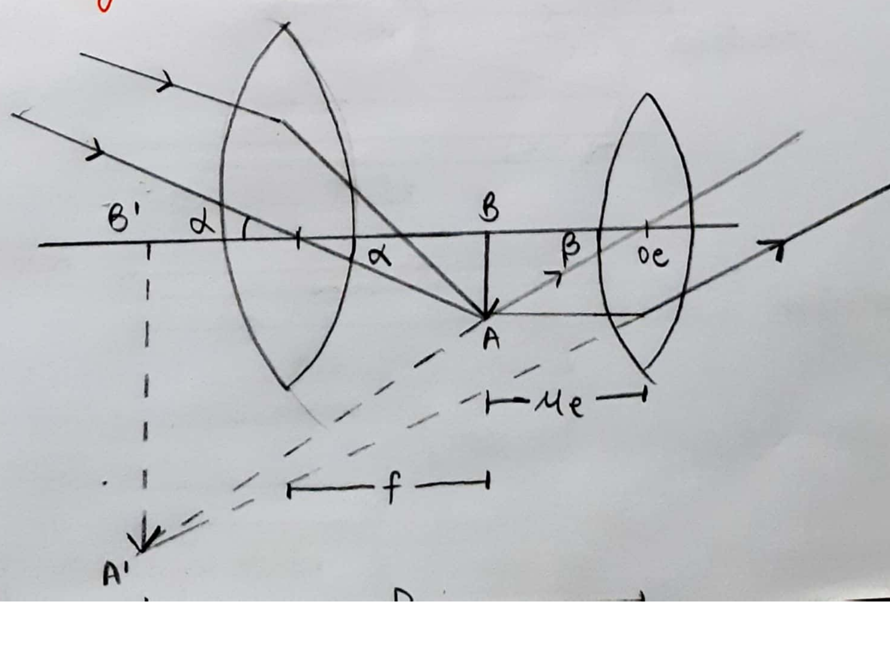
- when at $\infty$

$u_e = f_e$

$$\boxed{m = \frac{-v_o}{u_o} \times \frac{D}{f_e}}$$

## (3) Astronomical Telescope -

### (i) Refracting telescope



In $\Delta AOB$

$$\tan\alpha = \frac{AB}{f}$$

In $\Delta AO_eB$

$$\tan\beta = \frac{AB}{-u_e}$$

$$m = \frac{\beta}{\alpha}$$

$$m = \frac{-f_0}{u_0}$$

- when at LDDV

$$\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$$

$$\frac{1}{f_e} = \frac{1}{-D} - \frac{1}{u_e}$$

$$\frac{1}{f_e} + \frac{1}{D} = \frac{1}{u_e}$$

$$m = -f_0\left[\frac{1}{f_e} + \frac{1}{D}\right]$$

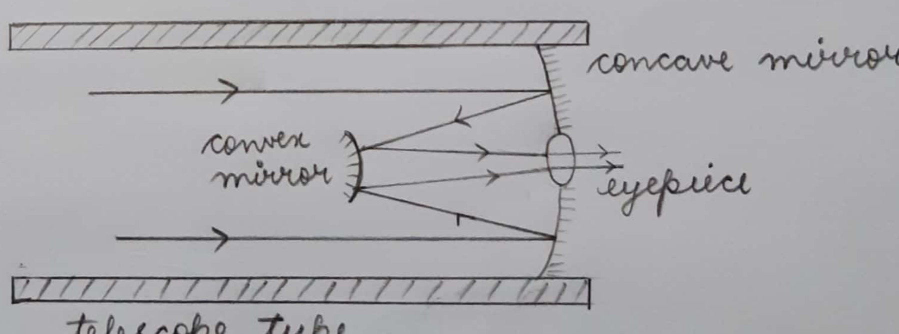
$$\boxed{m = -\frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]}$$

- when image is at $\infty$

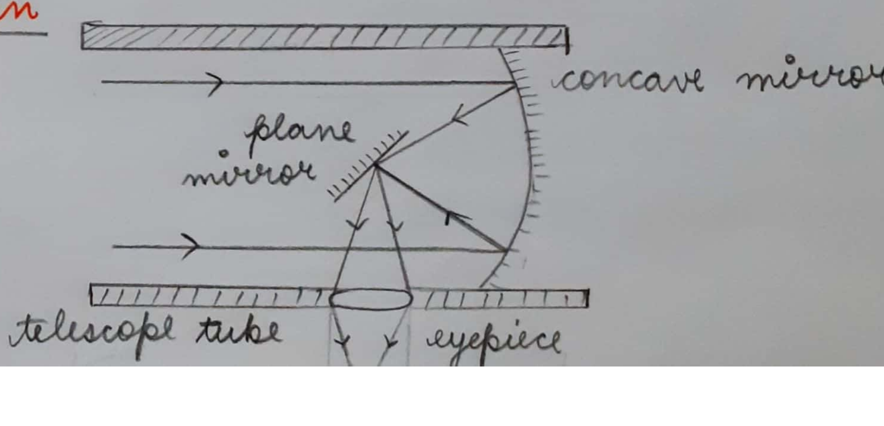
$$\boxed{m = \frac{-f_0}{f_e}}$$

## (ii) Reflecting telescope

- Cassegrain




- Newtonian



## Resolving power -

- The ability of an optical instrument to produce distinctly separate image of two close objects.
- The minimum distance between two objects which can be seen as separate is limit of resolution.

$$\text{Resolving power} \propto \frac{1}{\text{Limit of resolution}}$$

# ELECTROMAGNETIC WAVES

## DISPLACEMENT CURRENT-

Current that flows due to change in electric field is known as displacement current.

$$I = \frac{dq}{dt}$$

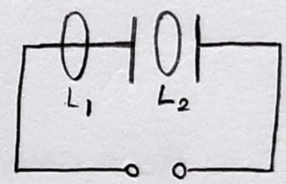
$$\phi_e = \frac{q}{\varepsilon_0} \qquad q = \phi_e \varepsilon_0$$

$$I = \frac{d}{dt} \phi_e \varepsilon_0$$

$$\boxed{I_D = \varepsilon_0 \frac{d\phi_e}{dt}}$$

- The magnitude of $I_D$ = magnitude of conduction current

## MAXWELL'S MODIFICATION OF ACL




$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (i_c + i_D)$$

$$\oint_{L_1} \vec{B} \cdot \vec{dl} = \mu_0 i_c \text{ [outside capacitor, } i_D = 0]$$

$$\oint_{L_2} \vec{B} \cdot \vec{dl} = \mu_0 \cdot \varepsilon_0 \frac{d\phi}{dt} \text{ [inside capacitor, } i_c = 0]$$

## FOUR EQUATIONS OF ELECTROMAGNETISM

① Gauss theorem for electrostats

→ To find flux and EF

$$\oint \vec{E} \cdot \vec{dA} = \frac{Q_{en.}}{\varepsilon_0}$$

② Gauss theorem for magnetism

→ To find flux and MF

$$\oint \vec{B} \cdot \vec{dA} = 0$$

3. Maxwell's Ampere Circuital Law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 \left( i_c + \varepsilon_0 \frac{d\phi_e}{dt} \right)$$

→ This proves changing electric flux creates MF.

4. Faraday's law of electromagnetic induction

→ $$\oint E \cdot dI = - \frac{d\phi_B}{dt}$$

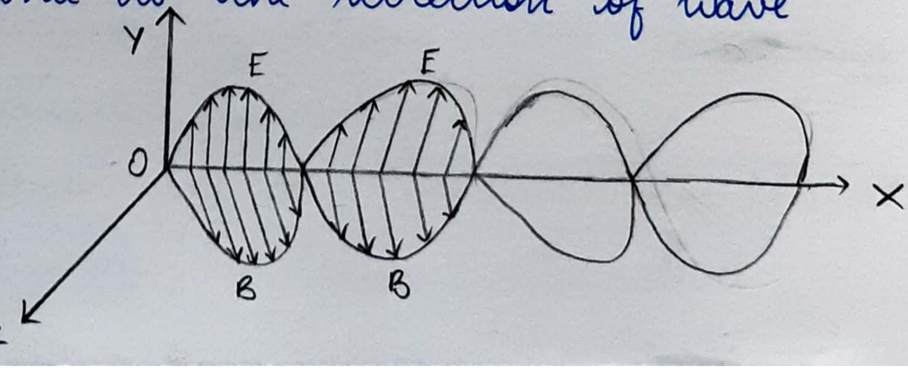
changing magnetic flux creates electric flux.

## ELECTROMAGNETIC WAVES

A wave radiated by an accelerated or oscillatory charge in which varying magnetic field is the source of electric field and varying electric field is the source of magnetic field.

## CHARACTERISTICS OF EM WAVES

1. The energy in EMW is divided on average equally between electric and magnetic fields.
2. The waves are transverse in nature.
3. EMW carry energy and exert force and pressure
4. EMW are not deflected by electric & magnetic field
5. EMW are transverse in nature i.e. electric field and magnetic fields are perpendicular to each other and to the direction of wave propagation.



# ELECTROMAGNETIC SPECTRUM

The systematic sequential distribution of EMW in ascending or descending order of frequency or wavelength is known as electromagnetic spectrum.

## 1. Radio Wave -

- Wavelength range - $>0.1$ m
- Frequency range - $10^4 - 10^9$ Hz
- Production - Rapid acceleration and decelaration of $e^-$.
- Detection - Receiver's aerials
- Uses - (i) In radio and TV communication
  (ii) In astronomical field

## 2. Microwaves

- Wavelength range - 0.1 m - 1 mm
- Frequency range - $10^9 - 10^{11}$
- Production - Klystron valve or magnetron valve
- Detection - Point contact diodes
- Uses - (i) In RADAR communication
  (ii) For cooking purpose

## 3. Infrared wave

- Wavelength range - 1 mm - 700 nm
- Frequency range - $3 \times 10^{11} - 4 \times 10^{14}$
- Production - Vibration of atoms and molecules
- Detection - Thermopile, Bolometer
- Uses - (i) In treatment of muscular complaints
  (ii) In knowing molecular structure

## 4. Visible rays

- Wavelength range - 700 nm to 400 nm
- Frequency range - $4 \times 10^{14} - 8 \times 10^{14}$ Hz
- Production - Electrons in atoms emit light when they move from one energy level to a lower energy level.
- Detection - The eye, photocells, photographic film
- Uses - (i) To see things
  (ii) In optical instruments.

## 5. Ultraviolet Rays

- Wavelength range - 400 nm - 1 nm
- Frequency range - $8 \times 10^{14} - 8 \times 10^{16}$ Hz
- Production - Inner shell $e^-$ in atoms moving from one energy level to a lower level.
- Detection - Photocells, photographic film
- Uses - (i) In burglar alarm
  (ii) To kill germs in minerals

## 6. X-Rays

- Wavelength range - 1 nm - $10^{-3}$ nm
- Frequency range - $1 \times 10^{16} - 3 \times 10^{21}$
- Production - X-ray tubes or inner shell $e^-$
- Detection - Photographic film, Geiger tubes
- Uses - (i) In medical diagnosis
  (ii) In detecting faults, cracks

## 7. Gamma Rays

- Wavelength range - $< 10^{-3}$ nm
- Frequency range - $5 \times 10^{18} - 5 \times 10^{22}$ Hz
- Production - Radioactive decay of the nucleus
- Detection - Photographic film, ionisation chamber
- Uses - (i) For food preservation by killing pathogenic microorganisms.
  (ii) In radiotherapy for treatment of tumour and cancer.

# IMPORTANT QUESTIONS

1. A radio can tune into any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength? NCERT

**Sol.** For 7.5 MHz band,

$$\text{Wavelength}, \lambda_1 = \frac{c}{\nu} = \frac{3\times10^8}{7.5\times10^6} = 40\ m$$

For 12 MHz band,

$$\text{Wavelength}, \lambda_2 = \frac{c}{\nu} = \frac{3\times10^8}{12\times10^6} = 25\ m$$

So, wavelength range is from 25 m – 40 m.

2. About 5% of the power of a 100 W light bulb is connected to visible radiation. What is the average intensity of visible radiation at
(i) distance of 1 m from the bulb
(ii) distance of 10 m?
Assume that the radiation is emitted isotropically and neglect reflection. NCERT

**Sol.** (i) Intensity, $I = \dfrac{\text{Power of visible light}}{\text{Area}}$

$$= \frac{100\times(5/100)}{4\pi(1)^2}$$

$$= 0.4\ W/m^2$$

(ii) $$I = \frac{100\times\left(\frac{5}{100}\right)}{4\pi(10)^2}$$

$$= 4\times10^{-3}\ W/m^2$$

3. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510\,nT$. What is the amplitude of the electric field part of the wave?

NCERT

Sol. $B_0 = 510\,nT = 510 \times 10^{-9}\,T$

Speed of light in vacuum, $c = \frac{E_0}{B_0}$

where, $E_0$ is the ~~of~~ amplitude of electric field part of the wave.

$$3 \times 10^8 = \frac{E_0}{510 \times 10^{-9}}$$

$$E_0 = 153\ N/C$$

Thus, the amplitude of the electric field part of wave is $153\ N/C$.

# ALTERNATING CURRENT

## ALTERNATING CURRENT -

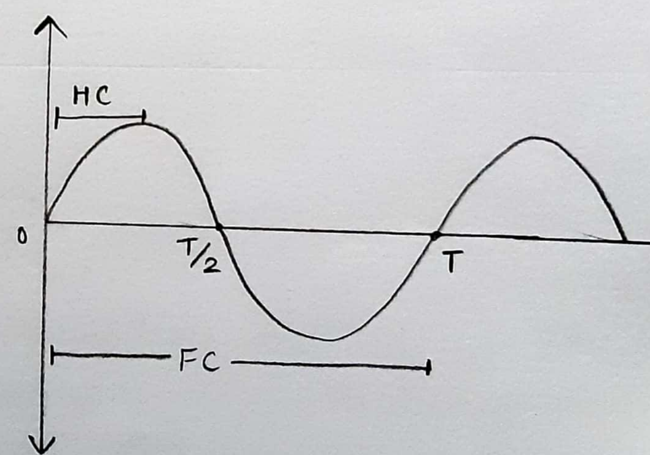
The current whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically.

$$\boxed{I = I_0 \sin \omega t} \Rightarrow I = I_0 \sin 2\pi \nu t$$

where, $\omega$ = angular frequency

$I_0$ = peak value of AC

## AVERAGE OR MEAN VALUE OF AC




$$I = I_0 \sin \omega t$$

$$I_{av.} = \int_0^{T/2} \frac{I dt}{T/2 - 0}$$

$$I_{av.} = \int_0^{T/2} \frac{I_0 \sin \omega t \, dt}{T/2}$$

$$I_{av.} = \frac{2}{T} I_0 \int_0^{T/2} \sin \omega t \, dt$$

$$I_{av.} = \frac{2}{T} I_0 \left[ -\frac{\cos \omega t}{\omega} \right]_0^{T/2}$$

$$I_{av.} = \frac{-2}{\omega T} I_0 \left[ \cos \omega t \right]_0^{T/2}$$

$$I_{av.} = -\frac{2 I_0 T}{2\pi T} \left[ \cos \frac{2\pi}{\cancel{T}} \times \frac{\cancel{T}}{2} - 1 \right]$$

$$I_{av.} = \frac{-I_0}{\pi} [-2]$$

$$\boxed{I_{av.} = \frac{2 I_0}{\pi}}$$

# ROOT MEAN SQUARE VALUE OF CURRENT -

$P = I^2 R$

$I = I_0 \sin \omega t$

$P = (I_0 \sin \omega t)^2 R$

$P = I_0^2 \sin^2 \omega t \, R$

$\frac{dH}{dt} = I_0^2 R \sin^2 \omega t$

$dH = I_0^2 R \sin^2 \omega t \, dt$

$H = \int_0^T I_0^2 R \sin^2 \omega t \, dt$

$H = I_0^2 R \int_0^T \sin^2 \omega t \, dt$

$\sin^2 \theta = \frac{1 - \cos^2 \theta}{2}$

$H = \frac{I_0^2 R}{2} \int_0^T (1 - \cos^2 \omega t) \, dt$

$H = \frac{I_0^2 R}{2} \left[ \int_0^T dt - \cancel{\int_0^T \cos 2\omega t \, dt} \right]$

$H = \frac{I_0^2 RT}{2}$

$AC \rightarrow H_{ac.} = \frac{I_0^2 RT}{2}$

$DC \rightarrow H_{ac.} = I_{RMS}^2 RT$

$\frac{I_0^2 RT}{2} = I_{RMS}^2 RT$

$\boxed{I_{RMS} = \frac{I_0}{\sqrt{2}}}$

$\boxed{V_{RMS} = \frac{V_0}{\sqrt{2}}}$

# INDUCTIVE REACTANCE ($X_L$) -

The effective resistance or opposition offered by the inductor to the flow of current is inductive reactance.

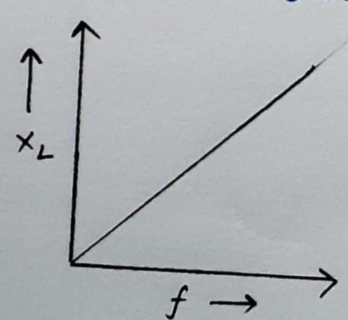
$X_L = \omega L = 2\pi f L$

$\boxed{X_L = (2\pi L) f}$

$X_L \propto f$

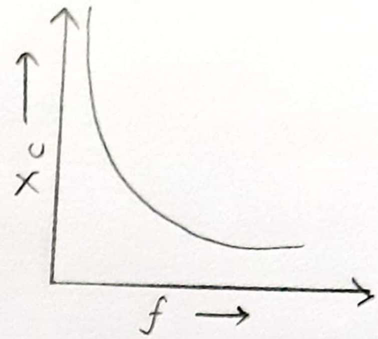
$L$ = self inductance

$[\because 2\pi L = \text{constant}]$



## CAPACITIVE REACTANCE ($X_c$)

The opposing nature of capacitor to the flow of alternating current is called capacitive reactance.




$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$X_c = \left(\frac{1}{2\pi C}\right) \cdot \frac{1}{f} \Rightarrow X_c \propto \frac{1}{f}$$

C = capacitance of AC

[$\because 1/2\pi C$ = constant]

## WATTLESS CURRENT–

The current in purely inductive or capacitive AC circuit when average power consumption in AC circuit is zero is wattless current.

## PHASOR DIAGRAM–

The representation of AC current and voltage by rotating vectors is called phasor diagram.

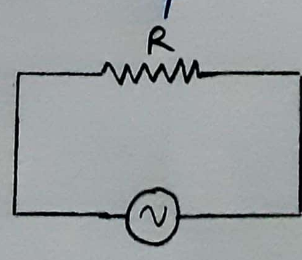
## TYPES OF AC CIRCUITS–

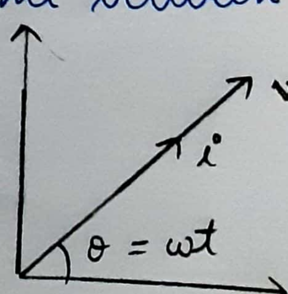
### 1. AC THROUGH RESISTOR

When AC is applied to resistor, I and V are in phase or there is no phase difference between I and V

$$V = V_0 \sin \omega t$$

$$I = \frac{V}{R}$$

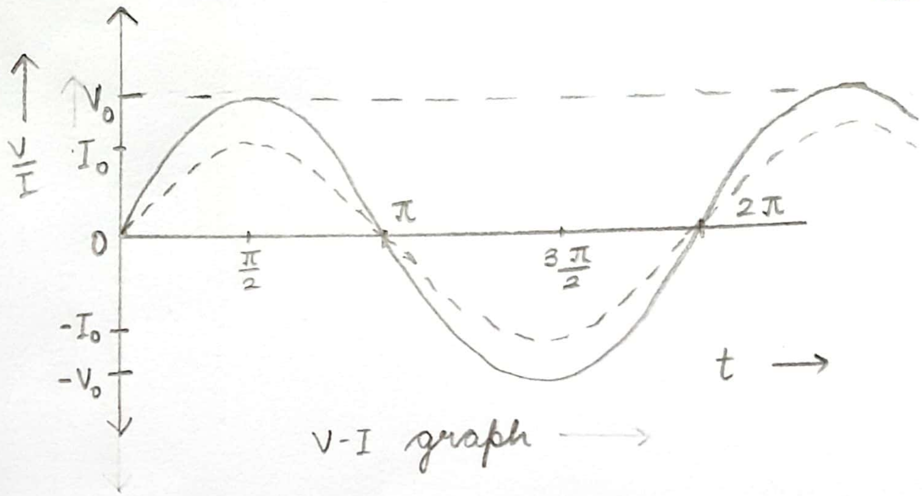


$$I = \frac{V_0 \sin \omega t}{R}$$

$$\boxed{I = I_0 \sin \omega t}$$

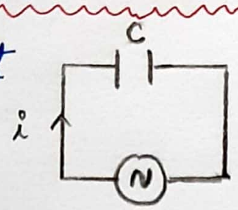
$$\left[ I_0 = \frac{V_0}{R} \right]$$




V-I graph →

## 2. AC THROUGH CAPACITOR

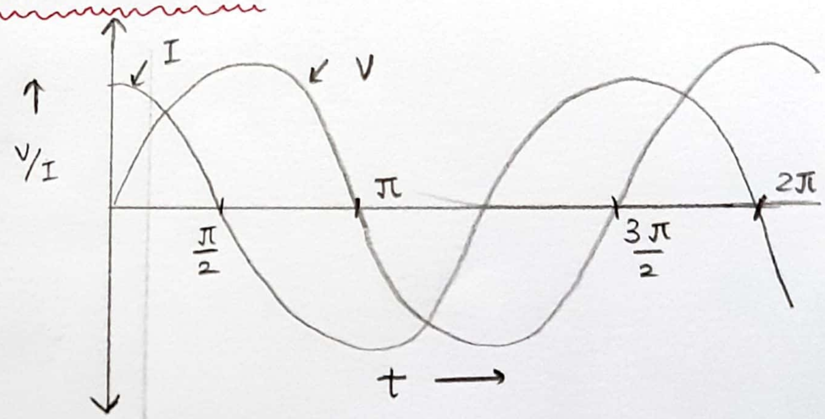
$$V = V_0 \sin \omega t$$




$$Q = CV$$

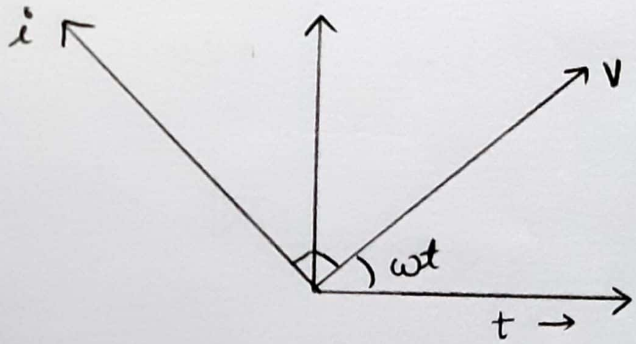
$$I = \frac{dq}{dt}$$

$$I = \frac{d}{dt} CV$$




$$I = C \frac{dV}{dt}$$

$$I = C \frac{d}{dt} (V_0 \sin \omega t)$$




$$I = V_0 C \omega \cos \omega t$$

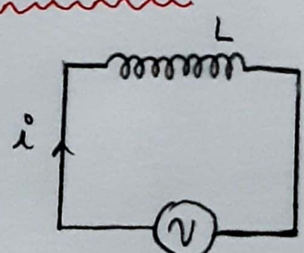
$$I = \frac{V_0}{1/\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)}$$

$$I_0 = \frac{V}{1/\omega C}$$

## 3. AC THROUGH INDUCTOR

$$V = L \frac{di}{dt}$$




$$V_0 \sin \omega t = L \frac{di}{dt}$$

$$\int di = \int \frac{V_0}{L} \sin \omega t \, dt$$

$$i = \frac{V_0}{\omega L}\left[-\cos \omega t\right]$$

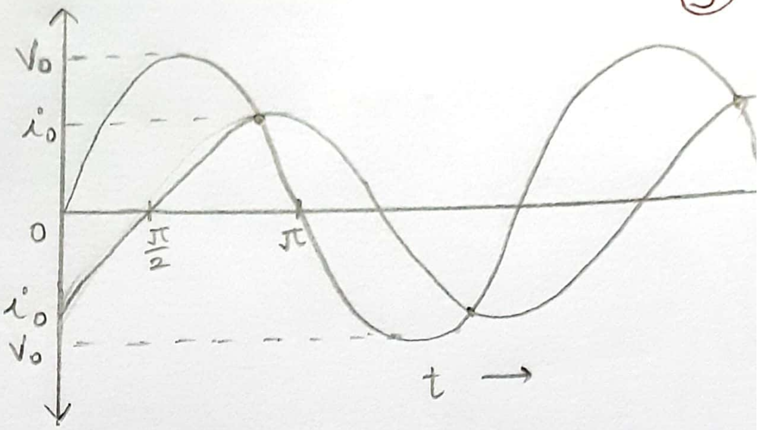
$$i = \frac{-V_0}{\omega L} \cos \omega t$$

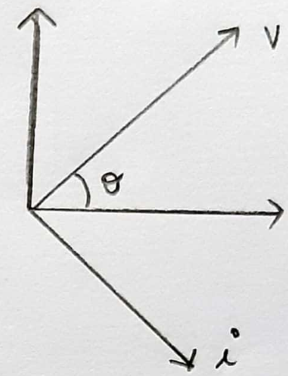
$$I = \frac{-V_0}{\omega L}\left[\sin 90 - \omega t\right]$$

$$I = \frac{V_0}{\omega L}\left[\sin \omega t - \frac{\pi}{2}\right]$$

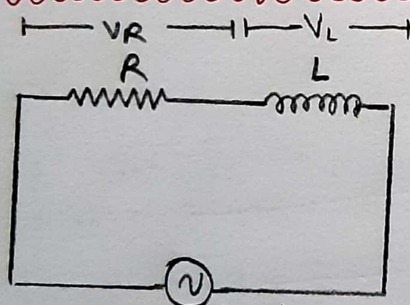
$$\boxed{I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)}$$

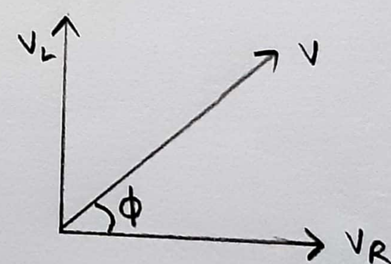
$$I_0 = \frac{V_0}{\omega L} \quad ; \quad I_0 = \frac{V_0}{X_L}$$




## L-R series AC circuit




Then,

(i) Impedance, $Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{R^2 + \omega^2 L^2} \quad [\because X_L = \omega L]$

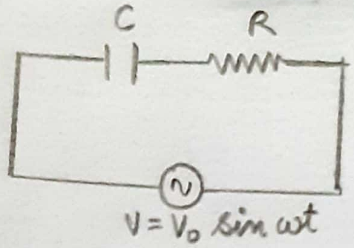
$= V_{rms} / I_{rms}$

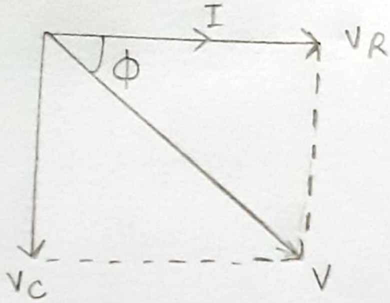
(ii) For the phase angle, $\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R}$

(iii) If $V = V_0 \sin \omega t$, then $I = I_0 \sin(\omega t - \phi)$

* Voltage leads current by phase $\phi$

## R-C series AC circuit




Then,

(i) Impedence, $Z = \dfrac{V_{rms}}{I_{rms}} = \sqrt{R^2 + X_C^2}$

$$= \sqrt{R^2 + \frac{1}{\omega^2 C^2}} \quad \left[\because X_C = \frac{1}{\omega C}\right]$$

(ii) For the phase angle, $\tan\phi = \dfrac{X_C}{R} = \dfrac{1}{\omega C R}$

(iii) If $V = V_0 \sin \omega t$, then $I = I_0 \sin(\omega t + \phi)$

(iv) Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + X_C^2}}$

* Current ahead of voltage by $\phi$

## L-C series AC circuit

(i) Impedence, $Z = \dfrac{V_{rms}}{I_{rms}}$

(ii) Applied voltage $= V_L - V_C$

(iii) Phase difference between voltage and current is $\pi/2$

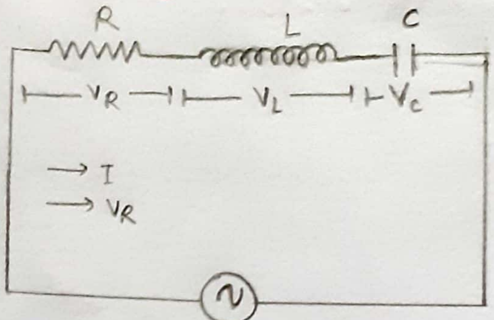
(iv) Power factor, $\cos\phi = 0$

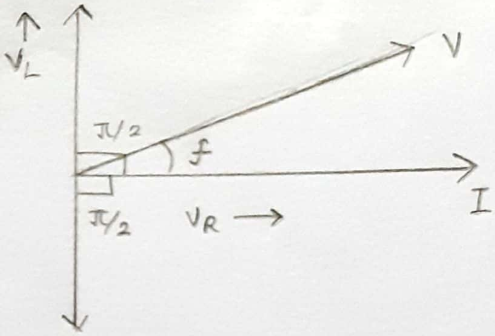
(v) Current, $I = I_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$

## Impedance -

It is the total resistance applied in the path of alternating current. It is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$

# L-C-R series AC Circuit




Then,

(i) Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \dfrac{V_{rms}}{I_{rms}}$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(ii) If net reactance is inductive, circuit behaves as L-R circuit.

(iii) If net reactance is capacitive, circuit behaves as C-R circuit.

# RESONANT LCR CIRCUIT

(i) $X_L = X_C$

(ii) Impedance, $Z = Z_{min.} = R$ i.e. circuit behaves as resistive circuit.

(iii) The phase difference between V and I is $0^\circ$.

(iv) Resonant angular frequency, $\omega_r = \dfrac{1}{\sqrt{LC}}$

(v) Average power consumption $P_{av}$ becomes maximum

# QUALITY FACTOR

The characteristic of a series resonant circuit is determinant by Q-factor. It indicates the sharpness of resonance in an L-C-R series AC circuit.

$$Q = \frac{V_L}{V_R} = \frac{V_C}{V_R} = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} \; ; \; \boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1}}$$

# RESONANCE

In series LCR circuit, when phase ($\phi$) between current and voltage is zero, the circuit is said to be resonant circuit.

The frequency at which $X_C$ and $X_L$ become equal is called resonant frequency.

$$V_L = I X_L$$

$$I X_L = I X_C$$

$$X_L = X_C$$

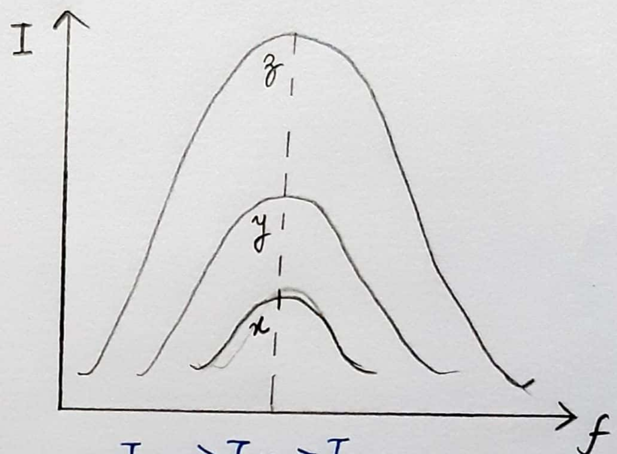
$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$




$I_z > I_y > I_x$

$R_x > R_y > R_z$

# AVERAGE POWER IN A SERIES LCR CIRCUI

$$P_{avg.} = V_{rms} I_{rms} \cos\phi$$

$$P = VI$$

Let $v = V_0 \sin \omega t$ —— (1)

$i = i_0 \sin(\omega t + \phi)$ —— (2)

$$P = V_0 \sin \omega t \, i_0 \sin(\omega t + \phi)$$

$$= V_0 \sin \omega t \, i_0 [\sin \omega t \cos\phi + \sin\phi \cos \omega t]$$

$$P = V_0 i_0 \left[\sin^2 \omega t \cos\phi + \frac{2 \sin \omega t \cos \omega t}{2} \sin\phi\right]$$

$$= V_0 i_0 \left[ \sin^2 \omega t \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi \right]$$

Average power over a complete cycle is equal to

$$= \int_0^T \frac{P\,dt}{T}$$

$$P = \int_0^T \frac{V_0 i_0 \left[ \sin^2 \omega t \cos\phi + \frac{\sin 2\omega t}{2} \sin\phi \right] dt}{T}$$

$$= \frac{V_0 i_0}{T} \left[ \int_0^T \sin^2 \omega t \cos\phi \, dt + \int_0^T \frac{\sin 2\omega t}{2} \sin\phi \, dt \right]$$

$$= \int_0^T \sin^2 \omega t \cos\phi = \frac{T}{2} \cos\phi$$

$$= \int_0^T \frac{\sin 2\omega t}{2} \sin\phi \, dt = 0$$

$$P_{avg.} = \frac{V_0 i_0}{\cancel{T}} \times \frac{\cancel{T}}{2} \cos\phi$$

$$P_{avg.} = \frac{V_0}{\sqrt{2}} \times \frac{i_0}{\sqrt{2}} \cos\phi$$

$$\boxed{P_{avg.} = V_{rms}\, i_{rms} \cos\phi}$$

$\cos\phi$ is the power factor

$V_{rms} \cdot i_{rms}$ = apparent / virtual power

$P$ = true power

$$\boxed{\cos\phi = \frac{\text{true power}}{\text{virtual power}}}$$

## CHOKE COIL -

A choke coil is an electrical device used for controlling current in an AC circuit, without wasting electrical energy in the form of heat.

# TRANSFORMER

It is a device which converts high voltage AC into low voltage AC and vice versa. It is based upon the principle of mutual induction.

## WORKING AND THEORY

When an AC is passed through the primary coil, the magnetic flux through the iron core changes, which does two things, produces emf in the primary coil and an induced emf is set up in the secondary coil. If we assume that resistance of primary coil is negligible, then the back emf will be equal to the voltage applied to the primary coil.

$$\boxed{V_1 = -N_1 \frac{d\phi}{dt}} \quad \text{and} \quad \boxed{V_2 = -N_2 \frac{d\phi}{dt}}$$

where, $N_1$ and $N_2$ are number of turns in the primary and the secondary coil respectively, while $V_1$ and $V_2$ are their voltages, respectively.

$$\therefore \frac{\text{Output emf}}{\text{Input emf}} = \frac{V_2}{V_1} = \frac{N_2}{N_1}$$

## Energy Losses in a transformer

1. Eddy Current Loss - Eddy current in iron core of transformer facilitate the loss of energy in the form of heat.

2. Flux leakage - Total fluxes linked with primary do not completely pass through the secondary which denotes the loss in the flux or flux leakage.

3. Copper loss - Due to heating, energy loss takes place in copper wires of primary and secondary coils.

4. Hysteresis Loss - The energy loss takes place in magnetising and demagnetising the iron core over every cycle.

5. Humming Loss - The magnetostruction effect leads to set the core in vibration which in turn produced the sound. This loss is referred as humming loss.

## TYPES OF TRANSFORMER

1. STEP-UP - ($N_1 > N_2$) It converts low ~~AC~~ alternating voltage into high alternating voltage.
2. STEP-DOWN - ($N_1 < N_2$) It converts high alternating voltage into low alternating voltage.

- For an ideal transformer

Input power = Output power

$$V_1 I_1 = V_2 I_2 \Rightarrow \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

- Transformation ratio ($r$)

$$r = \frac{N_S}{N_P} = \frac{V_S}{V_P} = \frac{I_P}{I_S}$$

- Efficiency

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100$$

## USES OF TRANSFORMER

1. In the induction furnaces
2. In voltage regulators for TV, computer, etc.
3. For welding purposes.

# IMPORTANT QUESTIONS

1. Obtain the resonant frequency ($\omega_r$) of a series L-C-R circuit with $L = 2.0\,H$, $C = 32\,\mu F$ and $R = 10\,\Omega$. What is the Q-value of this circuit? NCERT

Sol. Given, $L = 2.0\,H$, $C = 32 \times 10^{-6}\,F$ $R = 10\,\Omega$ $\omega_r = ?$ $Q = ?$

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = \frac{10^3}{8} = 125\ rad/s$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$$

$$= \frac{1}{10 \times 4 \times 10^{-3}} = 25$$

---

2. A 44 mH inductor is connected to 220 V, 50 Hz AC supply. Determine the rms value of the current in the circuit. What is the net power absorbed over a complete cycle? Explain. NCERT

Sol. $L = 44\,mH = 44 \times 10^{-3}\,H$ $V_{rms} = 220\,V$ $\nu = 50\,Hz$

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 44 \times 10^{-3} = 13.82\,\Omega$$

The rms value of current in the circuit,

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{13.82} = 15.9\,A$$

Power absorbed, $P = V_{rms} I_{rms} \cos\phi$

For pure inductive circuit, $\phi = 90°$ $P = 0$

Thus, power spent in one half cycle is retrieved in the other half cycle.

---

3. A coil of inductance 0.5 H and resistance 100 Ω is connected to a 240 V, 50 Hz AC supply.

(i) What is the maximum current in the coil?

(ii) What is the time lag between the voltage maximum and current maximum? NCERT

Sol. Given, $L = 0.5\,H$, $R = 100\,\Omega$

$\nu = 50\,Hz$, $V_{rms} = 240\,V$

(i) $I_0 = \dfrac{V_0}{\sqrt{R^2 + \omega^2 L^2}} = \dfrac{\sqrt{2} \times 240}{\sqrt{(100)^2 + (100 \times \pi \times 0.5)^2}}$

$= 1.82\,A$ $\quad [\because \omega = 2\pi\nu = 100\pi]$

(ii) $\tan\phi = \dfrac{\omega L}{R} = \dfrac{2\pi\nu L}{R} = \dfrac{2 \times 3.14 \times 50 \times 0.5}{100}$ =

$\phi = \tan^{-1}(3.19 \times 10^{-3}5)$

---

4. A series LCR circuit with $R = 20\,\Omega$, $L = 1.5\,H$ and $C = 35\,\mu F$ is connected to a ~~vehicle~~ variable frequency of 200 V AC supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Sol. When the frequency of the supply equals the natural frequency of the circuit, resonance occurs,

$Z_r = R = 20\,\Omega$

$I_{rms} = \dfrac{V_{rms}}{Z_r} = \dfrac{200}{20} = 10\,A$

$\therefore$ Average power transformed in one cycle

$P_{av.} = V_{rms} I_{rms} \cos\phi$

$= 200 \times 10 \times \cos 0°$ $\quad [\because \phi = 0°]$

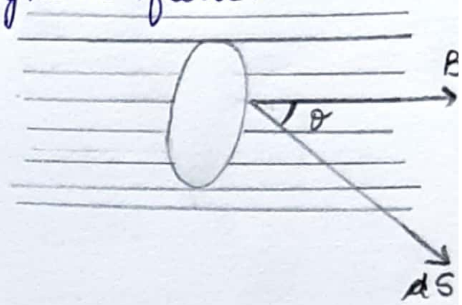
$= 2000\,W = \boxed{2\,KW}$

# ELECTROMAGNETIC INDUCTION

## MAGNETIC FLUX -

It represents total magnetic lines of force passing normally through a given area placed in a magnetic field.

$$\boxed{\phi_B = B \cdot S = BS \cos\theta}$$




Unit - Weber (Wb) = $Tm^2$ → SI unit
Maxwell → CGS unit

## ELECTROMAGNETIC INDUCTION -

The phenomenon to generate induced current or induced emf by changing the magnetic flux linked with a closed circuit is known as Electromagnetic Induction.

## FARADAY'S LAWS

① First Law - Whenever there is change in magnetic flux linked with the closed loop, an emf induces in the loop which lasts as long as the change in flux continues.

② Second Law - The induced emf in a closed loop or circuit is directly proportional to the rate of change of magnetic flux linked with the closed loop or circuit.

i.e. $e \propto (-)\dfrac{d\phi}{dt}$

$$\boxed{e = -\left(\frac{d\phi}{dt}\right)}$$

* The negative sign is due to Lenz Law.

## LENZ LAW-

Current induced in the loop due to changing magnetic flux is such that it tends to oppose the rate of change of magnetic flux.

- Lenz law is in accordance with law of conservation of energy.

## INDUCED CURRENT-

If N is the number of turns and R is the resistance of a coil, the magnetic flux linked with its each turn changes by $d\phi$ in short time interval $dt$, then induced current flowing through the coil is

$$\mathcal{E} = -\frac{d\phi}{dt} \qquad I = \frac{|\mathcal{E}|}{R} \quad \Rightarrow \quad I = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \quad — (1)$$

$$\because I = \frac{dq}{dt}$$

$$\frac{dq}{dt} = -\frac{1}{R}\left(\frac{d\phi}{dt}\right) \quad \text{using (1)}$$

$$\boxed{q = -\frac{1}{R}\int d\phi}$$

## MOTIONAL EMF-

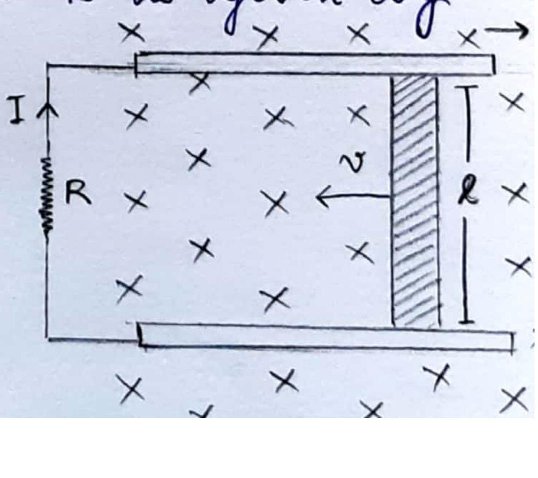
The potential difference induced in a conductor of length $l$ moving with ~~vel~~ velocity $v$ in a direction perpendicular to magnetic field B is given by

$$\mathcal{E} = \int (v \times B) \cdot dl = vBl$$

B → magnetic field

$l$ → length of conducting wire

$v$ → velocity



$$e = -\frac{d\phi}{dt}$$

$d\phi = BA$ $\qquad$ $e = -\frac{d}{dt}(BA)$

$A = lx$ $\qquad$ $e = -\frac{d}{dt}B(lx)$

$$\boxed{e = Blv}$$

- emf will not be induced if any two of $Blv$ are parallel.

## FORCE

$$I = \frac{Blv}{R}$$

$$F = BIl$$

$$\boxed{F = \frac{B^2l^2v}{R}}$$

## POWER

$$P = F \times v$$

$$\boxed{P = \frac{B^2l^2v^2}{R}}$$

## EDDY CURRENT

The current induced in bulk piece of conductor when magnetic flux linked with the conductor changes is known as eddy currents.

$$\boxed{i = \frac{e}{R}}$$

### Applications -

1. Magnetic Braking
2. Induction furnace
3. Speedometer
4. Electromagnetic damping
5. Energy meter

## Disadvantages -

1. Lot of heat energy is produced which damages the core of material.
2. Excessive heating may lead to fire.
3. Reduces the efficiency of the machine.

## Ways to minimize -

1. By laminating the core.
2. By making slots on the conductor surface.

## INDUCTANCE -

The flux linkage of a closely wound coil is directly proportional to the current I i.e. $\phi_B \propto I$. The flux linked with the coil having 'N' turns will be $N\phi_B \propto I$. The constant of proportionality in this relation is called inductance.

## SELF INDUCTANCE

The phenomenon of production of induced emf in a coil, when a current pass through it, undergoes a change.

$\because$ Total flux linked with coil, $N\phi \propto I$

$$\boxed{N\phi = LI}$$

where, $\phi$ = flux linked with each turn and
$L$ = coefficient of self - induction or self inductance

Also, induced emf, $e = -\frac{d\phi}{dt} = -L\frac{dI}{dt}$

SI unit - Henry (H)

where, $L = \frac{\varepsilon}{\ldots}$

## Self Inductance of Long Solenoid -

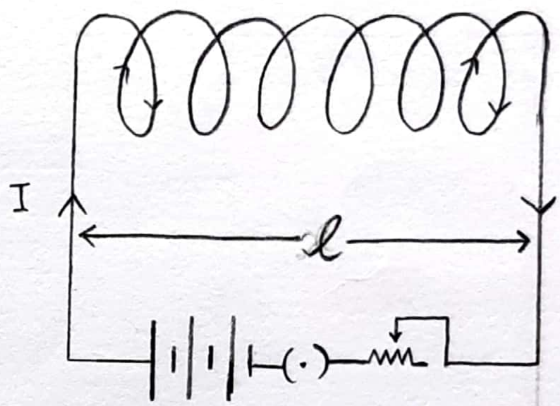
The magnetic field B at any point inside such a solenoid is constant;

$$B = \frac{\mu_0 N I}{l} = \mu_0 n I$$

where,
$\mu_0$ = magnetic permeability
N = total no. of turns
$l$ = length of the solenoid
$n$ = no. of turns per unit length

$\phi$ = B × area of each turn

$$\phi = \left(\mu_0 \frac{N}{l} I\right) A$$




$$N\phi = LI$$

where $L = \frac{\mu_0 N^2 A}{l}$

$$\boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$$

## MUTUAL INDUCTANCE -

The phenomenon according to which an opposing emf is produced in a coil as a result of change in current or magnetic flux linked with a neighbouring coil is called inductance.

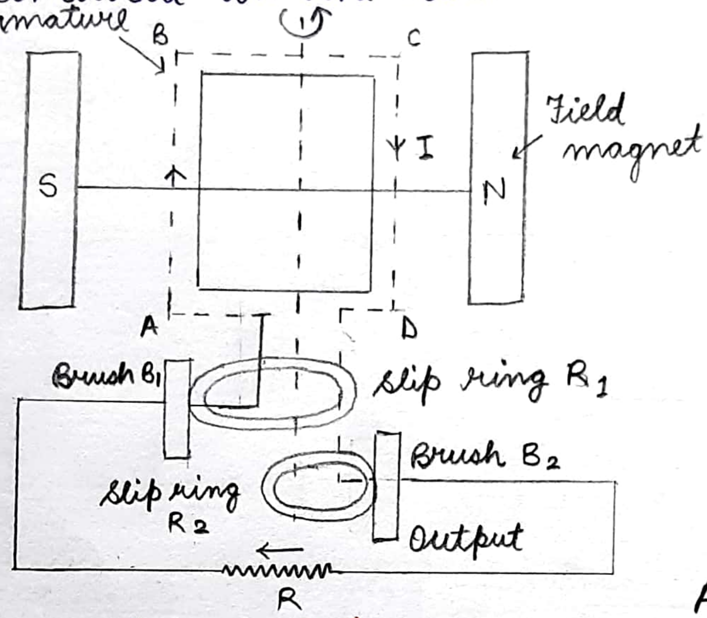
$$\phi \propto I$$

$$\phi = MI$$

$$\boxed{e = -M \frac{dI}{dt}}$$

# AC GENERATOR

An AC generator is based on the phenomenon of electromagnetic induction, which states that a coil is rotated in uniform magnetic field, the magnetic flux linked with a conductor changes and an emf is induced in the coil.




AC GENERATOR

## Theory and Working

As the armature of coil is rotated in uniform magnetic field, angle θ changes continuously. Therefore, magnetic flux changes and an emf is induced. If e is the emf induced in the coil, then

$$e = -\frac{N d\phi}{dt} \quad \text{or} \quad e = -\frac{d}{dt}(NBA \cos \omega t)$$

$$\varepsilon = NBA\omega \sin \omega t$$

N → no. of turns in the coil
B → strength of magnetic field
A → area of each turn of coil
ω → angular velocity of rotation of the coil

and $I = \frac{e}{R} = \frac{NBA\omega}{R} \sin \omega t$, R → resistance of the coil

# IMPORTANT QUESTIONS

1. A long solenoid with 15 turns per cm has a small loop of area 2.0 $cm^2$ placed inside, normal to the axis of solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced voltage in the loop, while the current is changing? (NCERT)

Sol. Here, no. of turns per unit length,

$$n = \frac{N}{l} = 15 \text{ turns/cm} = 1500 \text{ turns/m}$$

$$A = \cancel{\frac{N}{l}} = \cancel{15\ tu}\ 2.0\ cm^2 = 2 \times 10^{-4}\ m^2$$

$$\frac{dI}{dt} = \frac{4-2}{0.1} \quad \text{or} \quad \frac{dI}{dt} = 20\ A s^{-1}$$

$$|e| = \frac{d\phi}{dt} = \frac{d}{dt}(BA)$$

$$|e| = \frac{A\,d}{dt}\left(\mu_0 \frac{NI}{l}\right) = A\mu_0 \left(\frac{N}{l}\right)\frac{dI}{dt}.$$

$$|e| = (2 \times 10^{-4}) \times 4\pi \times 10^{-7} \times 1500 \times 20\ V$$

$$|e| = 7.5 \times 10^{-6}\ V$$

2. A 1 m long conducting rod rotates with an angular frequency of 400 rad/s about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring. (NCERT)

Sol. Length of rod, $l = 1$ m, $\omega = 400$ rad $s^{-1}$, $B = 0.5$ T, $e = ?$

Average linear velocity,

$$v = \frac{0 + l\omega}{2} = \frac{l\omega}{2}, \quad e = Blv$$

$$e = Bl\frac{l\omega}{2} = \frac{Bl^2\omega}{2} = \frac{0.5 \times (1)^2 \times 400}{2} = 100V$$

3. A line charge $\lambda$ per ~~p~~ unit length is lodged uniformly onto the rim of a wheel of mass M and radius R. The wheel has light non-conducting spokes and is free to rotate without friction about its axis as shown in the figure.

Sol. Given, linear charge density, $\lambda = \frac{\text{Total charge}}{\text{length}} = \frac{Q}{2\pi R}$

where, radius of rim = R and mass of rim = M

Magnetic field extends over a circular region,

$$B = -B_0\hat{k} \; (r \leq a, a < R) = 0$$

Let the angular velocity of the wheel be $\omega$, then the induced emf, $e = -\frac{d\phi}{dt}$

$$e = -\int E.\,dl = -\frac{d\phi}{dt}$$

$$E\int dl = -\frac{d}{dt}(\pi a^2 B)$$

$$E \times 2\pi a = -\pi a^2 \frac{dB}{dt} \; ; \; E = -\frac{a}{2} \cdot \frac{dB}{dt}$$

Force on charge, $F = QE = -\pi a^2 \lambda \frac{dB}{dt}$

$$F = \frac{dp}{dt} = M \cdot \frac{dv}{dt}$$

$$M\frac{dv}{dt} = -\pi a^2 \lambda \frac{dB}{dt} \Rightarrow MR\left(\frac{d\omega}{dt}\right) = -\pi a^2 \lambda \frac{dB}{dt}$$

$$d\omega = -\frac{\pi a^2 \lambda}{MR} dB$$

$$\omega = -\frac{\pi a^2 \lambda B}{MR} \Rightarrow \boxed{\omega = -\frac{\lambda a^2 \pi}{MR} B\hat{k}}$$

# MAGNETISM & MATTER

(CHAPTER-4)

The phenomenon of attraction of small bits of iron, steel, cobalt, nickel etc. towards the ore is called magnetism.

## CHARACTERSTICS OF MAGNET:-

1. Monopole doesnot exist.
2. Repulsion is a sure test of magnetisation
3. The distance between two poles of a magnet is called magnetic length and distance between two end of magnet is called geographic length.

$$\boxed{M \cdot L = \frac{7}{8} \times G \cdot L}$$

4. If we break a magnet, $\perp^r$ to the axis then pole strength remains unchanged.
5. If we break the magnet, along the axis into two equal part, pole strength becomes half.

## MAGNETIC FIELD LINES:-

Their properties are given below:-

1. Two magnetic field line cannot intersect each other.
2. They form continous closed loops.
3. The tangent at any point on the magnetic field represents the direction of the net magnetic field.
4. The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field.

## MAGNETIC DIPOLE:-

An arrangement of two equal and opposite magnetic pole separated by a small distance.

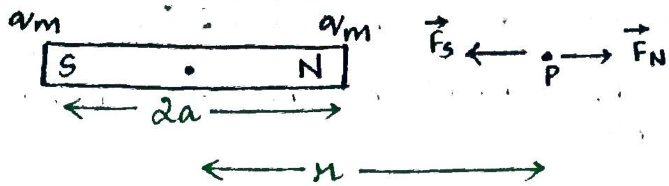
## MAGNETIC DIPOLE MOMENT:- (M)

It is defined as the product of its pole strength with the magnetic length of the magnet.

$$\boxed{M = q_m \cdot 2l}$$

* SI unit - $A/m^2$ or $J/T$
* It's direction is from south to north.

## MAGNETIC FIELD AT AXIAL POSITION :-




P is any point on the axial line at a distance $r$ from the centre.

$$\vec{F_N} = \frac{K q_m q_{m_0}}{(r-a)^2} (\hat{i})$$

$$\vec{F_S} = \frac{K q_m \cdot q_{m_0}}{(r+a)^2} (-\hat{i})$$

$$\vec{F}_{net} = (\vec{F_N} - \vec{F_S}) (\hat{i})$$

$$= \left[\frac{K q_m q_{m_0}}{(r-a)^2} - \frac{K q_m q_{m_0}}{(r+a)^2}\right] (\hat{i})$$

$$= K q_m q_{m_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] (\hat{i})$$

$$= K q_m q_{m_0} \left[\frac{4ar}{(r^2-a^2)^2}\right] (\hat{i})$$

$$= \frac{2 K m r q_{m_0}}{(r^2-a^2)^2} (\hat{i})$$

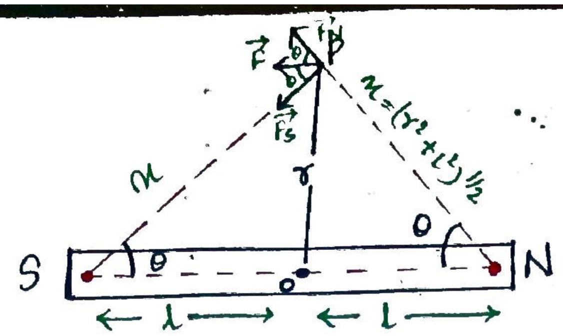
$$\vec{F}_{net} = \frac{2 K \vec{m} r q_{m_0}}{(r^2-a^2)^2} (\hat{i})$$

$$\boxed{\vec{B}_{net} = \frac{\vec{F}_{net}}{q_{m_0}} = \frac{2 K \vec{m} r}{(r^2-a^2)^2} (\hat{i})}$$

When the magnet is short, $a <<< r$, $a^2$ can be neglected.

$$\boxed{\vec{B}_{net} = \frac{2 K \vec{m} r}{r^4} = \frac{2 K \vec{m}}{r^3}}$$

## MAGNETIC FIELD AT AN EQUATORIAL POINT:-




P is any point on the equatorial line at a distance $r$ from the centre.

$$F_N = \frac{Kq_m}{x^2}, \text{ along NP}$$

$$F_S = \frac{Kq_m}{x^2}, \text{ along PS}$$

As the magnitudes of $F_N$ and $F_S$ are equal, so their vertical components get cancelled while the horizontal components add up.

$$B_{eq} = F_N\cos\theta + F_S\cos\theta$$

$$= 2F_N\cos\theta$$

$$= \frac{2Kq_m}{x^2} \cdot \frac{l}{x}$$

$$= \frac{Km}{x^3}$$

$$\boxed{B_{eq} = \frac{Km}{(r^2+l^2)^{3/2}}}$$

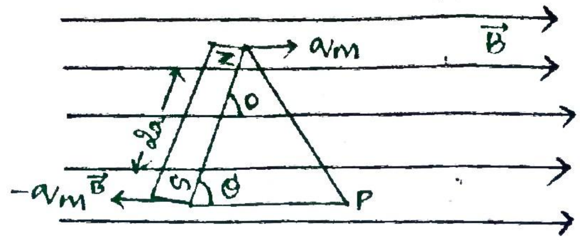
For short magnet, $l \ll r$, so $l^2$ can be neglected.

$$B_{eq} = \frac{Km}{r^3} \text{ along PR}$$

The magnetic field at any equitorial point of a magnetic dipole is in the direction opposite to that of its magnetic dipole moment.

$$\boxed{\vec{B}_{eq} = \frac{-K\vec{m}}{r^3}}$$

## TORQUE ON A MAGNETIC DIPOLE IN A MAGNETIC FIELD:-




In $\Delta$ SNP,

$$\sin\theta = \frac{NP}{NS}$$

$$\Rightarrow PN = 2a\sin\theta$$

The force acting on the south pole is towards left. The force acting on the north pole is towards right.

$$\vec{F}_{net} = q_m\vec{B} - q_m\vec{B} = 0$$

As, the force are not in same line of action. So net $\tau \neq 0$. So they constitute a couple due to which the dipole rotates.

$$\tau = \text{magnitude of force} \times \perp^r \text{dist.}$$
$$= q_m B \cdot 2a\sin\theta$$
$$= B(q_m \cdot 2a\sin\theta)$$
$$\tau = Bm\sin\theta$$
$$\vec{\tau} = \vec{m} \times \vec{B}$$

$\vec{\tau} \perp \vec{m}$ and $\vec{\tau} \perp \vec{B}$.

**Case-1**

when $\theta = 0$

$\boxed{\tau = 0}$ $\Rightarrow$ stable equilibrium.

**Case-2**

when $\theta = 180°$,

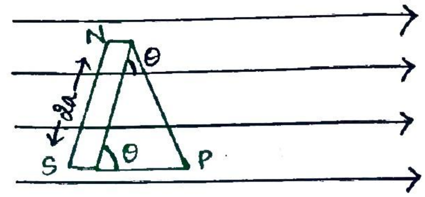
$\boxed{\tau = 0}$ $\Rightarrow$ unstable equilibrium.

**Case-3**

when $\theta = 90°$,

$\boxed{\tau = Bm}$ $\Rightarrow$ Maximum torque.

## POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD:-




Let the magnetic dipole moved through a small angle $d\theta$ and torque acting on dipole is $\tau$. Then the small work done in moving dipole $dW = \tau d\theta$

$$\Rightarrow \int_0^W dW = \int_{\theta_1}^{\theta_2} \tau d\theta$$

$$\Rightarrow W = \int_{\theta_1}^{\theta_2} mB\sin\theta \, d\theta$$

$$= mB \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= mB(-\cos\theta)_{\theta_1}^{\theta_2}$$

$$= -mB(\cos\theta_2 - \cos\theta_1)$$

$$\boxed{\begin{aligned} W &= -mB(\cos\theta_2 - \cos\theta_1) \\ W &= mB(\cos\theta_1 - \cos\theta_2) \end{aligned}}$$

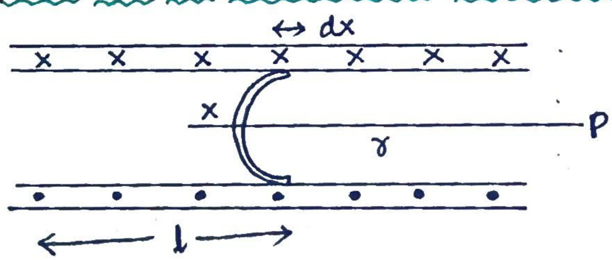
If initial angle, $\theta_1 = \pi/2$ and $\theta_2 = \theta$

$$W = mB(-\cos\theta)$$

$$\Rightarrow U = -mB\cos\theta$$

$$\Rightarrow \boxed{U = -\vec{m}\cdot\vec{B}}$$

## BAR MAGNET AS AN EQUIVALENT SOLENOID:-




$n$ = no. of turns per unit length
$l$ = length of solenoid
$R$ = radius of solenoid
$r$ = dist. of point P from the centre of the solenoid

Consider a circular loop at a dist $x$ from the solenoid.

Magnetic field at point P due to circular coil,

$$dB = \frac{\mu_0}{4\pi} \frac{2n\,dx \cdot IA}{\{R^2 + (r-x)^2\}^{3/2}}$$

Assuming, the point far away, $r >>> R$ and $r >>> x$

$$dB = \frac{\mu_0}{4\pi} \frac{2n\,dx\,IA}{r^3}$$

Integrating both sides with appropriate limits,

$$\int dB = \frac{\mu_0}{4\pi} \frac{2nIA}{r^3} \int_{-l/2}^{l/2} dx$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{2nIA}{r^3} [x]_{-l/2}^{l/2}$$

$$= \frac{\mu_0}{4\pi} \frac{2nIA}{r^3} \left[\frac{l}{2} + \frac{l}{2}\right]$$

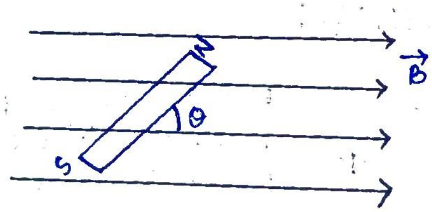
$$= \frac{\mu_0}{4\pi} \frac{2nIAl}{r^3}$$

$$= \frac{\mu_0}{4\pi} \frac{2NIA}{r^3}$$

$$\Rightarrow \boxed{B = \frac{\mu_0}{4\pi} \frac{2M}{r^3}}$$

This expression is similar to the expression of magnetic field due to bar magnet. So solenoid behaves like a bar magnet.

## OSCILLATIONS OF A FREELY SUSPENDED MAGNET:-/DIPOLE IN A UNIFORM MAGNETIC FIELD:-




When magnetic dipole is left in uniform magnetic field at any angle, it execute S.H.M

Proof:- When θ is the angle between dipole moment and $\vec{B}$, Restoring torque,

$$\tau = -MB\sin\theta$$

$$\Rightarrow I\alpha = -MB\sin\theta$$

$$\Rightarrow \alpha = -\frac{MB}{I}\theta \text{ (taking } \theta \text{ very small)}$$

$$\alpha \propto (-\theta)$$

So, dipole execute s.h.m, where,

$$\omega^2 = \frac{MB}{I}$$

$$\Rightarrow \omega = \sqrt{\frac{MB}{I}}$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{MB}{I}}$$

$$\Rightarrow T = \frac{\sqrt{\frac{MB}{I}}}{2\pi}$$

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{MB}}}$$

## THE ELECTROSTATIC ANALOG:-

| Physical Quantity | Electrostatics | Magnetism |
|---|---|---|
| Dipole moment | $P = q \times 2l$ | $m = q_m \times 2l$ |
| Axial field | $E_{axial} = \frac{2KP}{r^3}$ | $B_{axial} = \frac{2KM}{r^3}$ |
| Equatorial field | $E_{equa} = -\frac{KP}{r^3}$ | $B_{eq} = -\frac{KM}{r^3}$ |
| Torque in external field. | $\tau = PE\sin\theta$ | $\tau = mB\sin\theta$ |
| P.E in external field. | $U = -PE\cos\theta$ | $U = -mB\cos\theta$ |

## GAUSS'S LAW IN MAGNETISM :-

$$\phi_B = \oint \vec{B} \cdot \vec{ds} = 0$$

* The surface integral of magnetic field over a closed surface is always 0 as magnetic monopole never exist.
* The magnetic flux around a closed surface is 0.

## SOME DEFINATIONS IN CONNECTION WITH EARTH'S MAGNETISM :-

① GEOGRAPHIC AXIS :- The straight line passing through the geographic north and geographic south.

② MAGNETIC AXIS :- The straight line passing through magnetic north and south pole of the earth.

③ MAGNETIC EQUATOR :- It is a great circle on the earth perpendicular to the magnetic axis.

④ GEOGRAPHIC MEREDIAN :- The vertical plane passing through geographic north and south pole.

⑤ MAGNETIC MEREDIAN :- The vertical plane passing through magnetic axis of a freely suspended small magnet.

## ELEMENTS OF EARTH'S MAGNETIC FIELD :-

① Angle of dip / Angle of Inclination :- ($\delta$)

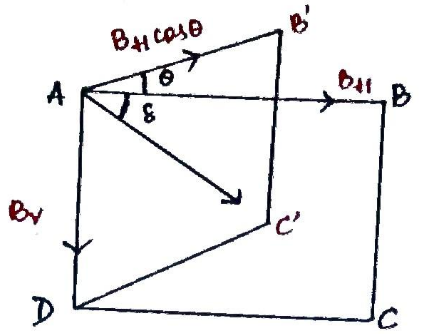
* It is the angle made by the resultant magnetic field of earth with horizontal in magnetic meredian.
* Its value is zero at the equator and $90°$ at the pole.

② HORIZONTAL COMPONENT :-

* It is the component of earth magnetic field along horizontal.
* It is zero at the pole and $90°$ at the equator.

③ DECLINATION / MAGNETIC DECLINATION :-

* It is the angle between Geographic meredian and magnetic meredian.
* It is measured as $\theta°$ east or $\theta°$ west.




ABCD → magnetic meridian
$\theta$ → declination
$\delta$ → angle of dip

$B_H = B\cos\delta$

$B_V = B\sin\delta$

$$B = \sqrt{B_H^2 + B_V^2}$$

$$\boxed{\tan\delta = \frac{B_V}{B_H}}$$

## SOME IMPORTANT TERMS USED TO DESCRIBE MAGNETIC PROPERTIES OF MATERIALS:-

### ① Intensity of magnetisation:- (I)

* It is defined as dipole moment of substance per unit volume.

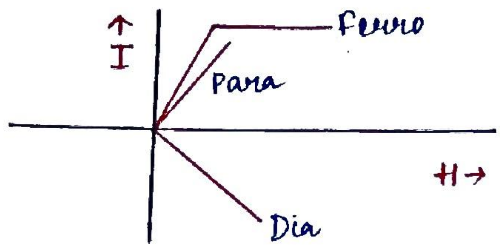
$$I = \frac{m}{\text{Volume}} = \frac{q_m \times 2l}{A \times 2l} = \frac{q_m}{A}$$

For diamagnetic substance, I is −ve
For paramagnetic substance, I is +ve
For ferromagnetic substance, I is highly +ve.

### ② Intensity of magnetic field:- ($\vec{H}$)

* It is defined as the ratio between the external applied magnetic field to the permeability.

- The variation of intensity of magnetisation and magnetising field :-




## ③ MAGNETIC SUSCEPTIBILITY :- ($\chi$)

* It is defined as the ratio of intensity of magnetisation to magnetising field

$$\boxed{\chi = \frac{I}{H}}$$

- For dia, it is -ve
- For paramagnetic substance, it is +ve
- For ferromagnetic substance, it is highly +ve

## ④ RELATIVE MAGNETIC PERMEABILITY :- ($\mu_r$)

* It is defined as the ratio of magnetic field inside a substance to applied magnetic field.

$$\boxed{\mu_r = \frac{B}{H}}$$

* It is unitless and dimensionless.

- For diamagnetic substance, $B < H$ so $\mu_r < 1$
- For paramagnetic substance, $B > H$, so $\mu_r > 1$
- For ferromagnetic substance, $B >> H$, so $\mu_r >> 1$.

**Relation between susceptibility and magnetic permeability :-**

$$B = B_0 + B_m$$

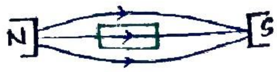
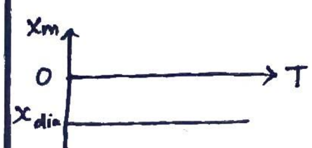
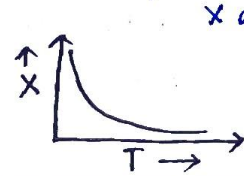
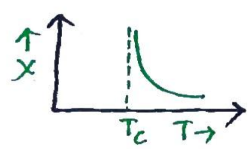
$$\Rightarrow \mu H = \mu_0 H + \mu_0 I$$

$$\Rightarrow \mu H = \mu_0 (H + I)$$

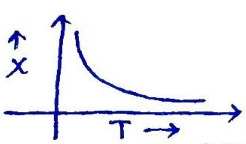
$$\Rightarrow \frac{\mu}{\mu_0} = 1 + \frac{I}{H}$$

$$\Rightarrow \boxed{\mu_r = 1 + \chi}$$

# COMPARATIVE STUDY OF MAGNETIC MATERIALS:-

| PROPERTY | DIAMAGNETIC | PARAMAGNETIC | FERROMAGNETIC |
|---|---|---|---|
| ① Effect of magnet | Feebly repelled  | Feebly attracted  | Strongly attracted  |
| ② External magnetic field | Acquire feeble magnetism opp. to the direction of magnetising field. $\vec{H}$ [M ←] | Acquire feeble magnetism in the direction of magnetising field $\vec{H}$ [→ M] | They are strongly magnetised in the direction of magnetising field $\vec{H}$ [→ M] |
| ③ Non uniform magnetic field | Tend to move slowly from stronger to weaker magnetic field. | Tend to move slowly from weaker to stronger magnetic field. | Tend to move rapidly from weaker to stronger magnetic field. |
| ④ Susceptibility ($\chi$) | Negative | Positive | Highly positive |
| ⑤ Permeability ($\mu_r$) | $\mu_r < 1$ | $\mu_r > 1$ | $\mu_r >> 1$ |
| ⑥ Effect of temp. | It is independent of temp.  | It is inversely prop to temp $\chi \propto \frac{1}{T}$  | It follow Curie-Weiss law.  |
| ⑦ Examples | Bi, Cu, Pb, Si, $H_2O$, NaCl, $N_2$ (STP) | Al, Na, Ca, $O_2$ (STP) etc. | Fe, Co, Ni, Alnico, Ticonal |

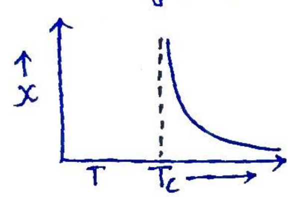
**CURIE-WEISS LAW:-** It states that susceptibility is inversely proportional to temp for paramagnetic substance.



(Paramagnetic)

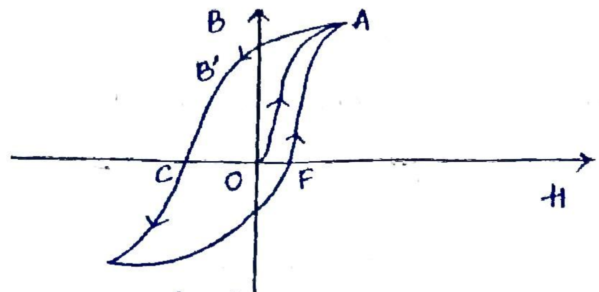
$\chi \propto \frac{1}{T}$

$\Rightarrow \boxed{\chi = \frac{C}{T}}$, C = curie's constant



$T_c$ (curie's temp) - temp above which ferromagnetic substance behaves like paramagnetic substance

## HYSTERESIS :-



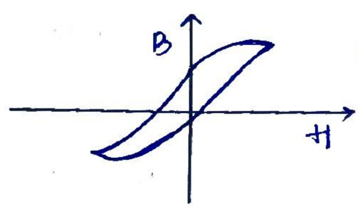

OB' = retentivity
OC = coercivity

The variation of magnetic field inside a ferromagnetic substance and applied magnetic field for a complete cycle of magnetisation and demagnetisation is called hysteresis loop.

Depending on graph :-

Ferromagnetic substances are classified into :-

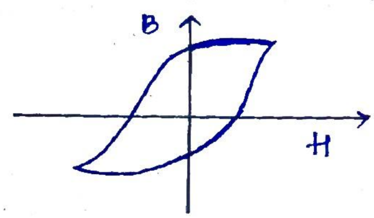
① soft iron :-




* low retentivity
* low coercivity
* high permeability

used for making electromagnet.

② hard iron :-




* high retentivity
* high coercivity
* high permeability

used for making permanent magnet

# MOVING CHARGES AND MAGNETISM

(CHAPTER-4)

## MAGNETIC FIELD:-

It is a region surrounding to a magnet within which another magnet experiences a force.

SI unit :- T (tesla)

CGS unit :- G (gauss)

$\boxed{1\,T = 10^4\,G}$

## BIOT-SAVART'S LAW:-

The magnitude of the magnetic field at any point around the current carrying conductor (i) is directly proportional to current.

(ii) is directly proportional to current element.

(iii) is directly proportional to sine of angle between current element and position vector.

(iv) and is inversely proportional to the square of distance between them.

$$dB \propto I$$
$$dB \propto dl$$
$$dB \propto \sin\theta$$
$$dB \propto \frac{1}{r^2}$$

$$dB \propto \frac{I\,dl\sin\theta}{r^2}$$

$$\Rightarrow \boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}}$$

$\mu_0$ = permeability of free space / magnetic permeability of vacuum.

$\boxed{\mu_0 = 4\pi \times 10^{-7}\ Tm/A}$

**Case-1**

when $\theta = 0°$ or $180°$

$\Rightarrow \boxed{dB = 0}$ (Along the conductor magnetic field is 0)

**Case-II**

when $\theta = 90°$

$\Rightarrow \boxed{dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}}$ (Maximum)

In vector form,

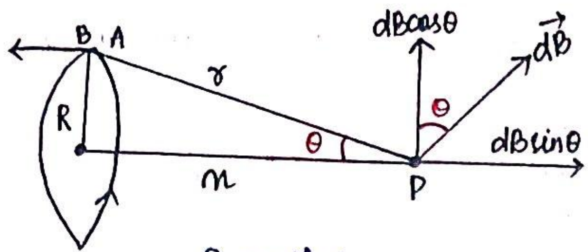
$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta \, r}{r^3}$$

$$\Rightarrow dB = \frac{\mu_0 I}{4\pi} \frac{|\vec{dl} \times \vec{r}|}{r^3}$$

$$\Rightarrow \boxed{\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{(\vec{dl} \times \vec{r})}{r^3}}$$

$\vec{dB} \perp^r \vec{dl}$ and $\vec{dB} \perp^r \vec{r}$.

## MAGNETIC FIELD AT ANY POINT ON THE AXIS OF CIRCULAR COIL:-




R = radius

P = any point on the axis at a dist x from the centre

AB = current element.

According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I dl \sin 90^\circ}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dl}{r^2}$$

$\vec{dB}$ can be resolved into 2 rectangular component. $dB\cos\theta$ is cancelled due to symmetry.

$$B = \int dB \sin\theta$$

$$= \int \frac{\mu_0}{4\pi} \frac{I dl}{r^2} \sin\theta$$

$$= \frac{\mu_0}{4\pi} \frac{I \sin\theta}{r^2} \int dl$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \cdot \frac{R}{r} \times 2\pi R$$

$$= \frac{\mu_0}{4\pi} \frac{2 I \pi R^2}{r^3}$$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2 I A}{(R^2 + x^2)^{3/2}}} \quad (A = \pi R^2 = \text{area of the coil})$$

For N no. of turns,

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2NIA}{(R^2+x^2)^{3/2}}}$$

Case-I

If $x >>> R$, $\boxed{B = \frac{\mu_0}{4\pi} \frac{2NIA}{x^3} = \frac{\mu_0}{4\pi} \frac{2M}{x^3}}$

This expression is similar to the expression of magnetic field due to magnetic dipole where dipole moment $M = NIA$.

Case-II

At the centre of circular coil, $x=0$

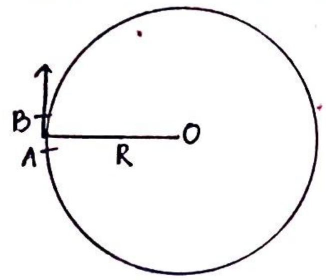
$$B = \frac{\mu_0}{4\pi} \frac{2NIA}{R^3}$$

$$= \frac{\mu_0}{4\pi} \frac{2I\pi R^2}{R^3} \quad (n=1)$$

$$= \frac{\mu_0}{4\pi} \frac{2I\pi}{R}$$

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

MAGNETIC FIELD AT THE CENTRE OF CIRCULAR CURRENT LOOP:-




$R$ = radius

$I$ = current passing through coil

using biot-savart's law:-

$$dB = \frac{\mu_0}{4\pi} \frac{Idl \sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{Idl}{R^2}$$

$$B = \int dB$$

$$= \frac{\mu_0 I}{4\pi R^2} \int dl$$

$$= \frac{\mu_0 I}{4\pi R^2} \times 2\pi R$$

$$B = \frac{\mu_0 I}{2R}$$

For N turns, $\boxed{B = \frac{\mu_0 NI}{2R}}$

CASE-I

For semicircle

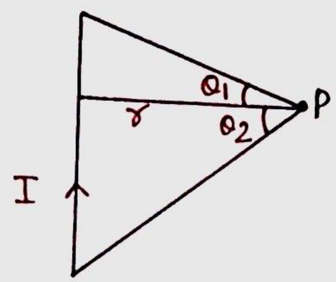
$$\boxed{B = \frac{\mu_0 I}{4R}}$$

CASE-II

For an arc,

$$\boxed{B = \frac{\mu_0 I}{2R} \cdot \frac{\theta}{360^\circ}}$$

## MAGNETIC FIELD DUE TO LONG STRAIGHT CURRENT CARRYING CONDUCTOR:-




$$B = \frac{\mu_0 I}{4\pi r}(\sin\phi_1 + \sin\phi_2)$$

CASE-I

Infinite straight

$\phi_1 = \phi_2 = 90^\circ$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

CASE-II

Semi-infinite

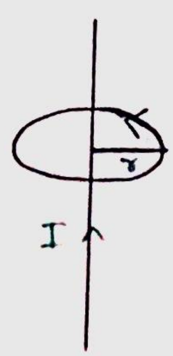
$$\boxed{B = \frac{\mu_0 I}{4\pi r}}$$

## AMPERE'S CIRCUITAL LAW:-

It states that the line integral of magnetic field over a closed loop is $\mu_0$ times current enclosed by the loop.

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{en}}$$

PROOF:-

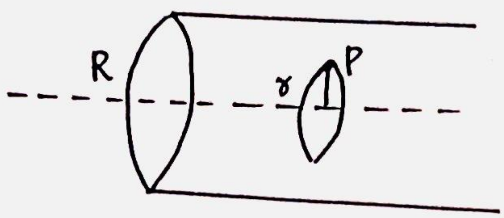



$r$ = radius of amperian loop.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos 0$$
$$= B\oint dl$$
$$= \frac{\mu_0 I}{2\pi r} \times 2\pi r$$
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

Q:- A straight thick long wire of uniform cross-section of radius 'R' is carrying stedy current I. use Ampere's circuital law to obtain a relation showing the variation of magnetic field inside and outside the wire with distance '$r$'. ($r \leq R$) and ($r > R$) of the field point from the centre of its cross-section. Plot a graph showing the variation of field B with distance $r$.




R = radius
I = current passing through conductor.

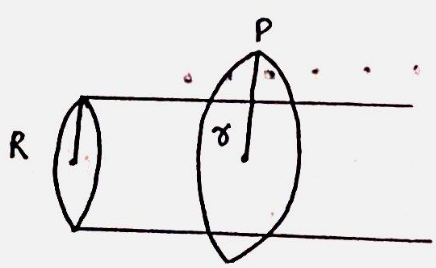
P is any point at a distance $r$ from the axis.

Case-1 ($r < R$)

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n$$
$$\Rightarrow B\,dl\cos 0 = \frac{\mu_0 I \pi r^2}{\pi R^2}$$
$$\Rightarrow B\cdot 2\pi r = \frac{\mu_0 I r^2}{R^2}$$
$$\Rightarrow B = \frac{\mu_0 I r^2}{2\pi r R^2}$$
$$\Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi R^2}} \qquad B \propto r$$

Case-2 ($r > R$)



using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n$$

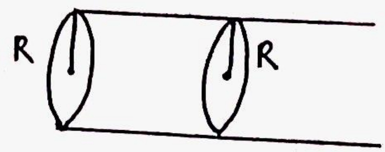
$$\Rightarrow B dl \cos 0 = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$B \propto \frac{1}{r}$$

Case-III

(on the conductor, $r = R$)

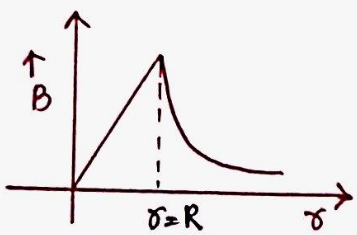



using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n$$

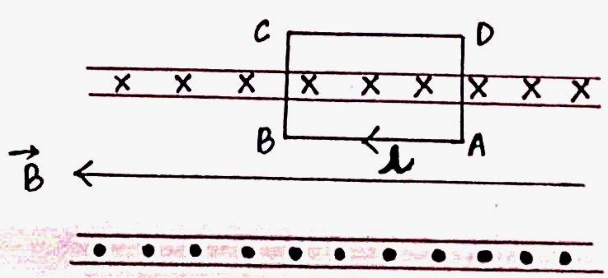
$$\Rightarrow B\cdot 2\pi R = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi R}} \text{ (Max)}$$




MAGNETIC FIELD INSIDE A STRAIGHT SOLENOID:-

Inside solenoid, magnetic field is uniform but just outside is 0.




$n$ = no. of turns per unit length = $N/l$

$I$ = current passing through each turn.

ABCD is the amperian loop of length $l$
N no of turns passes through it.

Using ampere's circuital law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_n$$

$$\Rightarrow \int_A^B Bdl\cos 0^\circ + \int_B^C Bdl\cos 90^\circ + \int_D^A Bdl\cos 90^\circ + 0 = \mu_0 NI$$

$$\Rightarrow BL = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{l}$$

$$\Rightarrow \boxed{B = \mu_0 nI}$$

CASE-I

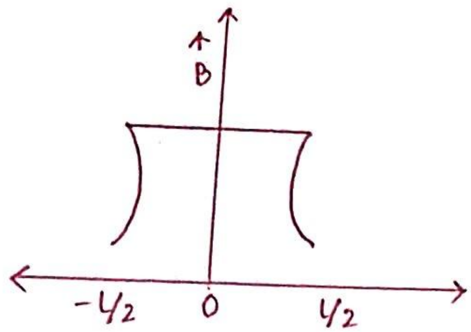
If the core contains material of relative magnetic permeability $\mu_r$.

$$\boxed{B = \mu_0 \mu_r nI}$$

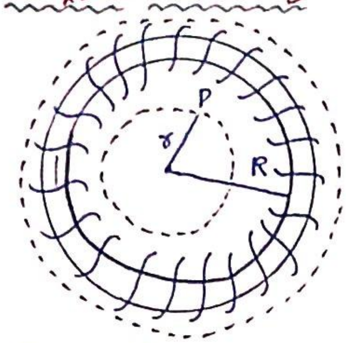
CASE-II

End of solenoid,

$$\boxed{B = \frac{\mu_0 nI}{2}}$$




MAGNETIC FIELD DUE TO TOROIDAL SOLENOID:-




R = radius

I = current passing through each turn

N = total no. of turns.

N.B:- SOLENOID:- It is defined as insulated copper wire wound in the form of helix.

TOROID:- when two ends of solenoid are joined then toroid is formed.

CASE-I

Space enclosed by toroid,

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n \quad (I_n = 0)$$

$$\Rightarrow \boxed{B = 0}$$

CASE-II

Outside the toroid,

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n \quad (I_n = 0)$$

$$\Rightarrow \boxed{B = 0}$$

CASE-III

Inside the toroid,

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n$$

$$\Rightarrow B\,dl\cos 0 = \mu_0 N I$$

$$\Rightarrow B = \frac{\mu_0 N I}{2\pi R}$$

$$\Rightarrow \boxed{B = \mu_0 n I}$$

SPECIAL CASE:-

If the core contain material of relative permeability $\mu_r$.

$$\boxed{B = \mu_0 \mu_r n I}$$

FORCE ON A MOVING CHARGE :-

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\Rightarrow |\vec{F}| = qvB\sin\theta$$

$$\vec{F} \perp \vec{v} \text{ and } \vec{F} \perp \vec{B}$$

Cause :- Moving charge is equivalent to current. Current produces its magnetic field and is affected by external magnetic field.

CASE-I

$\theta = 0°$ or $180°$

$\boxed{F=0}$ (min)

When charge moves either along or opposite to field then F is 0.

CASE-II

$\theta = 90°$

$\boxed{F=qvB}$ (max)

The direction of force can be determined by Fleming's left hand rule.

CASE-III

$v=0$

$\Rightarrow \boxed{F=0}$

Static charge experiences no force.

TRAJECTORY OF CHARGE PARTICLE:-

CASE-I

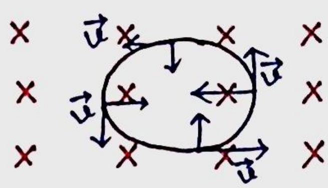
when $\theta = 0°$ or $180°$

$\Rightarrow \boxed{F=0}$

Trajectory is a straight line along the field or opposite to the field.

CASE-II

when $\theta = 90°$



Trajectory is circular.

The necessary centripetal force is provided by $\vec{B}$.

$qvB = \frac{mv^2}{r} \Rightarrow \boxed{r = \frac{mv}{qB}}$

Time period, $T = \frac{2\pi r}{v} = \boxed{\frac{2\pi m}{Bq}}$

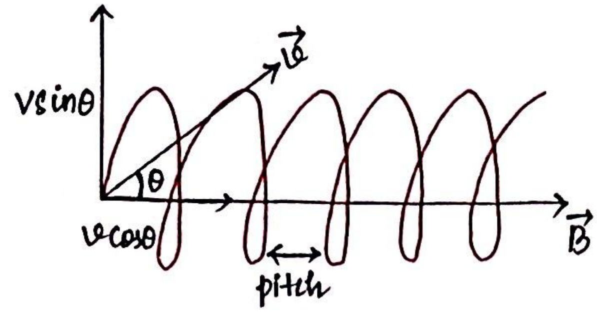
$f = \frac{1}{T} = \boxed{\frac{Bq}{2\pi m}}$

Case-III

when $\theta \neq 90°$

$\theta$ is the angle between $\vec{v}$ and $\vec{B}$

$\vec{v}$ can be resolved into 2 components.




(i) $v\cos\theta$ along the $\vec{B}$ which drag the particle in the forward direction.

(ii) $v\sin\theta$ $\perp^r$ to the $\vec{B}$ which make the particle to move in circular path.

So, ultimately trajectory is helical.

$$r = \frac{mv\sin\theta}{Bq}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{Bq}$$

Pitch:- The distance travelled by the particle between its period of revolution in the direction of magnetic field.

$$\text{Pitch} = v\cos\theta \times T = \frac{v\cos\theta \times 2\pi m}{Bq}$$

## SIMILARITIES AND DIFFERENCES BETWEEN BIOT-SAVART'S LAW AND COULOMB'S LAW:-

Similarities:-

1. Both are long range forces.
2. Both obey inverse square law.
3. Both obey superposition principle.

Differences:-

| ELECTRIC FIELD | MAGNETIC FIELD |
|---|---|
| ① apply force on static charge | cannot apply force on static charge |
| ② force is along the electric field | force is perpendicular to magnetic field. |
| ③ force is independent of angle | force depends on angle. |
| ④ It can change the speed of the particle. | It cannot change the speed of the particle. |

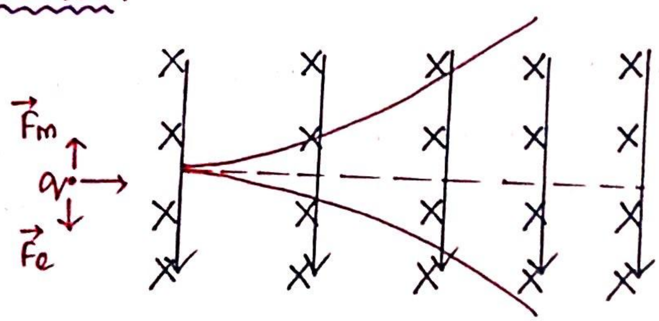
## LORENTZ FORCE:-

It is the net force experienced by a charge particle when it is in region of both electric field and magnetic field.

$$\vec{F}_L = \vec{F}_e + \vec{F}_m$$

$$\Rightarrow \boxed{\vec{F}_L = q\vec{E} + q(\vec{v} \times \vec{B})}$$

## VELOCITY SELECTOR:-




Particle gets undeflected if,

$$F_e = F_m$$

$$\Rightarrow qE = qvB$$

$$\Rightarrow \boxed{\frac{E}{B} = v}$$

CASE-I

$v > E/B$

$\Rightarrow F_m > F_e$

Particle is deflected upward.

CASE-II

$v < E/B$

$\Rightarrow F_m < F_e$

Particle is deflected downward

N.B :- Work done on the charge due to magnetic field.

$$W = Fs\cos\theta$$
$$= Fs\cos 90^\circ$$
$$\boxed{W = 0}$$

$\vec{F} \perp \vec{v}$ and $\vec{F} \perp \vec{s}$.

So, it cannot change K.E. i.e speed of particle.

## CYCLOTRON :-

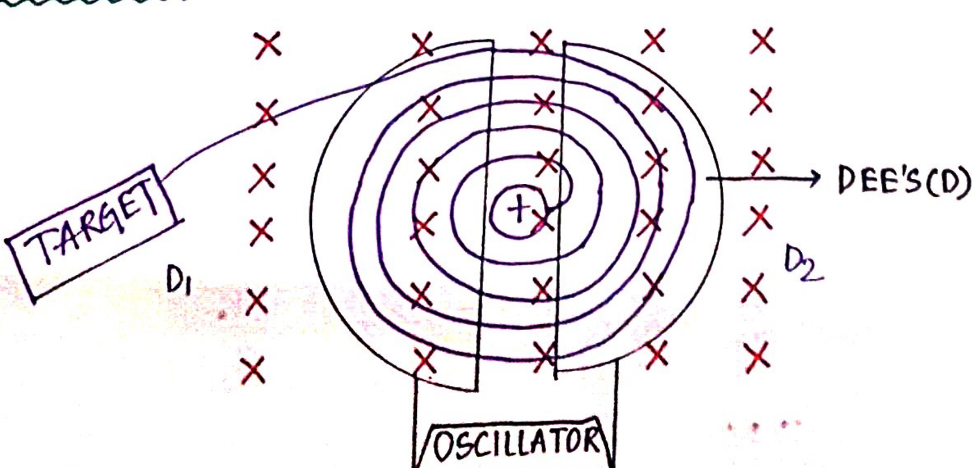
It is a device which is used to accelerate the charged particle like proton, α-particle except electron and neutron.

PRINCIPLE :-

① frequency of revolution of charge particle is independent of radius and velocity.

② when charged particle enter into crossed electric and magnetic field. Electric field increases kinetic energy and Magnetic field make it to move in a circular path.

CONSTRUCTION :-



**Dee's**:- It should be two small hollow metallic half cylinder in the shape of D.

**Magnetic Field**:- Dee's has to be kept in a plane perpendicular to the magnetic field. The whole device is in high vacuum and maintained at high pressure inside it

**Oscillator**:- Oscillator is a device that convert DC to AC. Here, it provide necessary electric field.

**WORKING**:-

Suppose a +ve ion enters the gap between two dee's and found $D_2$ to be negative and $D_1$ +ve. It gets accelarated towards $D_2$. As it enters the $D_2$, it doesnot experience any electric field due to the shielding effect. The perpendicular magnetic field throws it into a circular path. At the instant it comes out of $D_2$, it finds $D_1$ to be -ve and $D_2$ to be +ve. It now gets accelarated towards $D_1$ and the process is repeated. Every time the particle process the gap between the two dee's, its velocity increases. Hence, it moves with a greater radius and finally acquires a very large energy. The trajectory of the charged particle is spiral.

**THEORY**:-

Centripetal force,

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow \boxed{r = \frac{mv}{qB}}$$

More is the speed, more is the radius.

time taken to complete a semi-circle,

$$t = \frac{\pi r}{v} = \frac{\pi m v}{v q B} = \boxed{\frac{\pi m}{qB}}$$

According to resonance condition,

time taken to complete a semi-circle = half of the time period of the oscillator.

$$T = 2 \times t$$

$$\boxed{T = \frac{2\pi m}{Bq}}$$

Frequency,

$$\boxed{f = \frac{1}{T} = \frac{Bq}{2\pi m}}$$ It is called cyclotron frequency.

Max· K·E gained by the charged particle

$$K\cdot E_{max} = \frac{1}{2} m v_{max}^2$$

$$= \frac{1}{2} m \left(\frac{qBr_{max}}{m}\right)^2$$

$$\boxed{K\cdot E_{max} = \frac{q^2 B^2 r_{max}^2}{2m}}$$

LIMITATION :-

① electrons cannot be accelerated as it has lighter mass.

② neutrons cannot be accelerated as it has no charge.

USES :-

① atomic reactors.

② nuclear reactors.

③ synthesize of new particle.

④ used in hospital for diagnosis of cancer.

FORCE ON A CURRENT CARRYING CONDUCTOR :-

When current passes through a conductor, it behaves like a magnet and is affected by external magnetic field.

$$\vec{F} = I(\vec{l} \times \vec{B})$$

$$|\vec{F}| = BIl\sin\theta$$

$$\vec{F} \perp \vec{l} \text{ and } \vec{F} \perp \vec{B}$$

CASE-I

when $\theta = 0°$ or $180°$

$\Rightarrow \boxed{F = 0}$

CASE-II

when $\theta = 90°$

$\boxed{F = BIL}$.

The direction of force can be determined by Fleming's left hand rule.

CASE-III

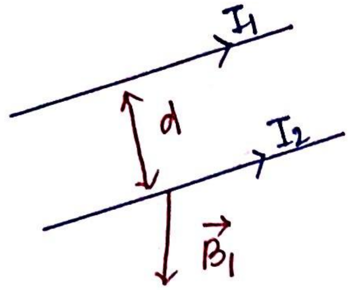
when $I = 0$

$\Rightarrow \boxed{F = 0}$

# FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTOR:-

Two parallel current apply force on each other because in the magnetic field of one current other current is present.

$I_1$ and $I_2$ are two || currents separated by distance d.




$\vec{B}$ due to $I_1$ on $I_2$,

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force on $I_2$ due to $I_1$.

$$F_{21} = B_1 I_2 l \sin 90^\circ$$
$$= \frac{\mu_0 I_1 I_2}{2\pi d} \cdot l$$
$$\Rightarrow \frac{F_{21}}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Similarly force on $I_1$ due to $I_2$

$$\frac{F_{12}}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

using fleming's left hand rule it is clear that, Two || currents attract each other and two anti || current repel each other.

**1 AMPERE :-** $I_1 = I_2 = 1A$, $d = 1m$

$$\frac{F}{l} = \frac{\mu_0}{2\pi} = \frac{10^{-7} \times 4\pi}{2\pi} = 2 \times 10^{-7}\ N/m$$

One ampere is defined as that current which when passes along two || conductors in same direction separated by 1m in vacuum exert force of attraction of $2 \times 10^{-7}$ N/m on each other.

# TORQUE ON A CURRENT CARRYING LOOP :-

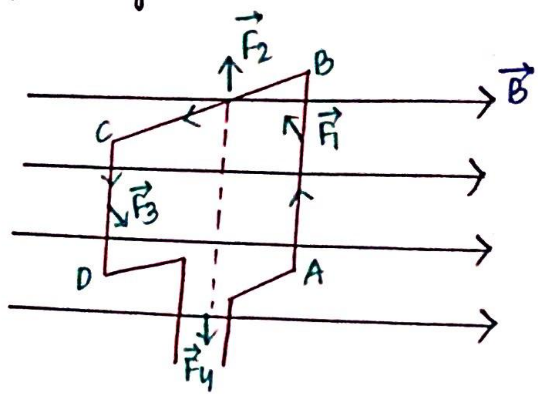


ABCD is a current carrying loop of length $l$ and breadth $b$

$I$ = current passing through it

area of the loop $A = l \times b$

$B$ = magnetic field strength




Force on AB,

$$\vec{F_1} = I(\vec{l} \times \vec{B}) \text{ inward}$$

Force on BC,

$$\vec{F_2} = I(\vec{b} \times \vec{B}) \text{ along the axis upward.}$$

Force on CD,

$$\vec{F_3} = I(\vec{l} \times \vec{B}) \text{ outward}$$

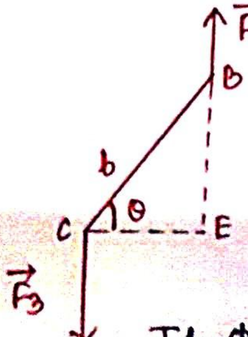
Force on AD,

$$\vec{F_4} = I(\vec{b} \times \vec{B}) \text{ downward}$$

$\vec{F_1}$ and $\vec{F_2}$ are cancelled by $\vec{F_3}$ and $\vec{F_4}$ respectively.

So net force = 0.

but $\vec{F_1}$ and $\vec{F_3}$ are not in same line of action. So they constitute a couple due to which loop rotate.




$$\tau = (BIl \sin 90^\circ) \times CE$$
$$= BIl \times b\cos\theta$$
$$= BIA\cos\theta.$$

for N turns, $\boxed{\tau = BINA\cos\theta}$

If $\phi$ is the angle made by axial vector of coil with $\vec{B}$

$$\theta + \phi = 90^\circ \Rightarrow \theta = 90^\circ - \phi$$

$$\tau = BINA\cos\theta = BINA\cos(90^\circ - \phi) = BINA\sin\phi$$

$$\tau = BM\sin\phi$$

$$\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$$

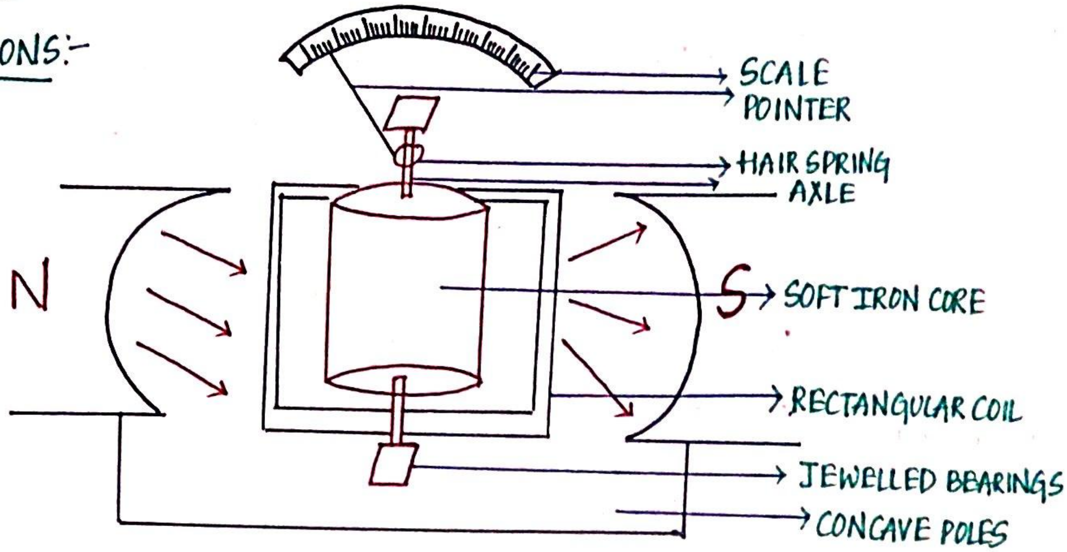
# MOVING COIL GALVANOMETER:-

**GALVANOMETER:-** It is a device that is used to detect small current in the circuit.

**PRINCIPLE:-**

When a current carrying coil is placed in uniform magnetic field, it experiences torque.

**CONSTRUCTIONS:-**




The moving coil galvanometer consists of a coil with many turns free to rotate about a fixed axis, in a uniform radial magnetic field to maintain the plane of the coil always remain parallel to field $\vec{B}$ and to have maximum torque. The soft iron cylinder is used as a core material due to its high permeability which intensifies the $\vec{B}$ and hence increases the sensitivity of the galvanometer. When a current flows through the coil, a torque acts on it.

**THEORY:-**

N = no. of turns
A = area of the coil
B = magnetic field strength

When current I passes through the coil, deflecting torque,

$$\boxed{\tau = BINA\cos\theta}$$

As the $\vec{B}$ is radial i.e angle between plane of the coil and $\vec{B}$ is 0

$$\boxed{\tau = BINA}$$

If K is restoring torque per unit angle of twist,
$\phi$ is the unit angle of twist.

In equilibrium,

$$BINA = K\phi$$

$$\Rightarrow \phi = \left(\frac{BAN}{K}\right) I$$

$$\Rightarrow \phi \propto I$$

When current passes through the coil, it deflects.

## SENSITIVITY OF GALVANOMETER :-

$(I_s) \rightarrow$ Current sensitivity

$(V_s) \rightarrow$ Voltage sensitivity

### (i) CURRENT SENSITIVITY $(I_s)$ :-

It is defined as angle of deflection per unit current.

$$I_s = \frac{\phi}{I} = \boxed{\frac{BAN}{K}}$$

It can be increased :-

(a) ↑ no of turns

(b) ↑ area of coil

(c) increasing $\vec{B}$

(d) ↓ restoring torque (torsion constant)

### (ii) VOLTAGE SENSITIVITY $(V_s)$ :-

It is defined as angle of deflection per unit voltage.

$$V_s = \frac{\phi}{V} = \boxed{\frac{BAN}{KR}}$$

It can be increased :-

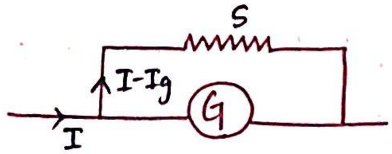
(a) increasing $\vec{B}$

(b) increasing area of coil

(c) decreasing torsion constant.

## CONVERSION OF GALVANOMETER :-

### (a) AMMETER




Galvanometer can be converted to ammeter by connecting a small resistance (shunt) ‖ to it

$I_g \rightarrow$ galvanometer range

$I \rightarrow$ Ammeter range

$G \rightarrow$ resistance of galvanometer.

As the connection is ‖,

$$I_g G = (I - I_g) S$$

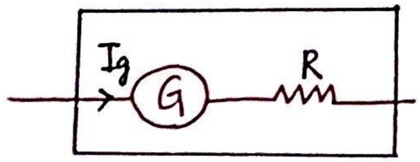
$$\Rightarrow \boxed{S = \frac{I_g G}{I - I_g}}$$

Resistance of ammeter.

$$\boxed{R = \frac{GS}{G+S}}$$

## (b) VOLTMETER:-

Galvanometer can be converted to voltmeter by connecting high resistance in series.




$I_g$ = galvanometer range

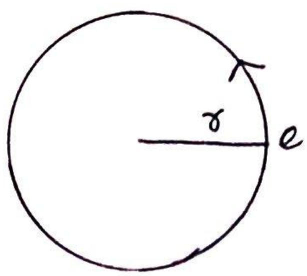
$G$ = galvanometer resistance

$R$ = high resistance.

Range of voltmeter, $\boxed{V = I_g(R+G)}$

Resistance of voltmeter = $\boxed{R+G}$

## MAGNETIC MOMENT OF REVOLVING ELECTRON:-




$$I = \frac{e}{t} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

Magnetic dipole moment,

$$\mu_L = I \times A \times 1$$

$$= \frac{ev}{2\pi r} \times \pi r^2$$

$$\mu_L = \frac{evr}{2}$$

$$\mu_L = \frac{emvr}{2m}$$

$$\Rightarrow \mu_L = \frac{eL}{2m}$$

According to Bohr's postulate,

$$\mu_L = \frac{e}{2m} = \frac{nh}{2\pi}$$

$$\boxed{\mu_L = \frac{neh}{4\pi m}}$$

So, magnetic moment of revolving electron is quantised

$$(\mu_L)_{min} = \frac{eh}{4\pi m} \text{ (Bohr's magneton)}$$

$$= \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} = 9.27 \times 10^{-24} \text{ A m}^2$$

**GYROMAGNETIC RATIO:-** Ratio of magnetic dipole moment of revolving $e^-$ to its angular momentum.

$$\frac{\mu_L}{L} = \frac{e}{2m} = \text{constant} = 8.8 \times 10^{10}$$

# CURRENT ELECTRICITY

(CHAPTER-3)

## ELECTRIC CURRENT (I)

It is defined as rate of flow of electric charge through any cross section of a conductor.

$$\boxed{I = \frac{\text{total charge}}{\text{time taken}} \qquad I = \frac{q}{t} = \frac{ne}{t} \qquad I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}}$$

→ Scalar quantity

SI unit → A

CGS unit → st A

## CURRENT DENSITY (J):-

It is the ratio of the current at that point in the conductor to the area of the cross section of the conductor at that point.

$$J = \frac{I}{A}$$

$$I = JA$$

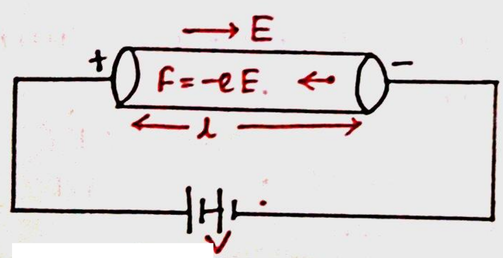
$$\rightarrow \boxed{I = \vec{J} \cdot \vec{A}}$$

→ Vector quantity.

## DRIFT VELOCITY ($v_d$)

It is defined as avg. velocity gained by the free $e^-$s of a conductor in the opposite direction of the externally applied electric field.

## DRIFT OF ELECTRONS:-



If $\vec{U}_1, \vec{U}_2, \ldots\ldots\ldots\ldots \vec{U}_N$ be the velocities of N no of free electrons,

Then, avg velocities of electrons $= \dfrac{\vec{U}_1 + \vec{U}_2 + \ldots\ldots + \vec{U}_N}{N} = 0$

Thus, there is no net flow of charge in any direction. In the presence of electric field, each $e^-$ experiences a force, $\vec{F} = -e\vec{E}$

The negative sign indicate $e^-$ are moving in the opp. direction of $\vec{E}$.

$$\vec{F} = -e\vec{E}$$

$$\Rightarrow m\vec{a} = -e\vec{E}$$

$$\Rightarrow \vec{a} = \frac{-e\vec{E}}{m}$$ , m = mass of the electron.

If n, no. of $e^-$ gain velocity component

$$\vec{V}_1, \vec{V}_2, \ldots\ldots\ldots \vec{V}_n$$

$$\vec{v}_1 = \vec{U}_1 + \vec{a}t_1$$

$$\vec{v}_2 = \vec{U}_2 + \vec{a}t_2$$

$$\vdots$$

$$\vec{v}_n = \vec{U}_n + \vec{a}t_n$$

$$\vec{v}_1 + \vec{v}_2 + \ldots + \vec{v}_n = \vec{U}_1 + \vec{U}_2 + \ldots + \vec{U}_n + \vec{a}(t_1 + t_2 + \ldots + t_n)$$

$$\Rightarrow \frac{\vec{V}_1 + \vec{V}_2 + \ldots + \vec{V}_n}{n} = \cancel{\frac{\vec{U}_1 + \vec{U}_2 + \ldots + \vec{U}_n}{n}}^{\,0} + \frac{\vec{a}(t_1 + t_2 + \ldots + t_n)}{n}$$

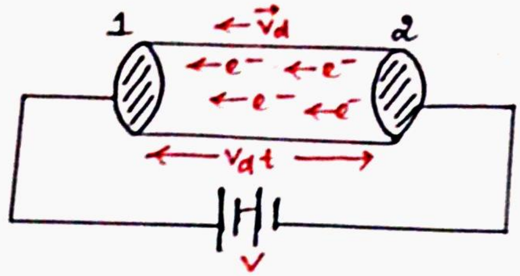
$$\Rightarrow \vec{V}_d = \vec{a}\tau$$ , $V_d$ = drift velocity

$\tau$ = relaxation time.

$\tau$ is the avg. time elapsed between 2 successive collision of the electron.

$$\boxed{\vec{V}_d = \frac{-e\vec{E}\tau}{m}}$$

## RELATION BETWEEN ELECTRIC CURRENT AND DRIFT VELOCITY :-




A = area of the cross-section
n = free electron density
t = time taken by electron to move from cross-section 1 to 2.

distance betn two cross-section = $v_d t$

volume bounded by two cross-section = $Al = Av_d t$

no. of electrons in that volume = $nAv_d t$

no. of electron passes through the cross-section 1 in time t = $nAv_d t$

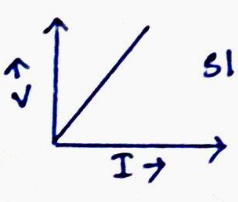
$$I = \frac{q}{t} = \frac{nAv_d t \cdot e}{t} = neAv_d$$

$$\boxed{I = neAv_d}$$

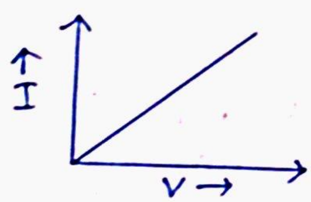
**OHM'S LAW :-** The potential difference between two ends of a conductor is directly proportional to current passing through it at constant temperature.

$$V \propto I$$

$$\Rightarrow \quad V = IR$$




Slope = $\frac{V}{I} = R$




Slope = $\frac{I}{V} = \frac{1}{R}$.

**DEDUCTION OF OHM'S LAW :-**

$$I = neAv_d$$

$$= neA\left(\frac{eV\tau}{ml}\right) = \left(\frac{ne^2A\tau}{ml}\right)V$$

$$V = \left(\frac{ml}{nAe^2\tau}\right)I$$

$\Rightarrow V = RI$ , $R = \frac{ml}{nAe^2\tau}$ , constant for a particular conductor at constant temp.

$\Rightarrow V \propto I$

## LIMITATIONS OF OHM'S LAW :-

1. only valid at constant temp.
2. some substances do not obey ohm's law.

## VECTOR FORM OF OHM'S LAW :-

$$J = \frac{I}{A}$$

$$= \frac{neAv_d}{A}$$

$$= n \cdot \frac{eE\tau}{m} \cdot e$$

$$= \left(\frac{ne^2\tau}{m}\right) E$$

$$J = \sigma E$$

In vector form.

$$\boxed{\vec{J} = \sigma \vec{E}}$$

## RESISTANCE (R) :-

It is defined as the opposition offered to the flow of current

SI unit → $\Omega$

CGS unit → St $\Omega$ / ab $\Omega$

$$\boxed{R = \frac{ml}{nAe^2\tau}}$$

**Resistance depends on :-**

1. Geometry of conductor
2. Nature of material
3. Temperature.

## CONDUCTANCE (G) :-

It is defined as the reciprocal of resistance.

$$G = \frac{1}{R} = \boxed{\frac{nAe^2\tau}{ml}}$$

SI unit.- $\Omega^{-1}$

## RESISTIVITY ($\rho$) :-

$$R = \frac{m l}{n A e^2 \tau}$$

$$= \left(\frac{m}{n e^2 \tau}\right) \frac{l}{A}$$

$$\Rightarrow R = \rho \frac{l}{A} \quad \text{where } \boxed{\rho = \frac{m}{n e^2 \tau}}$$

, which is constant for a particular material at constant temp.

**DEFINATION OF $\rho$ :-** $\rho = \frac{RA}{L}$ , $A = 1m^2$, $l = 1m$, $\rho = R$

It is defined as resistance of a rod of that material of length 1 m and area of cross section $1\,m^2$.

SI unit → $\Omega m$

$$R = \frac{\rho l}{A}$$

$R \propto l$ ( A is constant)

$R \propto \frac{1}{A}$ ( $l$ is constant)

## SPECIAL CASE :-

### CASE-I

When A is not constant

$$R = \rho \frac{l}{A} \times \frac{l}{l}$$

$$= \frac{\rho l^2}{Vol}$$

$$\Rightarrow \boxed{R \propto l^2}$$

### CASE-II

when $l$ is not constant

$$R = \rho \frac{l}{A} \times \frac{A}{A} = \frac{\rho l A}{A^2}$$

$$= \frac{\rho \times Vol}{A^2}$$

$$\Rightarrow \boxed{R \propto \frac{1}{A^2}}$$

## CONDUCTIVITY ($\sigma$) :-

It is defined as reciprocal of resistivity

$$\boxed{\sigma = \frac{1}{\rho} = \frac{n e^2 \tau}{m}}$$

SI unit :- $\Omega^{-1} m^{-1}$

## MOBILITY ($\mu$)

Mobility of a charge is defined as drift velocity per unit electric field.

$$\mu = \frac{v_d}{E}$$

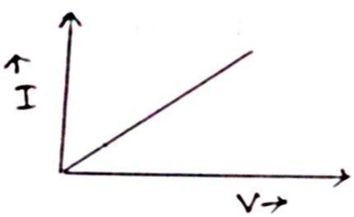
$$\mu = \frac{eE\tau}{m} \times \frac{1}{E}$$

$$\boxed{\mu = \frac{e\tau}{m}} \text{ (for electron)}$$

$$\boxed{\mu = \frac{q\tau}{m}} \text{ (general)}$$

* For a particular charge, $\mu \propto \tau \propto \frac{1}{\text{temp}}$.
* At constant temperature, $\mu \propto \frac{q}{m}$.

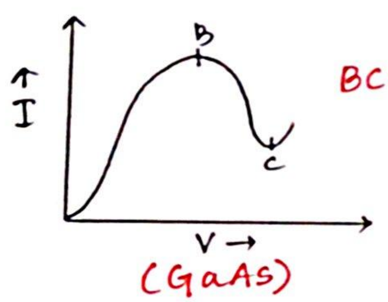
**OHMIC SUBSTANCE :-** Substance which obeys ohm's law.




eg :- all metals carrying low current.

**NON-OHMIC SUBSTANCE :-** Substance which doesn't obey ohm's law.

eg :- dil $H_2SO_4$, water voltameter, vacuum diode, GaAs




(GaAs)

BC = −ve region

**TEMPERATURE DEPENDANCE OF RESISTIVITY :-**

$\rho_0 \rightarrow$ initial velocity at temp $T_0$

$\rho \rightarrow$ final velocity at temp $T$

$\rho - \rho_0 \rightarrow$ change in resistivity.

$\Rightarrow \rho - \rho_0 \propto (T - T_0)$

$\Rightarrow \rho - \rho_0 \propto \rho_0$

$\Rightarrow \rho - \rho_0 \propto \rho_0 (T - T_0)$

$\Rightarrow \rho - \rho_0 = \alpha \rho_0 (T - T_0)$ , $\alpha$ = temp. coefficient of resistivity.

$\Rightarrow \boxed{\alpha = \dfrac{\rho - \rho_0}{\rho_0 (T - T_0)}}$

It is defined as the ratio between change in resistivity per original resistivity for degree rise of temp.

SI unit → $K^{-1}$

**CONDUCTOR :-** for conductor, $\alpha$ = +ve i.e $\rho$ increases with ↑ in temp.

**Cause :-** When temp ↑, K.E of free $e^-$ ↑ so no of collision per sec ↑. Hence, resistivity ↑.

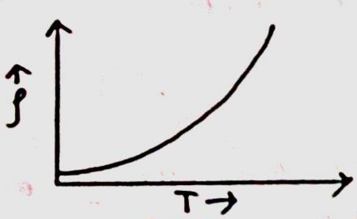
**SEMICONDUCTOR :-** $\alpha$ = −ve, with ↑ in temp, $\rho$ ↓

**Cause :-** When temp ↑, charge carrier density ↑ses which dominate the effect of $\tau$.

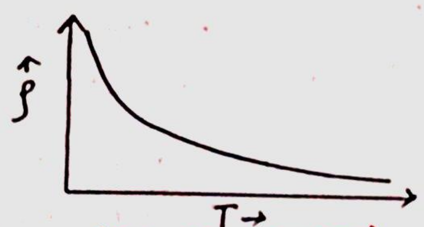
As $\rho = \dfrac{m}{ne^2\tau}$ , so $\rho$ decreases.

**INSULATOR :-** $\alpha$ = −ve, $\rho$ ↓ses with temperature.

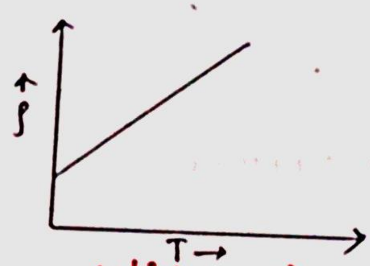
## $\rho \sim T$ GRAPHS :-



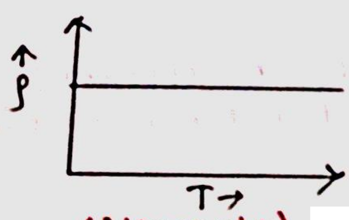

(Cu)




(Semiconductor)




(Nichrome)




(Manganin)

## USE OF ALLOY IN MAKING RESISTOR :-

① $\rho$ of alloy is very high

② They have very small temp. of coefficient.

③ least affected by atmospheric conditions such as air, moisture, pressure.

## COLOUR CODE OF CARBON RESISTOR :-

- B → Black → 0
- B → Brown → 1
- R → Red → 2
- O → Orange → 3
- Y → Yellow → 4
- G → Green → 5
- B → Blue → 6
- V → Violet → 7

G → Grey → 8
W → White → 9

G → Gold → 5% ⎫
S → Silver → 10% ⎬ Tolerance
N → No colour → 20% ⎭

**TRICK TO REMEMBER :-** B B ROY of Great Britain had a Very Good Wife Wearing Gold & Silver Necklace.

**INTERNAL RESISTANCE ($r$) :-** The opposition offered by electrolyte due to flow of electric current is called internal resistance.

**CAUSE :-** Due to collision of ions.

It depends on (1) nature of electrolyte and electrode.

(2) area of electrode dipped in electrolyte.

(more is the area, less is the internal resistance)

(3) distance between the two electrodes

(more is the separation, more is the internal resistance)

(4) temperature

(when temp ↑ses, internal resistance ↓ because viscosity decreases)

## RELATION BETWEEN EMF AND POTENTIAL DIFFERENCE :-

(A) DISCHARGING CONDITION

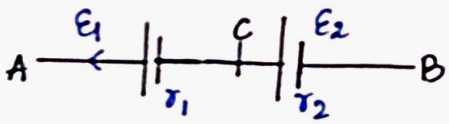
$$\boxed{\varepsilon = V + Ir}$$

(B) CHARGING CONDITION

$$\boxed{V = \varepsilon + Ir}$$

## COMBINATION OF CELL :-

(A) SERIES




for cell 1, $V_{AC} = \varepsilon_1 - Ir_1$

$\Rightarrow V_A - V_C = \varepsilon_1 - Ir_1$ —— (1)

for cell 2, $V_{CB} = \varepsilon_2 - Ir_2$

$\Rightarrow V_C - V_B = \varepsilon_2 - Ir_2$ —— (2)

Adding (1) & (2),

$V_A - V_B = \varepsilon_1 + \varepsilon_2 - I(r_1 + r_2)$ —— (3)

for the combination,

$V_{AB} = \varepsilon - Ir$

$\Rightarrow V_A - V_B = \varepsilon - Ir$ —— (4)

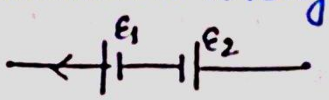
From eqn (3) & (4),

$\varepsilon_1 + \varepsilon_2 - I(r_1 + r_2) = \varepsilon - Ir$

$\Rightarrow \boxed{\varepsilon = \varepsilon_1 + \varepsilon_2}$ $\boxed{r = r_1 + r_2}$

SPECIAL CASE :-

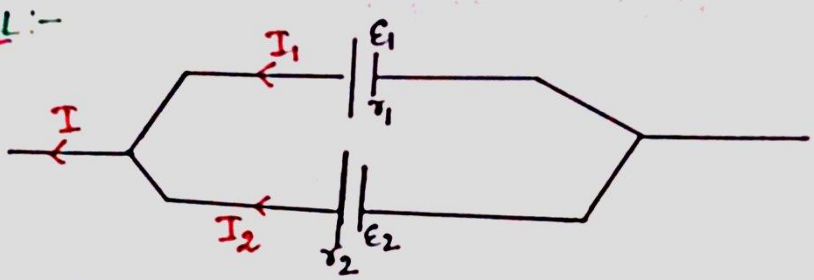
If the connection is wrong




$$\boxed{\varepsilon = \varepsilon_1 - \varepsilon_2} \quad (\text{If } \varepsilon_1 > \varepsilon_2)$$

$$\boxed{r = r_1 + r_2}$$

(B) PARALLEL :-




for cell 1, $V = \varepsilon_1 - Ir_1 \Rightarrow I_1 = \dfrac{\varepsilon_1 - V}{r_1}$ — ①

for cell 2, $I_2 = \dfrac{\varepsilon_2 - V}{r_2}$ — ②

Similarly for the combination,

$$I = \frac{\varepsilon - V}{r} \quad — ③$$

$$I = I_1 + I_2$$

$$\Rightarrow \frac{\varepsilon - V}{r} = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2}$$

$$\Rightarrow \frac{\varepsilon}{r} - \frac{V}{r} = \frac{\varepsilon_1}{r_1} - \frac{V}{r_1} + \frac{\varepsilon_2}{r_2} - \frac{V}{r_2} \quad \Rightarrow \frac{\varepsilon}{r} - \frac{V}{r} = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

Comparing both sides,

$$\frac{\varepsilon}{r} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \quad \& \quad \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2} \Rightarrow r = \frac{r_1 r_2}{r_1 + r_2}$$

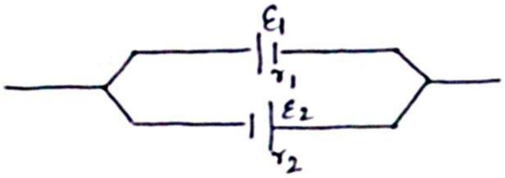
$$= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 \cdot r_2} \cdot r \quad = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \cdot \frac{r_1 r_2}{r_1 + r_2}$$

$$\boxed{\varepsilon = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}}$$

## SPECIAL CASE :-

### CASE-I

If connection is wrong




$$\boxed{\varepsilon = \frac{\varepsilon_1 r_2 - \varepsilon_2 r_1}{r_1 + r_2}}$$

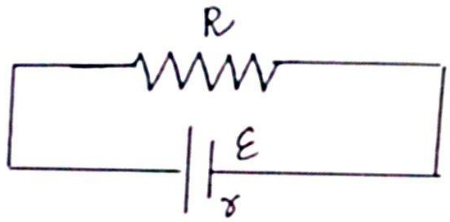
### CASE-II :-

If $\varepsilon_1 = \varepsilon_2 = \varepsilon$

$r_1 = r_2 = r$

$$\boxed{\varepsilon_{net} = \varepsilon}$$

## EXPRESSION OF CURRENT :-




$$\varepsilon = V + Ir$$

$$\Rightarrow \varepsilon = IR + Ir$$

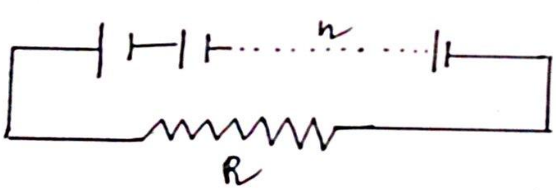
$$\Rightarrow \varepsilon = I(R + r)$$

$$\Rightarrow I = \frac{\varepsilon}{R + r}$$

$$\boxed{I = \frac{\text{net emf}}{\text{net resistance}}}$$

## COMBINATION OF IDENTICAL CELL :-

### (A) SERIES COMBINATION :-




$n$ = no. of cells connected in series

net emf $= n\varepsilon$

$$I = \frac{n\varepsilon}{nr + R} = \frac{n\varepsilon}{nr + R} = \frac{n\varepsilon}{nr + R}$$

CASE-I

If $R >>> nr$

$\boxed{I = \frac{n\varepsilon}{R}}$,

Current depends on no. of cells.

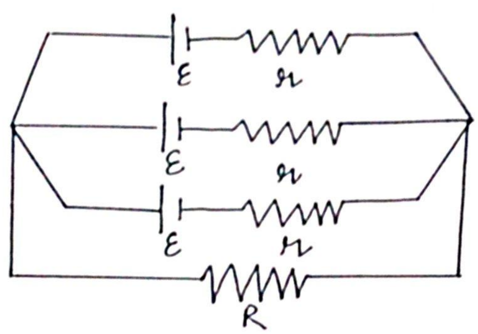
CASE-II

If $R <<< nr$

$\boxed{I = \frac{\varepsilon}{r}}$

Series connection is useful when external resistance is very large.

## (B) PARALLEL COMBINATION :-




Total emf = $\varepsilon$

Net. internal resistance = $r/n$

Net. resistance of entire network = $R + \frac{r}{n}$

$$I = \frac{\varepsilon}{R + \frac{r}{n}} = \frac{n\varepsilon}{nR + r}$$

CASE-I

If $R >>>> r$, $r$ can be neglected

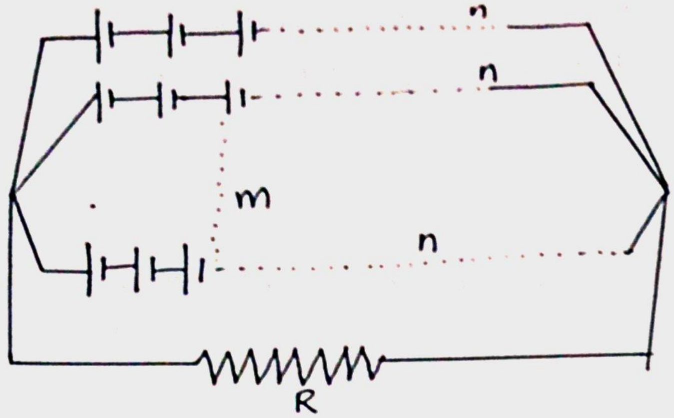
$$I = \frac{n\varepsilon}{nR} = \boxed{\frac{\varepsilon}{R}}$$

CASE-II

If $R <<< r$, R can be neglected

$$I = \frac{n\varepsilon}{r} = \boxed{n\left(\frac{\varepsilon}{R}\right)}$$

## MIXED CONNECTION :-




n = no. of cells in each row
m = no. of such rows

Net emf = $n\mathcal{E}$

Net internal resistance = $\frac{1}{R'} = \frac{1}{nr} + \frac{1}{nr} + \ldots\ldots m = \frac{m}{nr}$

$$R' = \frac{nr}{m}$$

$$I = \frac{n\mathcal{E}}{R + \frac{nr}{m}} = \frac{mn\mathcal{E}}{Rm + nr}$$

$$mR + nr = (\sqrt{mR})^2 + (\sqrt{nr})^2 = (\sqrt{mR} - \sqrt{nr})^2 + 2\sqrt{mnRr}$$

Current will be max. when $\sqrt{mR} = \sqrt{nr}$

$$\Rightarrow mR = nr$$

$$\Rightarrow \boxed{R = \frac{nr}{m}}$$

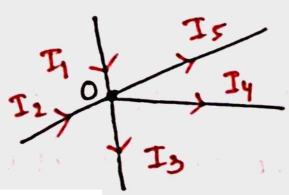
$\Rightarrow$ Total external resistance = Total internal resistance.

## KIRCHHOFF'S LAWS :-

### (a) KIRCHHOFF CURRENT LAW / JUNCTION LAW :-

It states that algebraic sum of currents meeting at a junction is zero.

$$\boxed{\Sigma I = 0}$$



The current coming towards the junction is taken as +ve.
The current going away from the junction is taken as −ve.

$$\Rightarrow I_1 + I_2 - I_3 - I_4 - I_5 = 0$$

$$\Rightarrow \boxed{I_1 + I_2 = I_3 + I_4 + I_5}$$

So, net current coming towards the junction = net current going out of the junction.

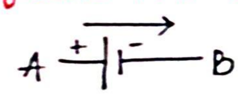
(b) KVL / Loop Law :-

It states that the algebraic sum of potential differences across cells and resistors in a close loop is 0.

$$\boxed{\Sigma \Delta V = 0}$$

SIGN CONVENTION:-

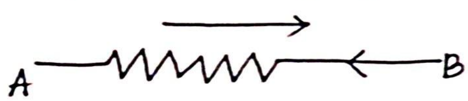
① If one moves from +ve to −ve of a cell, then emf is −ve




$$\Delta V = V_B - V_A$$

$$\boxed{\mathcal{E} = -ve}$$

② If one move opposite to direction of current then the product of current and resistance (IR) is taken as +ve.



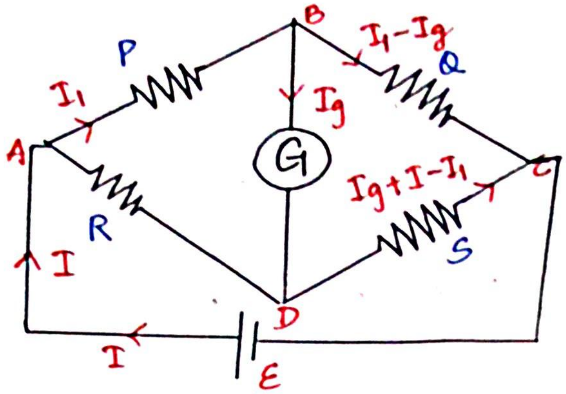

$$\Delta V = V_B - V_A$$

$$\Rightarrow \boxed{IR = +ve}$$

WHEATSTONE BRIDGE:-

P, Q, R, S are the 4 resistors connected in wheatstone bridge.

G → resistance of galvanometer.




Using KVL,

ABDA

$$-PI_1 - GI_g + R(I-I_1) = 0$$

$$-I_1P - I_gG + (I-I_1)R = 0 \quad \text{—— (I)}$$

BCDB

$$-Q(I_1-I_g) + S(I-I_1+I_g) + GI_g = 0 \quad \text{—— (II)}$$

The bridge is said to be balanced when no current passes through galvanometer

i.e $I_g = 0$

eqn (I) & (II) becomes,

$$-I_1P + (I-I_1)R = 0$$

$$\boxed{(I-I_1)R = I_1P} \quad \text{—— (III)}$$

$$-QI_1 + SI - SI_1 = 0$$

$$\boxed{I_1Q = (I-I_1)S} \quad \text{—— (IV)}$$

Dividing (III) & (IV),

$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

It is the balanced condition of wheatstone bridge.

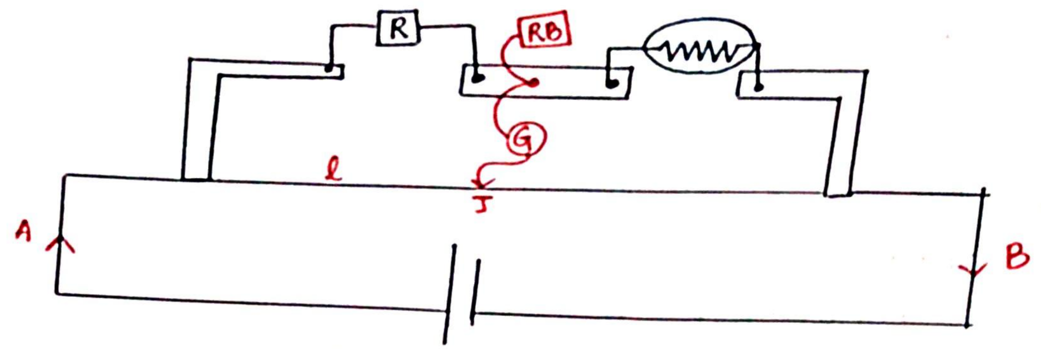
(Q):- What happens to the balanced condition if cell & galvanometer are interchanged?

No change.

## METER BRIDGE :-

It is an electrical device used to measure unknown resistance.

PRINCIPLE :- It works on the balanced condition of wheatstone bridge.




R= known resistance from resistance box

S= unknown resistance

J= null point such that AJ=l

According to balanced condition of wheatstone bridge.

$$\frac{R}{S} = \frac{R_{AJ}}{S_{BJ}}$$

$$\Rightarrow \frac{R}{S} = \frac{l}{100-l}$$

$$\Rightarrow \boxed{S = \frac{R(100-l)}{l}}$$

(Q) :- When is metre bridge most sensitive?
If It is obtained at the middle of the wire

(Q) :- Why thick copper strips are used?
Because of negligible resistance

(Q) :- What happens to balancing length if resistance R increases?
Increases.

## POTENTIOMETER

It is an electrical device which is used to measure emf of a cell.

# PRINCIPLE OF POTENTIOMETER :-

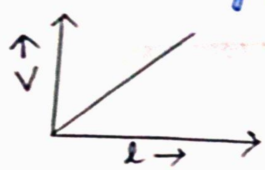
$$V = IR$$

$$\Rightarrow V = I\rho l/A$$

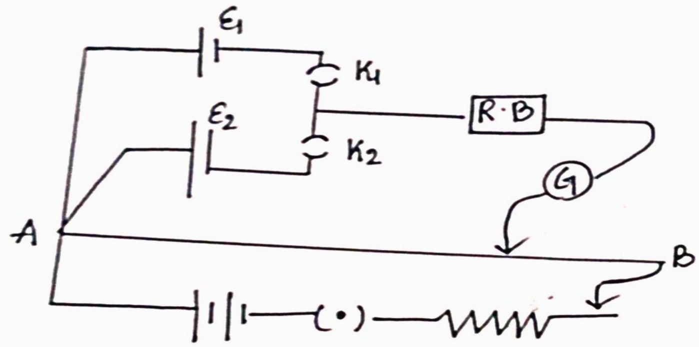
$$\Rightarrow V = \left(\frac{I\rho}{A}\right) l$$

$$\Rightarrow \boxed{V = Kl} \quad , \quad K = \rho I/A$$

The principle is that when a constant current flows through a wire of uniform cross-section and composition, the potential drop across any length of the wire is directly proportional to that length.




## ① Comparison of emf :-




$\varepsilon_1$ & $\varepsilon_2$ → are two primary cells

$K_1$ & $K_2$ → Two way key

RB → Resistance box

$\varepsilon$ → Driving cell

K → Key of Auxillary / primary circuit

R → Rheostat

AB → potentiometer wire

Close $K_1$, $K_2$ is open

$$\varepsilon_1 \propto l_1$$

$$\Rightarrow \varepsilon_1 = Kl_1 \quad \text{— (I)}$$

Close $K_2$, $K_1$ is open.

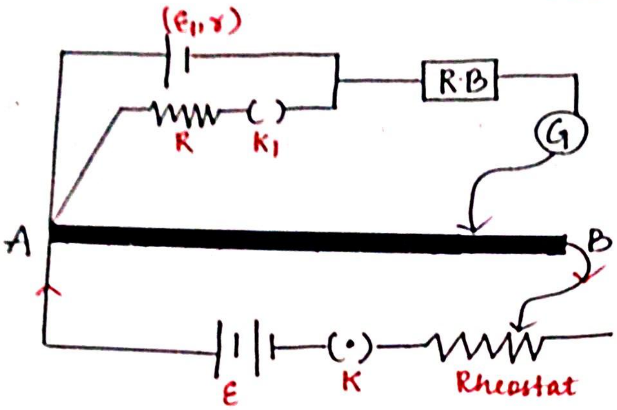
$$\varepsilon_2 \propto l_2$$

$$\Rightarrow \varepsilon_2 = Kl_2 \quad \text{— (II)}$$

Eqⁿ (I) by Eqⁿ (II),

$$\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}}$$

## (2) DETERMINATION OF INTERNAL RESISTANCE OF GIVEN PRIMARY CELL:-




CASE-I

$K_1$ is open

$E_1 \propto l_1 \Rightarrow \boxed{E_1 = K l_1}$ —— (I)

CASE-II

$V_1$ is closed.

$V \propto l_2 \Rightarrow \boxed{V = K l_2}$ —— (II)

$$\frac{\text{eqⁿ (I)}}{\text{eqⁿ (II)}} = \frac{E_1}{V} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{I(R+r)}{IR} = \frac{l_1}{l_2}$$

$$\Rightarrow 1 + \frac{r}{R} = \frac{l_1}{l_2}$$

$$\Rightarrow \frac{r}{R} = \frac{l_1}{l_2} - 1$$

$$\Rightarrow \boxed{r = \left(\frac{l_1 - l_2}{l_2}\right) R}$$

$l_1$ = balancing length when only $E_1$ is connected

$l_2$ = balancing length when R is connected

# ELECTROSTATIC POTENTIAL & CAPACITANCE (1)

(CHAPTER-2)

**ELECTROSTATIC POTENTIAL:-** It is defined as the amount of work done to bring unit +ve charge from $\infty$ to that point along any path without any acceleration.

$$\boxed{V = \frac{W}{q_0}}$$

→ It is a scalar quantity

→ SI unit- J/C or V

→ Dimension- $[M^1L^2T^{-3}A^{-1}]$

**POTENTIAL DIFFERENCE:-** It is defined as the amount of work done to bring unit +ve charge from one point to another point along any path without any acceleration.

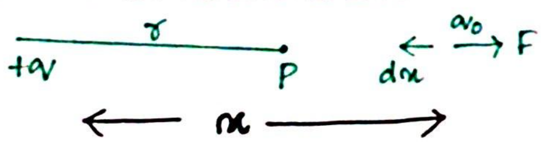
$$\Delta V = \frac{W}{q_0}$$

$$\Rightarrow W = q_0 \Delta V$$

if $q_0 = q$

$$\boxed{W = q\Delta V}$$

**POTENTIAL DUE TO A POINT CHARGE:-**




P is any point at a dist. r from +q charge, work done to bring $q_0$ from infinity to point P is equal to

$$W = \int \vec{F} \cdot \vec{dx}$$

$$= \int_{\infty}^{r} F\,dx \cos 180^\circ$$

$$= -\int_{\infty}^{r} K \frac{q q_0}{x^2} \, dx$$

$$= -K q q_0 \left[ \frac{x^{-1}}{-1} \right]_{\infty}^{r}$$

$$= K q q_0 \left[ \frac{1}{x} \right]_{\infty}^{r}$$

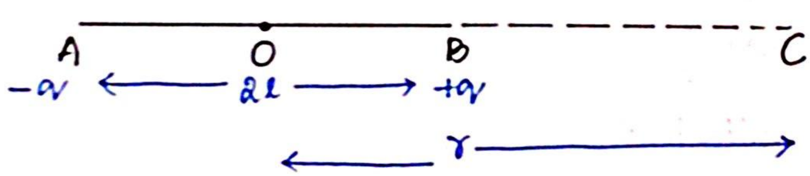
$$= K q q_0 \left( \frac{1}{r} - \frac{1}{\infty} \right)$$

$$W = \frac{K q q_0}{r}$$

$$V = \frac{W}{q_0}$$

$$\Rightarrow \boxed{V = \frac{Kq}{r}}$$

## POTENTIAL DUE TO DIPOLE ON THE AXIAL LINE:-




Due to +q, charge potential,

$$V_1 = \frac{Kq}{(r-l)}$$

Due to -q, charge potential,

$$V_2 = \frac{-Kq}{(r+l)}$$

Net potential,

$$= \frac{Kq}{r-l} - \frac{Kq}{r+l}$$

$$= Kq\left(\frac{1}{r-l} - \frac{1}{r+l}\right)$$

$$= Kq\left[\frac{r+l-r+l}{(r-l)(r+l)}\right]$$

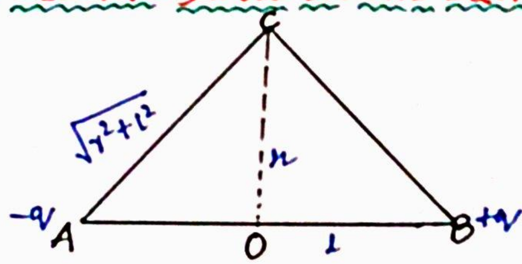
$$= \frac{Kq \cdot 2l}{r^2-l^2}$$

$$\boxed{V = \frac{KP}{r^2-l^2}}$$

For ideal dipole, $l <<<< r$, so $l^2$ can be neglected.

$$\boxed{V = \frac{KP}{r^2}}$$

## POTENTIAL DUE TO AN ELECTRIC DIPOLE ON THE EQUITORIAL LINE:-




Due to $+q$ charge, potential

$$V_1 = \frac{Kq}{\sqrt{r^2+l^2}}$$

Due to $-q$ charge, potential

$$V_2 = \frac{K(-q)}{\sqrt{r^2+l^2}}$$

Net potential,

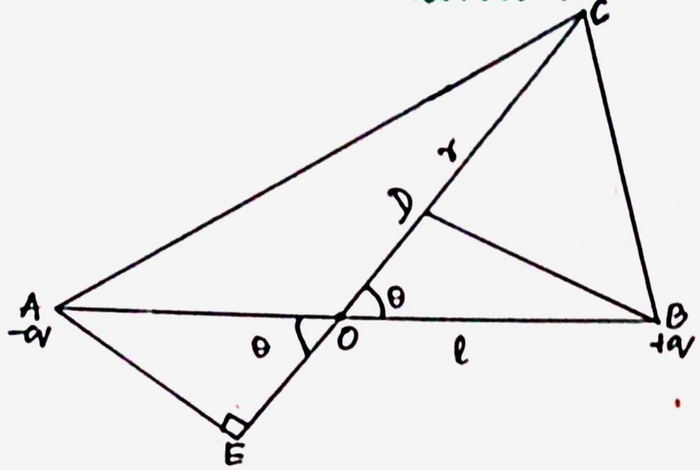
$$V = V_1 + V_2$$

$$= \frac{Kq}{\sqrt{r^2+l^2}} - \frac{Kq}{\sqrt{r^2+l^2}}$$

$$\boxed{V = 0}$$

# POTENTIAL DUE TO AN ELECTRIC DIPOLE AT ANY POINT:-






C is any point at a dist r from the centre of the dipole making an angle θ.

Due to +q,

$$V_1 = \frac{Kq}{BC}$$

$$= \frac{Kq}{CD} \quad (\because BC \sim CD)$$

$$= \frac{Kq}{OC-OD}$$

$$V_1 = \frac{Kq}{r-l\cos\theta}$$

Due to -q,

$$V_2 = \frac{K(-q)}{AC}$$

$$= \frac{K(-q)}{CE} \quad (\because AC \sim CE)$$

$$= \frac{K(-q)}{OC+OE}$$

$$V_2 = \frac{-Kq}{r+l\cos\theta}$$

Total potential,

$$V = V_1 + V_2$$

$$= \frac{Kq}{r - l\cos\theta} - \frac{Kq}{r + l\cos\theta}$$

$$= Kq\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$= Kq\left[\frac{r + l\cos\theta - r + l\cos\theta}{(r - l\cos\theta)(r + l\cos\theta)}\right]$$

$$= \frac{Kq\,2l\cos\theta}{r^2 - l^2\cos^2\theta} = \frac{KP\cos\theta}{r^2 - l^2\cos^2\theta}$$

$$\boxed{V = \frac{KP\cos\theta}{r^2 - l^2\cos^2\theta}}$$

for ideal dipole. $l \lll r$, $l^2$ can be neglected.

$$\boxed{V = \frac{KP\cos\theta}{r^2}}$$

## POTENTIAL DUE TO SYSTEM OF CHARGES:-

**POTENTIAL ENERGY:-** It is defined as the amount of work done to bring the charges from $\infty$ to their respective positions to constitute the systems

**(1) 2 particles system:-**

$$q_1 \overset{r_{12}}{\text{———}} q_2$$

To bring $q_1$, work done $W_1 = 0$

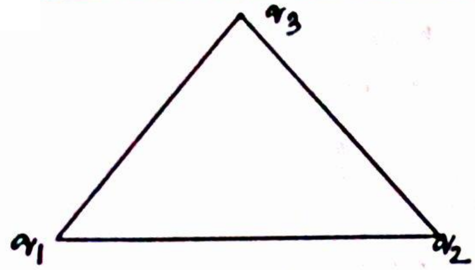
To bring $q_2$. work done $W_2 = \frac{Kq_1 q_2}{r_{12}}$

Potential Energy,

$U = W = W_1 + W_2$

$$\rightarrow \boxed{U = \frac{Kq_1 q_2}{r_{12}}}$$

**(2) 3 particles system:-**




to bring $q_1$, work done $W_1 = 0$

to bring $q_2$, work done $W_2 = \frac{Kq_1q_2}{r_{12}}$

to bring $q_3$, work done $W_3 = \frac{Kq_2q_3}{r_{23}} + \frac{Kq_1q_2}{r_{12}}$

Total potential energy,

$$U = \frac{Kq_1q_3}{r_{13}} + \frac{Kq_2q_3}{r_{23}} + \frac{Kq_1q_2}{r_{12}}$$

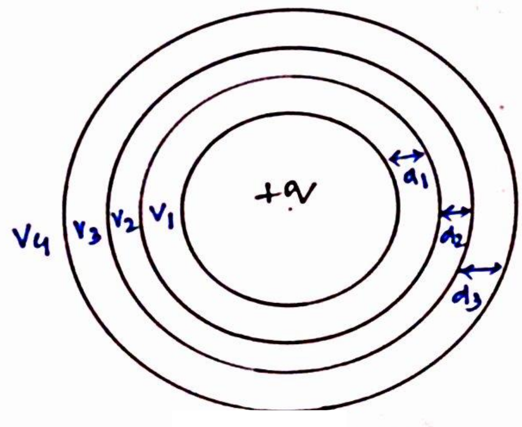
$$\boxed{U = K\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_2}{r_{12}}\right)}$$

(3) 4 particle system

$$\boxed{U = \frac{Kq_1q_2}{r_{12}} + \frac{Kq_1q_3}{r_{13}} + \frac{Kq_1q_4}{r_{14}} + \frac{Kq_2q_3}{r_{23}} + \frac{Kq_2q_4}{r_{24}} + \frac{Kq_3q_4}{r_{34}}}$$

## EQUIPOTENTIAL SURFACES:

Any surface which has same electrostatic potential at every point on it is called an equipotential surfaces.




$d_1 < d_2 < d_3$

$V_1 > V_2 > V_3 > V_4$

## PROPERTIES:-

1. Along the electric field, potential decreases
2. Work done to move a charge on equipotential surface

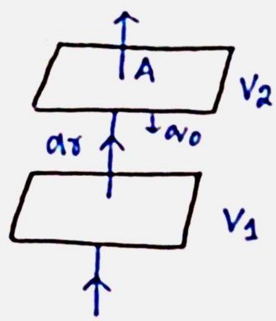
$$W = q\Delta V = q\cdot 0 = 0$$

3. Two equipotential surface never cross each other.

**Reason**:- If they cross at every point on the line of intersection two potential appear which is not possible.

4. Electric field is always $\perp^r$ to the equipotential surface.

## RELATION BETWEEN ELECTRIC FIELD INTENSITY AND POTENTIAL:-




Work done to move $q_0$ from A to B.

$$dw = \vec{F}\cdot\vec{dr}$$
$$= q_0\vec{E}\cdot\vec{dr}$$
$$= q_0 E dr \cos 180^\circ$$
$$= -q_0 E dr$$

Again. $dw = q_0 dv$

Now, $-q_0 E dr = q_0 dv$

$\Rightarrow -E dr = dv$

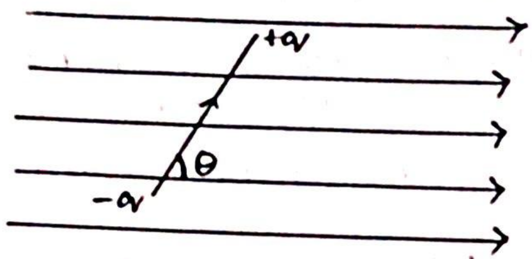
$\Rightarrow \boxed{E = -\frac{dv}{dr}}$

Electric field intensity is negative of potential gradient.

In compenent form,

$$E_x = \frac{-\partial v}{\partial x}, \quad E_y = \frac{-\partial v}{\partial y}, \quad E_x = \frac{-\partial v}{\partial x}$$

## POTENTIAL ENERGY OF DIPOLE IN UNIFORM ELECTRIC FIELD:-




θ is the angle between dipole moment and electric field intencity.

Work done to rotate the dipole from $\theta_1$ angle to $\theta_2$ angle.

$$W = \int_{\theta_1}^{\theta_2} \tau \cdot d\theta$$

$$= \int_{\theta_1}^{\theta_2} PE\sin\theta \cdot d\theta$$

$$= PE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= PE \left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= PE \left[-\cos\theta_2 + \cos\theta_1\right]$$

$$\boxed{W = PE(\cos\theta_1 - \cos\theta_2)}$$

When $\theta_1 = 90^\circ, \theta_2 = \theta$

$$W = PE(\cos 90^\circ - \cos\theta) = -PE\cos\theta$$

$$\therefore \boxed{U = -PE\cos\theta}$$

**CASE-1**

When $\theta = 0$

$\boxed{U = -PE}$

(minimum)

It is in stable equilibrium.

**CASE-2**

When $\theta = 180^\circ$

$\boxed{U = PE}$

(maximum)

It is in unstable equilibrium.

**CASE-3**

When $\theta = 90^\circ$

$\boxed{U = 0}$

## POTENTIAL ENERGY OF 2 PARTICLE SYSTEM: IN EXTERNAL ELECTRIC FIELD:-


Work done to bring $q_1$,

$$W = q_1(V_1 - V_\infty)$$

$$W = q_1 V_1$$

Work done to bring $q_2$,

$$W = q_2 V_2 + \frac{K q_1 q_2}{r_{12}}$$

Potential Energy,

$$\boxed{U = q_1 V_1 + q_2 V_2 + \frac{K q_1 q_2}{r_{12}}}$$

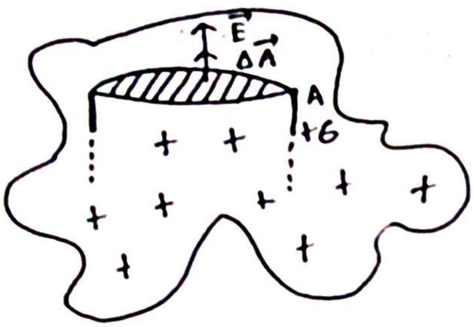
## BEHAVIOUR OF CONDUCTOR IN ELECTROSTATICS:-

1. Net electrostatic field is zero in the interior of a conductor.
2. Just outside the surface of a charged conductor, electric field is normal to the surface.
3. The net charge in the interior of the conductor is 0.
4. Potential is constant within and on the surface of the conductor:

$$E = -\frac{dV}{dr} \Rightarrow -\frac{dV}{dr} = 0 \Rightarrow V = \text{constant}$$

(Q) Prove that electric field at the surface of the charged conductor is directly proportional to the surface charge density.

In order to calculate electric field inside the conductor, let us assume pill box shaped gaussian surface.




According to gauss law,

$$\oint \vec{E} \cdot \vec{dA}$$
$$= \oint E dA \cos\theta$$
$$= EA$$

According to gauss's law,

$$\phi = \frac{q_{en}}{\epsilon_0} = EA = \frac{6A}{\epsilon_0}$$

$$\Rightarrow E = \frac{6}{\epsilon_0}$$

$$\Rightarrow E \propto 6$$

$$\boxed{\vec{E} = \frac{6}{\epsilon_0}\hat{n}}$$

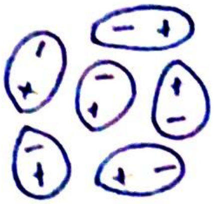
**ELECTROSTATICS SHIELDING:-** The phenomenon of making a region free from any $e^-$ field is called electrostatics shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

**DI-ELECTRICS AND POLARISATION:-**

| POLAR-DIELECTRIC | NON-POLAR DI-ELECTRIC |
|---|---|
| ① The di-electric in which centre of +ve charge doesn't coincide with the centre of −ve charge is called polar-di-electric. | The dielectric in which centre of +ve charge exactly coincide with the centre of −ve charge is called non-polar dielectric |
| For eg :- $H_2O$, $HCl$, $CH_3OH$, $CH_3COOH$ etc. | For eg:- $CO_2$, $N_2$, $O_2$, $H_2$ etc. |
| ② unsymmetrical shape. | symmetrical shape |
| ③ It posses a permanent dipole moment of the order $10^{-30}$ cm. | There is no permanent dipole moment. |

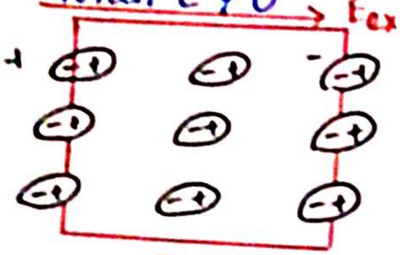
## POLARISATION IN EXTERNAL FIELD:-

When $\vec{E}=0$



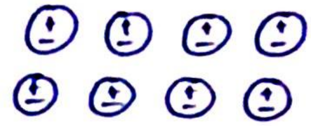
$\Sigma P=0$ because of random orientation of the individual atom.

When $\vec{E}\neq 0$ $\quad \vec{E}_{ext}$



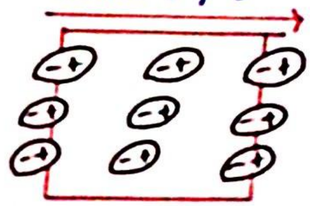
$\Sigma P\neq 0$

$\vec{E}=0$



$\Sigma P=0$ as the centre coincide.

When $\vec{E}\neq 0$ $\quad \vec{E}_{ext}$



$\Sigma P\neq 0$

## CAPACITANCE:-

The device that can store charge or energy is called capacitor. The capacity of a capacitor is called capacitance.

Representation:- 

$$Q \propto V$$

$$\Rightarrow Q = CV$$

$$\Rightarrow \boxed{C=\frac{Q}{V}}$$ , C is capacitance.

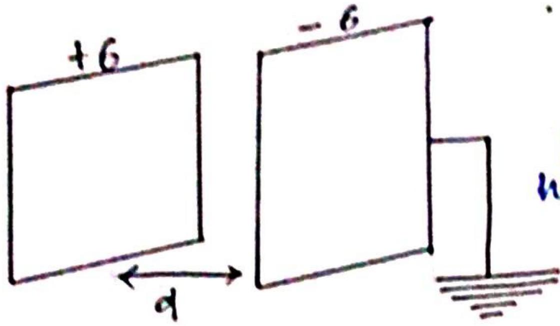
It is defined as the ratio between amount of charge stored in the plates to the potential maintained across its plate.

SI unit → F

CGS unit → stF, abF

Dimension → $[M^{-1}L^{-2}T^{4}A^{2}]$

# PARALLEL PLATE CAPACITOR :-




It consists of 2 parallel metal plates separated by some distance having some insulating medium between them. 1 plate is given +ve charge and other plate is connected to earth.

A = common area
d = separation betn 2 plates
$\sigma$ = surface charge density = $q/A$

$\vec{E}$ = electric field betn 2 plates

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$= \frac{\sigma}{2\epsilon_0}\hat{i} + \frac{\sigma}{2\epsilon_0}\hat{i}$$

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{i}$$

We know, $E = \frac{-dv}{dr}$ $\Rightarrow -dv = \vec{E}\cdot\vec{dr}$

$$\Rightarrow -dv = E\,dr\cos 180^\circ$$

$$\Rightarrow dv = E\,dr$$

$$\Rightarrow \int_0^v dv = \int_0^d E\,dr$$

$$\Rightarrow V = E[r]_0^d$$

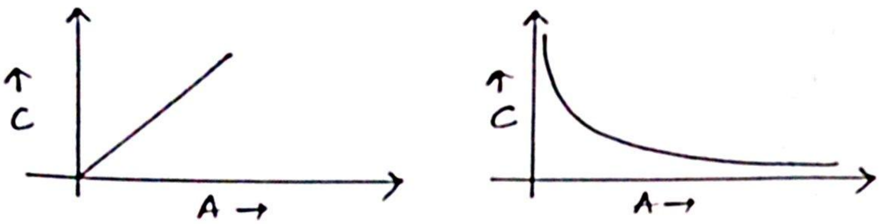
$$\Rightarrow V = Ed$$

$$= \frac{\sigma}{\epsilon_0}d$$

$$\Rightarrow \boxed{V = \frac{\sigma}{\epsilon_0}d = \frac{q}{A\epsilon_0}d}$$

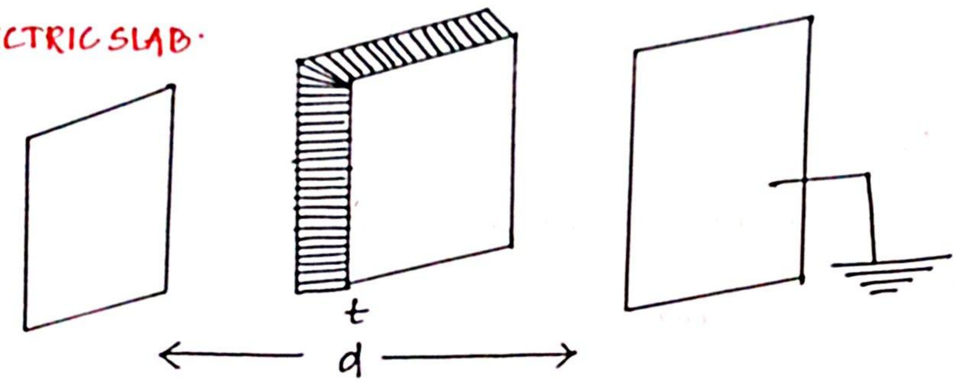
$$\boxed{C = \frac{q}{V} = \frac{A\epsilon_0}{d}}$$

$C \propto A$ , $C \propto \frac{1}{d}$




## CASE-I

**DI-ELECTRIC SLAB.**




t = thickness of di-electric slab

electric field,

$$E_{air} = \frac{\sigma}{\epsilon_0} \quad , \quad E_{dil} = \frac{\sigma}{\epsilon_0 K}$$

$$V = E \cdot d$$

$$= E_{air}(d-t) + E_{dil}\, t$$

$$= \frac{\sigma}{\epsilon_0}(d-t) + \frac{\sigma}{\epsilon_0 K} t$$

$$= \frac{\sigma}{\epsilon_0}\left(d - t + \frac{t}{K}\right)$$

$$= \frac{q}{A\epsilon_0}\left(d - t + \frac{t}{K}\right)$$

$$C = \frac{q}{V} = \frac{q}{\frac{q\left(d-t+\frac{t}{K}\right)}{A\epsilon_0}} = \boxed{\frac{A\epsilon_0}{d-t+\frac{t}{K}}}$$

If the whole space is di-electric.

$$t = d$$

$$C = \frac{A\epsilon_0}{d/K}$$

$$C_{dil} = \frac{K\epsilon_0 A}{d}$$

$$\boxed{C_{dil} = K\, C_{air}}$$

## CASE-II :-

## CONDUCTING SLAB :-

$t$ = thickness of conducting slab

$$E_{air} = \frac{\sigma}{\epsilon_0} \quad , \quad E_{cond} = 0$$

$$V = E \cdot d$$

$$= E_{air}(d-t) + E_{cond} \cdot t$$

$$= \frac{\sigma}{\epsilon_0}(d-t) + 0$$

$$V = \frac{\sigma}{\epsilon_0}(d-t)$$

$$V = \frac{q}{A\epsilon_0}(d-t)$$

$$C = \frac{q}{V} = \frac{q}{\frac{q}{A\epsilon_0}(d-t)} = \boxed{\frac{\epsilon_0 A}{d-t}}$$

If the whole space is conductor,

$$d = t$$

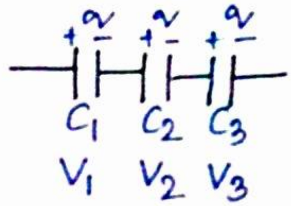
$$C = \frac{\epsilon_0 A}{t-t}$$

$$\boxed{C = \infty}$$

# COMBINATION OF CAPACITORS:-

## (a) SERIES

In this connection, negative part of one capacitor is connected to positive plate of other




In this connection, charge remains same but potential gets divided.

$$V = V_1 + V_2 + V_3$$

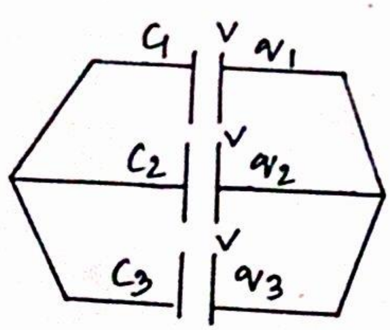
$$\Rightarrow \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

## (b) PARALLEL

In this connection, +ve terminals of all capacitors are connected at one point and -ve terminal are connected at one point

In this connection, potential diff remains constant but charge divides.




$$q = q_1 + q_2 + q_3$$

$$\Rightarrow CV = C_1V + C_2V + C_3V$$

$$\Rightarrow \boxed{C = C_1 + C_2 + C_3}$$

# ENERGY STORED IN A CAPACITOR :-

The amount of work done to add charge to a capacitor is stored in the form of electric potential energy in the space between the two plates.

Let $q$ is the charge given to a capacitor and $V$ is the potential difference at any instant.

$$C = \frac{q}{V}$$

Add $dq$ amount of charge, work done

$$dW = dq\,V$$

Integrating both sides,

$$\int_0^W dW = \int_0^Q dq \cdot V$$

$$\Rightarrow \quad W = \int_0^Q \frac{q}{C}\,dq$$

$$\Rightarrow \quad W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2C}\left[Q^2 - 0^2\right]$$

$$\Rightarrow \quad \boxed{W = \frac{Q^2}{2C}}$$

Potential Energy, $\boxed{U = \frac{Q^2}{2C}}$

$$U = \frac{1}{2}(CV)^2/C = \frac{1}{2}\frac{C^2V^2}{C}$$

$$\boxed{U = \frac{1}{2}CV^2}$$

$$U = \frac{1}{2}\frac{Q}{V}\cdot V^2$$

$$\boxed{U = \frac{1}{2}QV}$$

Energy density = $\frac{\text{Energy}}{\text{Volume}}$

$$u = \frac{\frac{1}{2}CV^2}{Ad}$$

$$= \frac{\frac{1}{2}\frac{\epsilon_0 A}{d}(Ed)^2}{Ad}$$

$$= \frac{\epsilon_0 A}{2d} \times E^2 d^2 \times \frac{1}{Ad}$$

$$= \frac{E^2 \epsilon_0}{2}$$

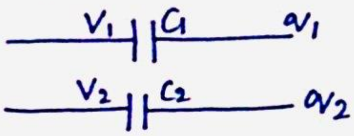
$$u = \frac{1}{2}\epsilon_0 E^2$$

## ENERGY STORED IN COMBINATION :-

$$\boxed{U = U_1 + U_2 + U_3 + \cdots}$$

## COMMON POTENTIAL :-

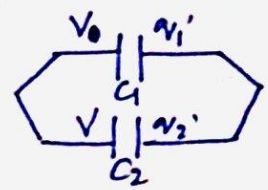
Before connection




$q_1 = C_1 V_1$

$q_2 = C_2 V_2$

After connection




$q_1' = C_1 V$

$q_2' = C_2 V$

According to conservation of charge.

$$Q_1 + Q_2 = Q_1' + Q_2'$$

$$\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$$

$$\Rightarrow \boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

## ENERGY LOSS:-

Before combination,

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

After combination,

$$U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$= \frac{1}{2}(C_1 + C_2)V^2 \quad = \frac{1}{2}(C_1 + C_2)\left\{\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right\}^2.$$

$$= \frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}$$

$$\Delta u = U_i - U_f$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \left\{\frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2}\right\}$$

$$= \frac{C_1C_2}{2(C_1 + C_2)}(V_1 - V_2)^2 > 0$$

So, $U_i > U_f$.

So energy is lost. The lost energy appears in the form of heat in connecting wire

# ELECTRIC CHARGES AND FIELDS (CHAPTER-1)

CHARGE:- It is the property of the body by virtue of which it shows both electric and magnetic behaviour.

REPRESENTATION – Q or q

- Charge is a scalar quantity
- SI unit – coulomb (C)
- CGS unit – st C (electrostatic unit of charge) $1C = 3 \times 10^9$ st C
  ab C (electromagnetic unit of charge) $1C = \frac{1}{10}$ ab C

## SPECIFIC PROPERTIES OF CHARGE:-

1. According to Benjamin Franklin, charges are of two types, positive and negative.
2. Like charges repel and unlike charges attract. (Fundamental law of electrostatics)
3. Charge is always associated with mass.
   i.e Charge cannot exist without mass whereas mass can exist without charge.
4. when a body is positively charged → lose electrons → mass decreases
   when a body is negatively charged → gains electrons → mass increases
5. Charge is conserved:- The charge of an isolated system remains constant. That means, charge can neither be created nor be destroyed
6. charge is quantised:- Total charge of a body is equal to the integral multiple of fundamental charge 'e'
   i.e $Q = \pm ne$, n = an integer (1, 2, 3, ....)

★ $\boxed{\text{Minimum possible charge} = \pm e = \pm 1.6 \times 10^{-19} C}$

7. Charge is invariant:- Charge is independent of frame of reference. That is, charge on a body doesnot change whatever may be its speed.
8. Charge is additive:- Total charge on an isolated system is equal to the algebraic sum of charges on individual bodies of the system.
   i.e If a system contain three charges, $q_1$, $q_2$ & $q_3$ then total charge on the system $\boxed{Q = q_1 + q_2 + q_3}$.

JM

Difference between charge and mass:- 

| CHARGE | MASS |
| --- | --- |
| ① Charge cannot exist without mass. | Mass can exist without charge. |
| ② Force between the charges can either be attractive or repulsive. | Gravitational force between two mass is always attractive |
| ③ Charge doesnot depend on the speed of the body. | Mass of a body changes according to the formula, $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where, $c$ = speed of light in vacuum, $m$ = mass of a body moving with velocity $v$, $m_0$ = rest mass of the body. |
| ④ Charge can be either positive, negative or zero | Mass is a positive quantity. |

## METHODS OF CHARGING

There are three methods of charging:-

① Friction

② Electrostatic induction

③ conduction

① **FRICTION**:- If we rub one body with another body, then transfer of electrons take place from one body to another body.

The transfer of $e^-$ take place from lower work function body to the higher work function body.

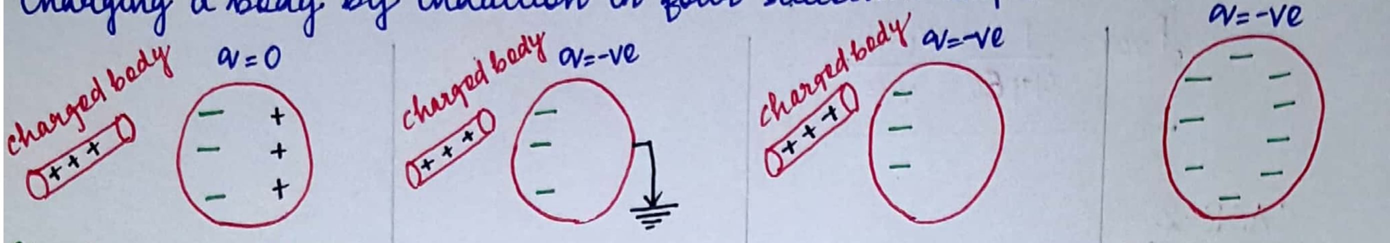
| Positive | Negative |
| --- | --- |
| Glass rod | Silk cloth |
| Wollen cloth | Plastic objects, rubber shoes, amber |
| Cat skin | Ebonite rod |
| Dry hair | Comb |

- clouds become charged by friction.

## ② ELECTROSTATIC INDUCTION (without direct contact betⁿ 2 bodies)

The phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its farthest end in the presence of a nearby charged body is called electrostatic induction.

Charging a body by induction in four successive steps:-




Step-1:- charged body is brought near an uncharged body.

Step-2:- uncharged body is connected to earth.

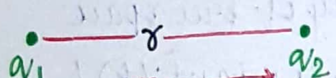
Step-3:- uncharged body is disconnected from earth.

Step-4:- charging body is removed.

## ③ CONDUCTION:-
The process of transfer of charge by direct contact betⁿ 2 bodies is called conduction.

## COULOMB'S LAW

The force of attraction or repulsion between any two point charges at rest is directly proportional to product of magnitude of charges and inversely proportional to square of distance between them and acts along the line joining the 2 charges.




$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$\Rightarrow \boxed{F = \frac{K q_1 q_2}{r^2}}$$

where K = proportionality constant

K depends on two factors:-

(i) nature of medium between the two charges.

(ii) system of units chosen.

$$\boxed{K = 9 \times 10^9 \ Nm^2/c^2}$$

In SI unit, $\boxed{K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/c^2}$

$\epsilon_0$ = permittivity of free space.

**PERMITTIVITY:-** Permittivity is the quantity that determines how far the medium permits the electrical interaction between two charged bodies.

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \ C^2/Nm^2}$$

CASE-1

In air/vacuum/free space:-

In SI:- $K = \frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2/C^2$

$$\boxed{F_{vac} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}}$$

CASE-2

In any medium/dielectric medium

In SI:- $K = \frac{1}{4\pi\epsilon} = \frac{1}{4\pi\epsilon_0\epsilon_r} = \frac{1}{4\pi\epsilon_0 k}$

$$\boxed{F_{med} = \frac{1}{4\pi\epsilon_0 k}\frac{q_1 q_2}{r^2}}$$

$$\boxed{\epsilon = \epsilon_0 \cdot \epsilon_r}$$

$$F_{med} = \frac{F_{vac}}{k}$$

where, $\epsilon$ = permittivity of the medium/ electrical permittivity

$\epsilon_0$ = permittivity of free space

$\epsilon_r$ = dielectric constant ($k$)/ relative permittivity.

## ELECTRICAL PERMITTIVITY/ PERMITTIVITY OF THE MEDIUM:-

It is a constant defined for all mediums to know how far the medium permits the electrical interaction between two charged bodies.

- Symbol:- $\epsilon$
- Dimension:- $[M^{-1}L^{-3}T^4A^2]$

★ Also called as absolute permittivity

## RELATIVE PERMITTIVITY ($\epsilon_r$) / DIELECTRIC CONSTANT ($k$)

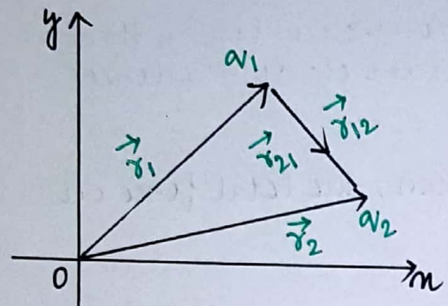
The ratio of the permittivity of the medium to the permittivity of the free space is called relative permittivity ($\epsilon_r$) or dielectric constant ($k$):

$$\boxed{\epsilon_r \text{ or } k = \frac{\epsilon}{\epsilon_0}}$$

JM

- Relative permittivity or dielectric constant has no unit and dimensionless.
- Symbol:- $\varepsilon_r$ or $k$
- for vacuum, $k = 1$
- For metal, $k = \infty$
- For water, $k = 80$

## COULOMB'S LAW IN VECTOR FORM :-




Force on $q_1$ due to $q_2$,

$$\vec{F}_{12} = \frac{K q_1 q_2}{r_{21}^2}\hat{r}_{21} = \frac{K q_1 q_2}{r_{21}^2}\frac{\vec{r}_{21}}{r_{21}} = \frac{K q_1 q_2}{r_{21}^3}\vec{r}_{21}$$

$$= \frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

Force on $q_2$ due to $q_1$,

$$\vec{F}_{21} = \frac{K q_1 q_2}{r_{12}^2}\hat{r}_{12}$$

$$= \frac{K q_1 q_2}{r_{12}^2}\frac{\vec{r}_{12}}{r_{12}} = \frac{K q_1 q_2}{r_{12}^3}\vec{r}_{12}$$

$$= \frac{K q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1) = -\frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

This means that, the two charges exert equal & opposite force on each other. So, they obey Newton's third law of motion.

## CHARACTERSTICS OF COULOMB'S FORCE :-

1. Applicable or valid only for point charges which are at rest.
2. Obeys inverse square law $\left(F \propto \frac{1}{r^2}\right)$
3. It is a long range force.
4. Coulomb's force is inactive when the separation between two charges is less than one fermi ($10^{-15}$ m)
5. It is a central force. i.e it act along the line joining the centres of the two bodies.

6. coulomb force depends on the medium within which charges are placed.

7. coulomb force is not affected by the presence of other charged bodies near it

8. It obeys newton's third law of motion

## FORCE BETWEEN MULTIPLE CHARGES :- THE SUPERPOSITION PRINCIPLE :-

When a number of charges are interacting among each other, then the force acting on one charge will be the vector sum of all the forces acting on it due to all other charges.

Then, according to the principle of superposition, the total force on charge $q_1$ is given by

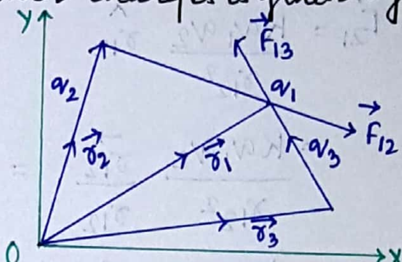
$$F_1 = F_{12} + F_{13} + \cdots + F_{1n} \quad \text{——— (1)}$$

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

Similarly, the force on charge $q_1$ due to other charges is given by

$$F_{13} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31}$$

$$F_{1n} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_n}{r_{n1}^2} \hat{r}_{n1}$$




Substituting these values in eqn (1) we get,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2}\hat{r}_{31} + \cdots + \frac{q_1 q_n}{r_{n1}^2}\hat{r}_{n1}\right]$$

$$\boxed{F_{1i} = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{i1}^2} \hat{r}_{i1}}$$

## ELECTRIC FIELD :-

The region surrounding to a charged body within which another charge experiences a force is called electric field.

## TEST CHARGE

→ The charge which produces the electric field is called source charge and the charge which experiences the effect of source charge is called test charge.

→ unit positive charge is taken as test charge.

→ its magnitude is very small in comparison to source charge because its own field should not affect the field of source charge.

## ELECTRIC FIELD INTENSITY

It is defined as the force experienced per unit positive test charge placed at that point, without disturbing the source charge.

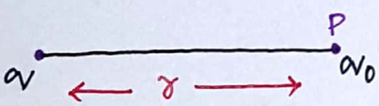
It is expressed as, $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$ , where $\vec{E}$ = electric field intensity
$q_0$ = test charge
$\vec{F}$ = force experienced by the test charge $q_0$.

- It is a vector quantity
- SI unit :- N/C or V/m
- CGS unit :- D/stC or D/abC

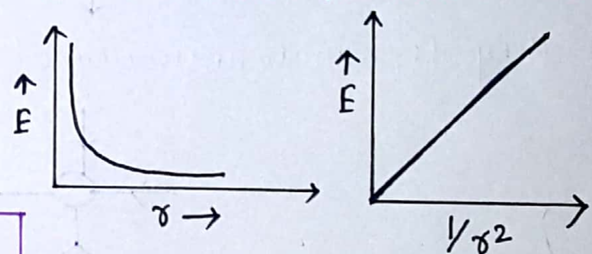
Dimension :- $\frac{M^1L^1T^{-2}}{AT} = [M^1L^1T^{-3}A^{-1}]$

* Electric field due to positive charge is always away from it while due to negative charge is always towards it

## ELECTRIC FIELD INTENSITY DUE TO POINT CHARGE:-




P is any point at a distance r from the source charge q.




Force on $q_0$,

$$\vec{F} = \frac{Kqq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{Kq}{r^2}\hat{r}$$

$$\boxed{|\vec{E}| = \frac{Kq}{r^2}}$$

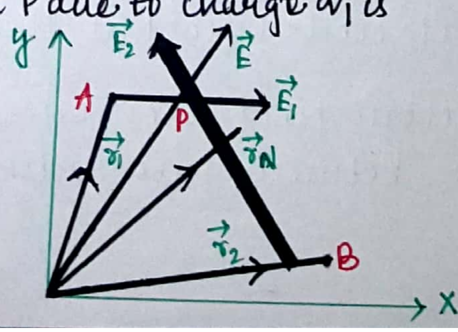
## ELECTRIC FIELD DUE TO MULTIPLE CHARGES:-

Consider $q_1, q_2, \ldots\ldots, q_N$ charges are placed at a dist $r_1, r_2, \ldots r_N$ from origin in vacuum. Hence, the electric field at point P due to charge $q_1$ is

$$\vec{E}_1 = \frac{\vec{F}_1}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P}$$

Similarly,

$$\vec{E}_2 = \frac{\vec{F}_2}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P}$$

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$$\vec{E}_N = \frac{1}{4\pi\varepsilon_0} \frac{q_N}{r^2_{NP}} \hat{r}_{NP}$$

According to the superposition principle,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots\cdots + \vec{E}_N$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r^2_{1P}}\hat{r}_{1P} + \frac{q_2}{r^2_{2P}}\hat{r}_{2P} + \cdots\cdots + \frac{q_N}{r^2_{NP}}\hat{r}_{NP}\right]$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r^2_{iP}}\hat{r}_{iP}}$$

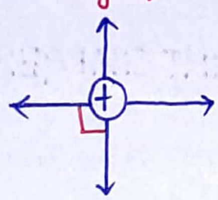
## ELECTRIC FIELD LINES / LINES OF FORCE:-

A curve along which the test charge would tend to move when force to do so in an electric field due to a source charge. These imaginary lines are called electric field lines.

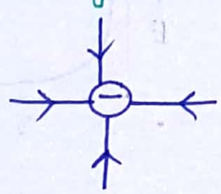
### PROPERTIES:-

① They start from positive charge and end at negative charge.

② They emerge normally from the surface of a positive charge.



③ They terminate normally on the surface of a negative charge.



④ The field lines have a tendency to expand laterally so as to exert a lateral pressure. This explains repulsion between two like charges.

⑤ Tangent to any point on electric field lines shows the direction of electric field at that point.

⑥ Electric field lines contract lengthwise to represent attraction between two unlike charges.

7. Two field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point, which is not possible.

8. They are continous smooth curve without any breaks

9. They donot form closed loops.

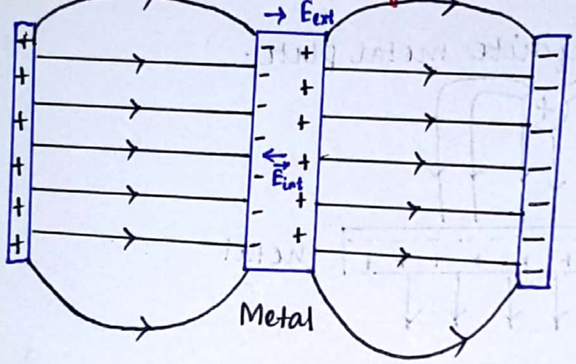
10. The region where lines of force are crowded, its intensity is more.

11. The number $\Delta N$ of lines per unit cross sectional area perpendicular to the field lines is directly proportional to the magnitude of the intensity of electric field in that region.

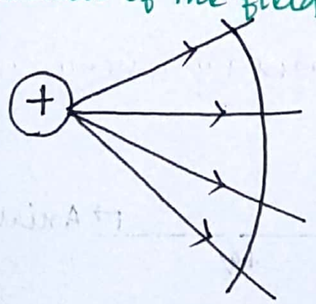
$$\frac{\Delta N}{\Delta A} \propto E$$

12. They donot pass through a conductor

13. Field lines cannot pass through a metallic slab.



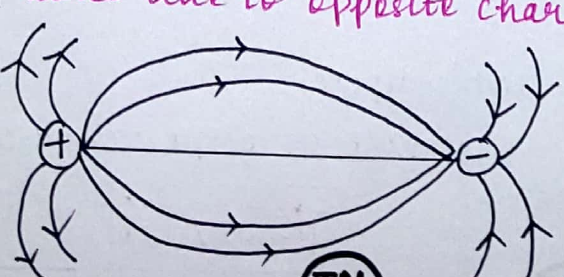

14. The relative closeness of the field lines gives the strength of electric field



- Fields close to each other indicate strong field.
- Fields lines away to each other indicate weak field.

## REPRESENTATION OF ELECTRIC FIELD:-

Electric field lines due to opposite charges are equal in magnitude.



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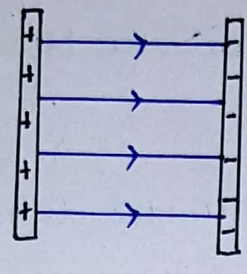
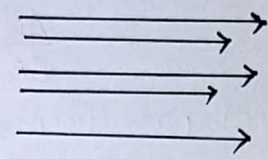
- In case of uniform field, the field lines are parallel (to have same direction) and are equidistance (to have same magnitude) to each other

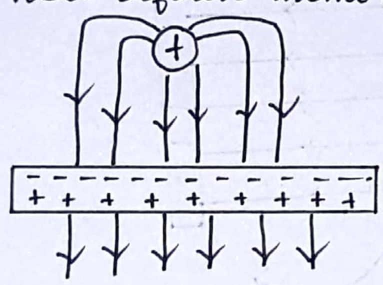


- In case of non-uniform field, the field lines are not parallel and are not equidistance to each other.



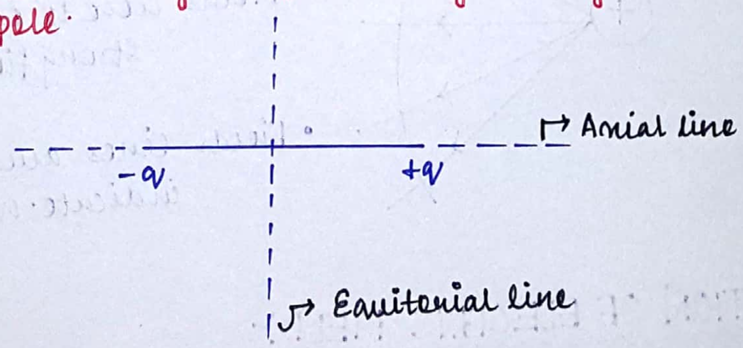
parallel but not equidistance

- Fixed point charge near infinite metal plate.




## ELECTRIC DIPOLE :-

Two equal and opposite charges separated by a very small distance constitute a dipole.




## ELECTRIC DIPOLE MOMENT :-

- It determines the strength of electric dipole
- It is define as the product of magnitude of either charge and separation of distance between them.

$$\boxed{\vec{P} = q \times \vec{2l}}$$

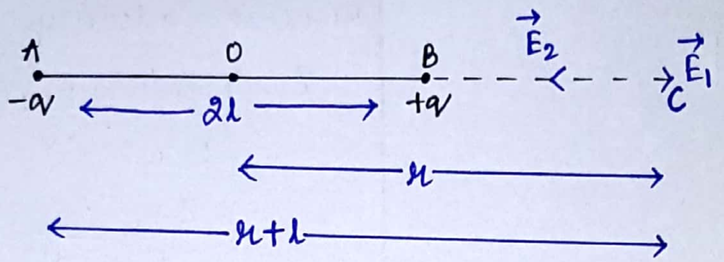
$$\boxed{|P| = q(2l)}$$

- vector quantity
- direction is always from negative charge to positive charge.
- Dimension - [ATL]
- SI unit - Cm

## IDEAL DIPOLE / POINT DIPOLE :-

Suppose, $q \to \infty$, $2l \to 0$ such that p is finite. Such a dipole of negligibly small size is called as ideal dipole or point dipole.

## ELECTRIC FIELD INTENSITY DUE TO DIPOLE AT THE AXIAL POSITION/END ON POSITION :-




C is any point on the axial line at a distance r from the centre of the dipole

Due to +q,

$$\vec{E}_1 = \frac{Kq}{BC^2}\hat{i}$$

$$= \frac{Kq}{(r-l)^2}\hat{i}$$

Due to -q,

$$\vec{E}_2 = \frac{Kq}{AC^2}(-\hat{i})$$

$$= \frac{Kq}{(r+l)^2}(-\hat{i})$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{Kq}{(r-l)^2}\hat{i} + \frac{Kq}{(r+l)^2}(-\hat{i})$$

$$= Kq\left[\frac{(r+l)^2 - (r-l)^2}{(r-l)^2\,(r+l)^2}\right]\hat{i}$$

$$= Kq\left[\frac{4rl}{(r^2-l^2)^2}\right]\hat{i}$$

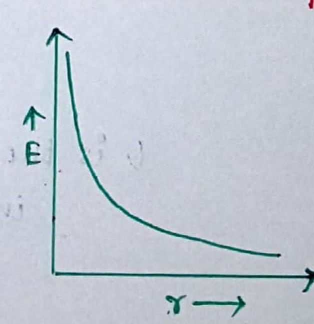
$$= Kq\,\frac{2r \cdot 2l}{(r^2-l^2)^2}\hat{i}$$

$$\boxed{\vec{E} = \frac{2Kpr}{(r^2-l^2)^2}\hat{i}}$$

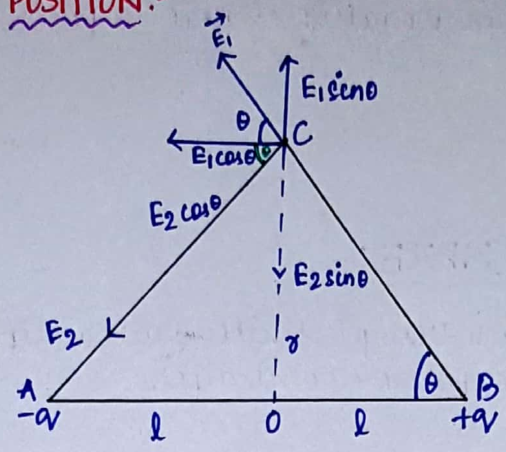
For ideal dipole, $l <<<< r$, so $l^2$ can be neglected

$$\vec{E} = \frac{2Kpr}{r^4}\hat{i}$$

$$\boxed{\vec{E} = \frac{2KP}{r^3}\hat{i}}$$



## ELECTRIC FIELD INTENSITY DUE TO DIPOLE AT AN EQUITORIAL POSITION OR BROAD SIDE ON POSITION:-




C is any point on the equitorial line at a distance r from the centre of the dipole.

Due to +q charge,

$$E_1 = \frac{Kq}{BC^2} = \frac{Kq}{r^2+l^2}$$

Due to -q charge,

$$E_2 = \frac{Kq}{AC^2} = \frac{Kq}{r^2+l^2}$$

$E_1 = E_2$

$E_1 \sin\theta$ and $E_2 \sin\theta$ cancel each other

Resultant Intensity,

$$\vec{E} = (E_1 \cos\theta + E_2 \cos\theta)(-\hat{i})$$

$$= 2E_1 \cos\theta(-\hat{i})$$

$$= 2\frac{Kq}{r^2+l^2}\frac{l}{\sqrt{r^2+l^2}}(-\hat{i})$$

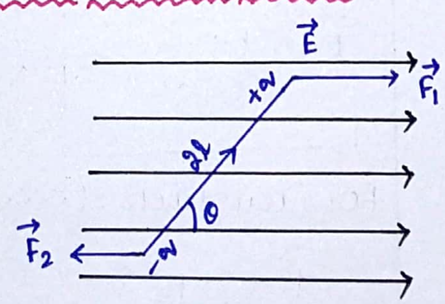
$$= \frac{2Kql}{(r^2+l^2)^{3/2}}(-\hat{i})$$

$$\boxed{\vec{E} = \frac{KP}{(r^2+l^2)^{3/2}}(-\hat{i})}$$

for ideal dipole, $l <<< r$, $l^2$ can be neglected

$$\boxed{\vec{E} = \frac{KP}{r^3}(-\hat{i})}$$

## DIPOLE IN UNIFORM ELECTRIC FIELD:-




$\theta$ is the angle between dipole moment and intensity.

Force on $+q$.

$\vec{F}_1 = q\vec{E}$

Force on $-q$,

$\vec{F}_2 = -q\vec{E}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 = 0$

So no translatory motion

As two forces are not in same line of action, so they constitute a couple due to which dipole rotate.

$\tau = (2l) F \sin\theta$

$= 2lqE \sin\theta$

$= PE \sin\theta$

$\boxed{\tau = PE\sin\theta}$

In vector form,

$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$

$\vec{\tau} \perp \vec{P}$ and $\vec{\tau} \perp \vec{E}$

These are two pairs of perpendicular vector.

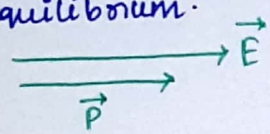
The direction of torque is $\perp^r$ to the plane inward according to figure.

**Case-1**

when $\theta = 0$

$\boxed{\tau = 0}$

It is a condition of stable equilibrium.

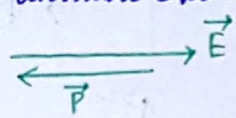



**Case-2**

when $\theta = 180°$

$\boxed{\tau = 0}$

It is a condition of unstable equilibrium




**Case-3**

when $\theta = 90°$

$\boxed{\tau = PE}$

Maximum torque

## NEUTRAL POINT :-

It is apoint in an electric field where when any charge is placed experience no force.

**CASE-1**

- for 2 like charges, neutral point lies between them. $+q_1$ N $+q_2$
- when similar charge is placed at neutral point, it is in unstable equilibrium along Y-axis and stable equilibrium along X-axis
- when dissimilar charge is placed at neutral point, it is in stable equilibrium along Y-axis and unstable equilibrium along X-axis.

**CASE-2**

- for two unlike charges, neutral point lies at the side of less magnitude charge.
- If $q_1 = q_2$, then neutral point is not possible.

# ELECTRIC FLUX :- ($\phi$)

## PHYSICAL SIGNIFICANCE :-

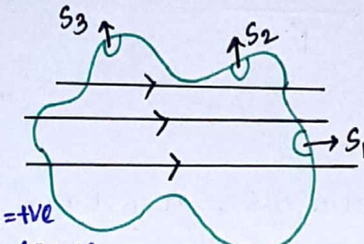
It determines the amount of electric field lines linked with the surface.

## DEFINATION :-

- It is defined as the dot product of electric field intensity with the arial vector of a surface.
- Electric flux at any point can be defined as the number of field line passing normally through that area placed inside an electric field.
- SI unit – $Nm^2/C$ or $Vm$
- Scalar quantity
- Dimension – $[ML^3T^{-3}A^{-1}]$

$$\phi = \vec{E}\cdot\vec{S} \Rightarrow \boxed{\phi = ES\cos\theta}$$

$$\phi = \int \vec{E}\cdot\vec{ds} \Rightarrow \boxed{\phi = \int E\,ds\cos\theta}$$




### Special case

**Case-1** (for surface $S_1$)

$\theta = <90^\circ$, $\cos\theta = +ve$, flux $= +ve$

when the lines of force leave the surface, the flux is positive

**Case-2** (for $S_2$)

$\theta = 90^\circ$, $\cos\theta = 0$

flux $= 0$

**Case-3** (for $S_3$)

$\theta > 90^\circ$, $\cos\theta = -ve$,

flux $= -ve$

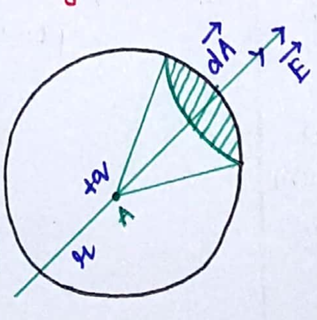
So, when lines of force enter to the surface, flux is negative

# GAUSS LAW :-

It states that the electric flux linked with a closed surface in vacuum is $\frac{1}{\varepsilon_0}$ times the total charge enclosed within it.

### PROOF :-

Take a charge $+q$ at point A. Take a gaussian surface in the shape of a sphere of radius $r$ centred at $+q$




$$\phi = \oint E\,ds\cos\theta$$

$$= \oint E\,ds\cos 0$$

$$= E\oint ds$$

$$= \frac{kq}{r^2} \times 4\pi r^2$$

$$= kq4\pi$$

$$= \frac{1}{4\pi\varepsilon_0} q4\pi$$

$$= \frac{q}{\varepsilon_0}$$

$$\boxed{\phi = \frac{q_{en}}{\varepsilon_0}}$$

## DERIVATION OF COULOMB'S LAW FROM GAUSS LAW :-

According to gauss law,

$$\phi = \oint \vec{E} \cdot \vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow \oint E ds \cos 0 = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow E \oint ds = \frac{q}{\epsilon_0}$$

$$\Rightarrow E 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{4\pi r^2 \epsilon_0} \quad \Rightarrow E = \frac{Kq}{r^2}$$

Suppose, a charge is placed on the periphery of the gaussian surface then force exerted on it will be,

$$F = E q_0$$

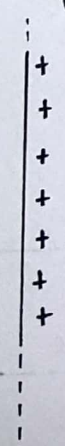
$$\boxed{F = \frac{K q q_0}{r^2}}$$

## IMPORTANT POINTS ON GAUSS LAW :-

- Gauss law is applicable for any closed surface, whatever its shape and size is.
- The surface in which gauss law is applied is called gaussian surface.
- Flux linked with closed surface is independent of area of the surface.
- If the medium is di-electric, then $\phi = \frac{q_{en}}{\epsilon_0 k}$

## CONTINOUS CHARGE DISTRIBATION :-

(a) Linear charge distrubation :-



linear charge density,

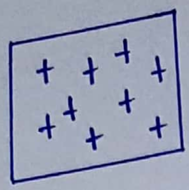
$$\lambda = \frac{q}{l}$$

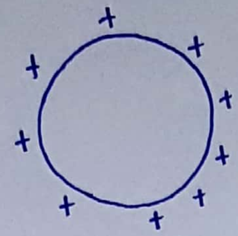
$$\boxed{\lambda = \frac{dq}{dl}}$$

DIMENSION - $[AT L^{-1}]$

SI unit - C/m

(b) Surface charge distribution:-





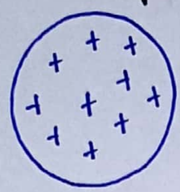
Surface charge density,

$$\sigma = \frac{q}{S} = \frac{dq}{dS}$$

DIMENSION - $[ATL^{-2}]$

SI unit - $C/m^2$

(C) Volume charge distribution:-



Volume charge density,

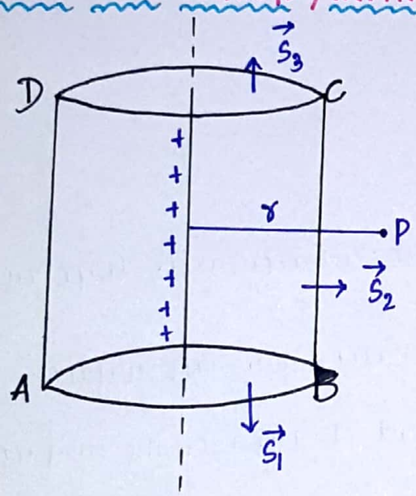
$$\rho = \frac{q}{V} = \frac{dq}{dV}$$

DIMENSION - $[ATL^{-3}]$

SI unit - $C/m^3$

## APPLICATION-1

(INFINITE LINE CHARGE / INFINITELY LONG CHARGED WIRE)




$\lambda$ = linear charge density = $\frac{q}{l}$

P is any point at a distance r from the line charge.

using gauss law,

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\varepsilon_0}$$

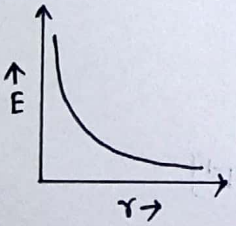
$$\Rightarrow \int_{S_1} \vec{E}\cdot\vec{ds} + \int_{S_2} \vec{E}\cdot\vec{ds} + \int_{S_3} \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow \int_{S_1} E\,ds\cos 90^\circ + \int_{S_2} E\,ds\cos 0 + \int_{S_3} E\,ds\cos 90^\circ = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow E\cdot 2\pi r l = \frac{q_{en}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q_{en}}{2\pi r l \varepsilon_0}$$

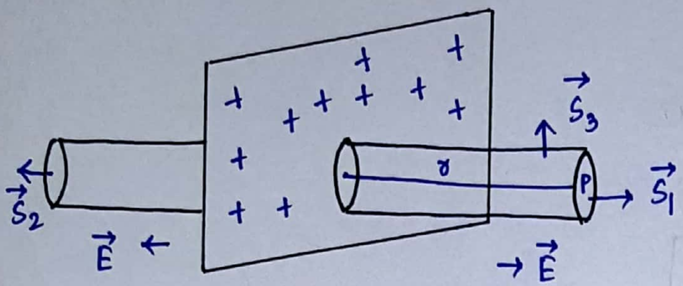
$$\Rightarrow \boxed{E = \frac{\lambda}{2\pi r \varepsilon_0}}$$




In vector form, $\boxed{\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0}\hat{r}}$

## APPLICATION-2

### INFINITE THIN PLANE SHEET :-




$\sigma$ = surface charge density

P is any point at a distance r from the plane sheet.

using gauss law,

$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow \int_{S_1} E\,ds\cos 0 + \int_{S_2} E\,ds\cos 0 + \int_{S_3} E\,ds\cos 90^\circ = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow ES + ES = \frac{q_{en}}{\epsilon_0}$$

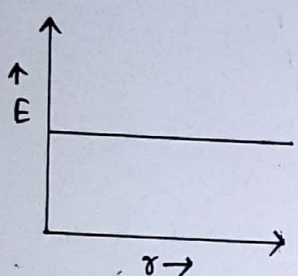
$$\Rightarrow 2ES = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow 2ES = \frac{\sigma S}{\epsilon_0}$$

$$\Rightarrow 2E\cdot\epsilon_0 = \sigma$$

$$\Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

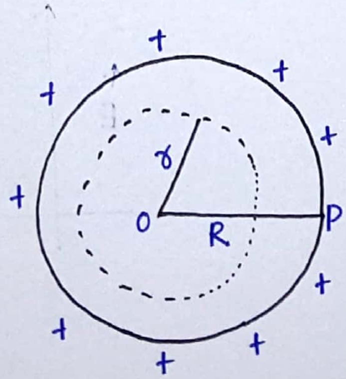
In vector form, $\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{n}}$




Electric field is independent of distance.

## APPLICATION-3

### SPHERICAL SHELL (SOLID CONDUCTING SPHERE)




R = radius

$\sigma$ = surface charge density $= \dfrac{q}{4\pi R^2}$

Case-I

Inside

P is any point at a distance $r$ from the centre $(r<R)$

using gauss law,

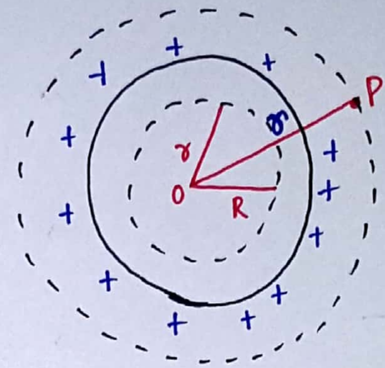
$$\phi = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow \oint \vec{E}\cdot\vec{ds} = 0$$

$$\Rightarrow \boxed{E=0}$$

Case-II $(r>R)$

Outside




Applying gauss law,

$$\phi = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow \oint \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

$$\Rightarrow \oint E\,ds\cos 0 = \frac{q}{\epsilon_0}$$

$$\Rightarrow E4\pi r^2 = \frac{q}{\epsilon_0} \quad \Rightarrow \boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}}$$

This expression is same as the expression used in intensity due to point charge. So charges resides on the surface of the spherical shell behave as if they are concentrated at the centre.

Case-III

On the surface $(r=R)$

using gauss law,

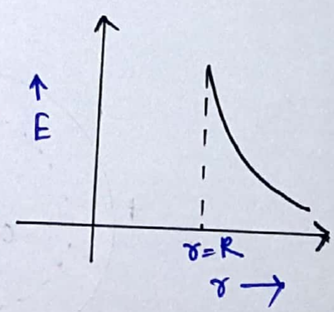
$$\oint \vec{E}\cdot\vec{ds} = \frac{q_{en}}{\epsilon_0}$$

$$\Rightarrow \oint E\,ds\cos 0 = \frac{q}{\epsilon_0}$$

$$\Rightarrow E4\pi R^2 = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{4\pi R^2\epsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$

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