ELECTRIC CHARGES AND FIELDS (CHAPTER-1)

CHARGE:- It is the property of the body by virtue of which it shows both electric and magnetic behaviour.

REPRESENTATION– Q or q

- Charge is a scalar quantity
- SI unit– coulomb (C)
- CGS unit– st C (electrostatic unit of charge) $1C = 3\times10^{9}$ st C
  ab C (electromagnetic unit of charge) $1C = \frac{1}{10}$ ab C

SPECIFIC PROPERTIES OF CHARGE:-

① According to Benjamin Franklin, charges are of two types, positive and negative.

② Like charges repel and unlike charges attract. (Fundamental law of electrostatics)

③ Charge is always associated with mass.
i.e Charge cannot exist without mass where as mass can exist without charge.

④ when a body is positively charged → lose electrons → mass decreases
when a body is negatively charged → gains electrons → mass increases

⑤ Charge is conserved:- The charge of an isolated system remains constant. That means, charge can neither be created nor be destroyed

⑥ charge is quantised:- Total charge of a body is equal to the integral multiple of fundamental charge 'e'
i.e $Q = \pm ne$, n = an integer (1, 2, 3, ...)

★ Minimum possible charge $= \pm e = \pm 1.6\times10^{-19}C$.

⑦ Charge is invariant:- Charge is independent of frame of reference. That is, charge on a body doesnot change whatever may be its speed.

⑧ Charge is additive:- Total charge on an isolated system is equal to the algebraic sum of charges on individual bodies of the system.
i.e If a system contain three charges, q_1, q_2 & q_3 then total charge on the system $Q = q_1 + q_2 + q_3$.

Difference between charge and mass:-

CHARGE	MASS
① Charge cannot exist without mass.	Mass can exist without charge.
② Force between the charges can either be attractive or repulsive.	Gravitational force between two mass is always attractive
③ Charge doesnot depend on the speed of the body.	Mass of a body changes according to the formula, $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ where, C = speed of light in vacuum, m = mass of a body moving with velocity v, m_0 = rest mass of the body.
④ Charge can be either positive, negative or zero	Mass is a positive quantity.

METHODS OF CHARGING

There are three methods of charging:-

① Friction

② Electrostatic induction

③ conduction

① **FRICTION**:- If we rub one body with another body, then transfer of electrons take place from one body to another body.

The transfer of e^- take place from lower work function body to the higher work function body.

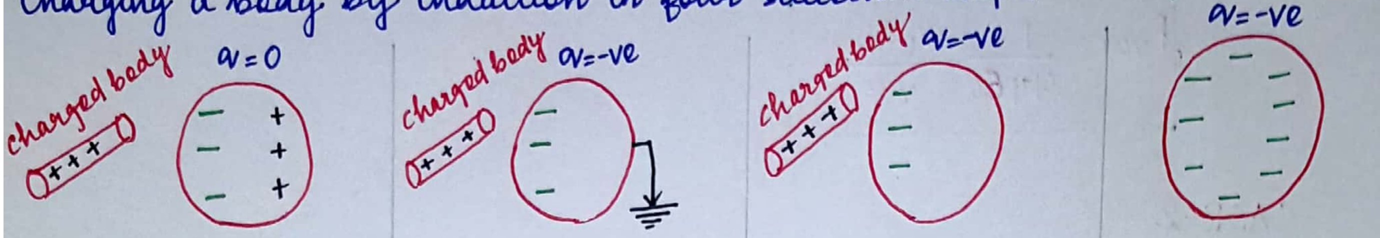
Positive	Negative
Glass rod	Silk cloth
Wollen cloth	Plastic objects, rubber shoes, amber
Cat skin	Ebonite rod
Dry hair	Comb

- Clouds become charged by friction.

② ELECTROSTATIC INDUCTION (without direct contact betⁿ 2 bodies)

The phenomenon of temporary electrification of a conductor in which opposite charges appear at its closer end and similar charges appear at its farthest end in the presence of a nearby charged body is called electrostatic induction.

Charging a body by induction in four successive steps:-



Step-1:- charged body is brought near an uncharged body.

Step-2:- uncharged body is connected to earth.

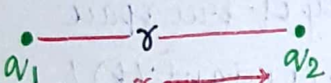
Step-3:- uncharged body is disconnected from earth.

Step-4:- charging body is removed.

③ CONDUCTION:- The process of transfer of charge by direct contact betⁿ 2 bodies is called conduction.

COULOMB'S LAW

The force of attraction or repulsion between any two point charges at rest is directly proportional to product of magnitude of charges and inversely proportional to square of distance between them and acts along the line joining the 2 charges.



$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$\Rightarrow \boxed{F = \frac{K q_1 q_2}{r^2}}$$

where K = proportionality constant

K depends on two factors:-

(i) nature of medium between the two charges.

(ii) system of units chosen.

$$\boxed{K = 9 \times 10^9 \ Nm^2/C^2}$$

In SI unit, $\boxed{K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2/C^2}$

ϵ_0 = permittivity of free space.

PERMITTIVITY:- Permittivity is the quantity that determines how far the medium permits the electrical interaction between two charged bodies.

$$\boxed{\epsilon_0 = 8.854 \times 10^{-12} \ C^2/Nm^2}$$

CASE-1

In air/vacuum/free space:-

In SI:- $K = \frac{1}{4\pi\epsilon_0} = 9\times10^9\ Nm^2/C^2$

$$F_{vac} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$$

CASE-2

In any medium/dielectric medium

In SI:- $K = \frac{1}{4\pi\epsilon} = \frac{1}{4\pi\epsilon_0\epsilon_r} = \frac{1}{4\pi\epsilon_0 k}$

$$F_{med} = \frac{1}{4\pi\epsilon_0 k}\frac{q_1 q_2}{r^2}$$

$$\epsilon = \epsilon_0 \cdot \epsilon_r$$

$$F_{med} = \frac{F_{vac}}{k}$$

where, ϵ = permittivity of the medium/ electrical permittivity

ϵ_0 = permittivity of free space

ϵ_r = dielectric constant (k)/ relative permittivity.

ELECTRICAL PERMITTIVITY/ PERMITTIVITY OF THE MEDIUM:-

It is a constant defined for all mediums to know how far the medium permits the electrical interaction between two charged bodies.

- Symbol:- ϵ
- Dimension:- $[M^{-1}L^{-3}T^4A^2]$

* Also called as absolute permittivity

RELATIVE PERMITTIVITY (ϵ_r) / DIELECTRIC CONSTANT (k)

The ratio of the permittivity of the medium to the permittivity of the free space is called relative permittivity (ϵ_r) or dielectric constant (k):

$$\epsilon_r \text{ or } k = \frac{\epsilon}{\epsilon_0}$$

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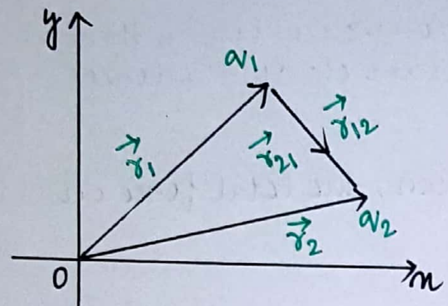
- Relative permittivity or dielectric constant has no unit and dimensionless.
- Symbol:- ε_r or k
- for vacuum, $k = 1$
- For metal, $k = \infty$
- For water, $k = 80$

COULOMB'S LAW IN VECTOR FORM:-



Force on q_1 due to q_2,

$$\vec{F}_{12} = \frac{K q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

$$= \frac{K q_1 q_2}{r_{21}^2} \frac{\vec{r}_{21}}{r_{21}} = \frac{K q_1 q_2}{r_{21}^3} \vec{r}_{21}$$

$$= \frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

Force on q_2 due to q_1,

$$\vec{F}_{21} = \frac{K q_1 q_2}{r_{12}^2} \hat{r}_{12}$$

$$= \frac{K q_1 q_2}{r_{12}^2} \frac{\vec{r}_{12}}{r_{12}} = \frac{K q_1 q_2}{r_{12}^3} \vec{r}_{12}$$

$$= \frac{K q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1) = -\frac{K q_1 q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2)$$

$$\boxed{\vec{F}_{21} = -\vec{F}_{12}}$$

This means that, the two charges exert equal & opposite force on each other. So, they obey Newton's third law of motion.

CHARACTERSTICS OF COULOMB'S FORCE :-

1. Applicable or valid only for point charges which are at rest.
2. Obeys inverse square law ($F \propto \frac{1}{r^2}$)
3. It is a long range force.
4. Coulomb's force is inactive when the separation between two charges is less than one fermi (10^{-15} m)
5. It is a central force. i.e it act along the line joining the centres of the two bodies.

⑥ coulomb force depends on the medium within which charges are placed.

⑦ coulomb force is not affected by the presence of other charged bodies near it

⑧ It obeys newton's third law of motion

FORCE BETWEEN MULTIPLE CHARGES :- THE SUPERPOSITION PRINCIPLE :-

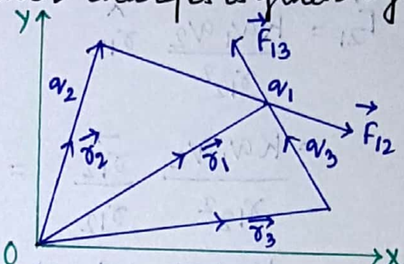
When a number of charges are interacting among each other, then the force acting on one charge will be the vector sum of all the forces acting on it due to all other charges.

Then, according to the principle of superposition, the total force on charge q_1 is given by

$$F_1 = F_{12} + F_{13} + \cdots + F_{1n} \quad \text{——— (1)}$$

$$F_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{r}_{21}$$

Similarly, the force on charge q_1 due to other charges is given by



$$F_{13} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31}$$

$$F_{1n} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_n}{r_{n1}^2} \hat{r}_{n1}$$

Substituting these values in eqn (1) we get,

$$F_1 = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{21}^2} \hat{r}_{21} + \frac{q_1 q_3}{r_{31}^2} \hat{r}_{31} + \cdots + \frac{q_1 q_n}{r_{n1}^2} \hat{r}_{n1} \right]$$

$$\boxed{F_{1i} = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{i1}^2} \hat{r}_{i1}}$$

ELECTRIC FIELD :-

The region surrounding to a charged body within which another charge experiences a force is called electric field.

TEST CHARGE

→ The charge which produces the electric field is called source charge and the charge which experiences the effect of source charge is called test charge.

→ unit positive charge is taken as test charge.

→ its magnitude is very small in comparison to source charge because its own field should not affect the field of source charge.

ELECTRIC FIELD INTENSITY

It is defined as the force experienced per unit positive test charge placed at that point, without disturbing the source charge.

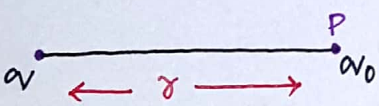
It is expressed as, $\boxed{\vec{E} = \frac{\vec{F}}{q_0}}$, where $\vec{E}$ = electric field intensity
q_0 = test charge
$\vec{F}$ = force experienced by the test charge q_0.

- It is a vector quantity
- SI unit :- N/C or V/m
- CGS unit :- D/stC or D/abC

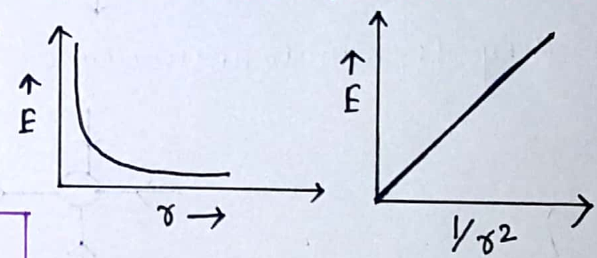
Dimension :- $\frac{M^1L^1T^{-2}}{AT} = [M^1L^1T^{-3}A^{-1}]$

* Electric field due to positive charge is always away from it while due to negative charge is always towards it

ELECTRIC FIELD INTENSITY DUE TO POINT CHARGE :-



P is any point at a distance r from the source charge q.



Force on q_0,

$$\vec{F} = \frac{Kqq_0}{r^2}\hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{Kq}{r^2}\hat{r}$$

$$\boxed{|\vec{E}| = \frac{Kq}{r^2}}$$

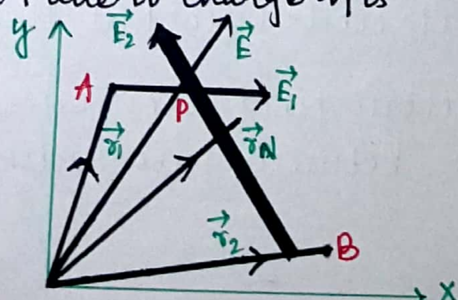
ELECTRIC FIELD DUE TO MULTIPLE CHARGES :-

Consider $q_1, q_2, \ldots\ldots, q_N$ charges are placed at a dist $r_1, r_2, \ldots r_N$ from origin in vacuum. Hence, the electric field at point P due to charge q_1 is

$$\vec{E_1} = \frac{\vec{F_1}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{1P}^2}\hat{r}_{1P}$$

Similarly,

$$\vec{E_2} = \frac{\vec{F_2}}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_{2P}^2}\hat{r}_{2P}$$



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$$\vec{E}_N = \frac{1}{4\pi\epsilon_0} \frac{q_N}{r^2_{NP}} \hat{r}_{NP}$$

According to the superposition principle,

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \dots\dots + \vec{E}_N$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r^2_{1P}}\hat{r}_{1P} + \frac{q_2}{r^2_{2P}}\hat{r}_{2P} + \dots\dots + \frac{q_N}{r^2_{NP}}\hat{r}_{NP}\right]$$

$$\Rightarrow \boxed{\vec{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r^2_{iP}}\hat{r}_{iP}}$$

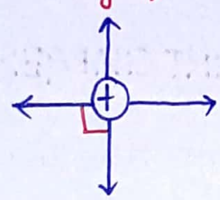
ELECTRIC FIELD LINES / LINES OF FORCE :-

A curve along which the test charge would tend to move when force to do so in an electric field due to a source charge. These imaginary lines are called electric field lines.

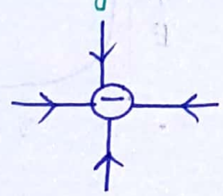
PROPERTIES :-

① They start from positive charge and end at negative charge.

② They emerge normally from the surface of a positive charge.



③ They terminate normally on the surface of a negative charge.



④ The field lines have a tendency to expand laterally so as to exert a lateral pressure. This explains repulsion between two like charges.

⑤ Tangent to any point on electric field lines shows the direction of electric field at that point.

⑥ Electric field lines contract lengthwise to represent attraction between two unlike charges.

⑦ Two field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point, which is not possible.

⑧ They are continous smooth curve without any breaks

⑨ They donot form closed loops.

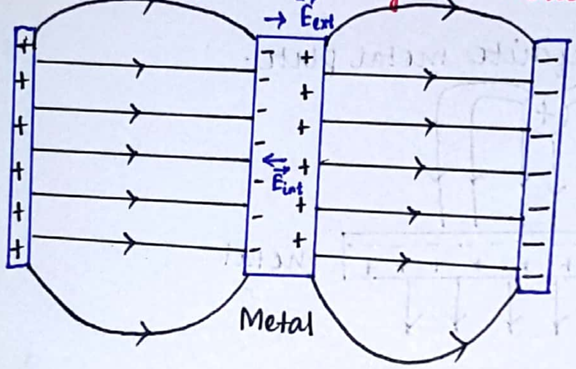
⑩ The region where lines of force are crowded, its intensity is more.

⑪ The number ΔN of lines per unit cross sectional area perpendicular to the field lines is directly proportional to the magnitude of the intensity of electric field in that region.

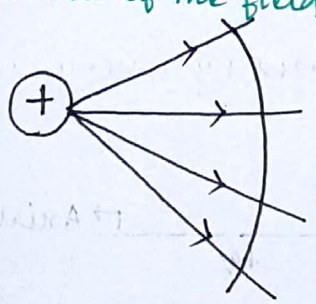
$$\frac{\Delta N}{\Delta A} \propto E$$

⑫ They donot pass through a conductor

⑬ Field lines cannot pass through a metallic slab.



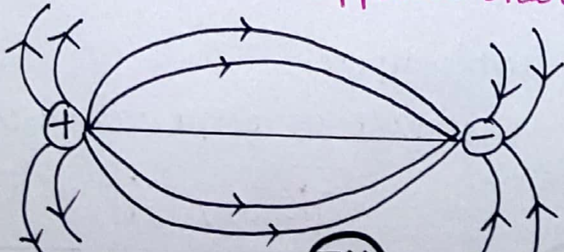
⑭ The relative closeness of the field lines gives the strength of electric field



- Fields close to each other indicate strong field.
- Fields lines away to each other indicate weak field.

REPRESENTATION OF ELECTRIC FIELD:-

Electric field lines due to opposite charges are equal in magnitude.



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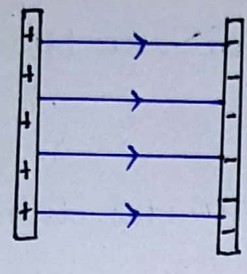
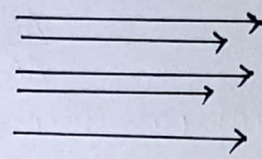
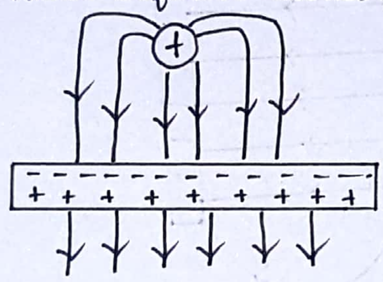
- In case of uniform field, the field lines are parallel (to have same direction) and are equidistance (to have same magnitude) to each other



- In case of non-uniform field, the field lines are not parallel and are not equidistance to each other.

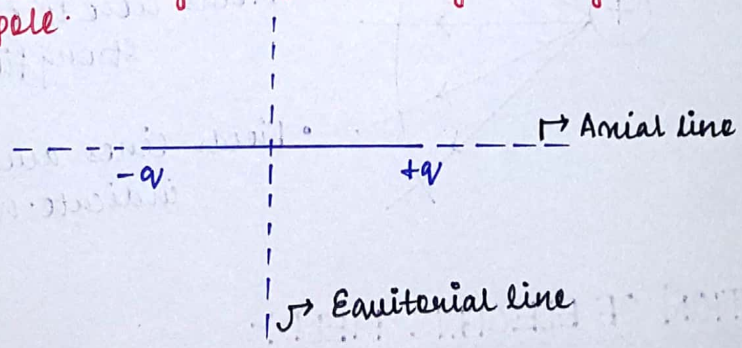


- Fixed point charge near infinite metal plate.



ELECTRIC DIPOLE :-

Two equal and opposite charges separated by a very small distance constitute a dipole.



ELECTRIC DIPOLE MOMENT :-

- It determines the strength of electric dipole
- It is define as the product of magnitude of either charge and separation of distance between them.

$$\vec{P} = q \times \vec{2l}$$

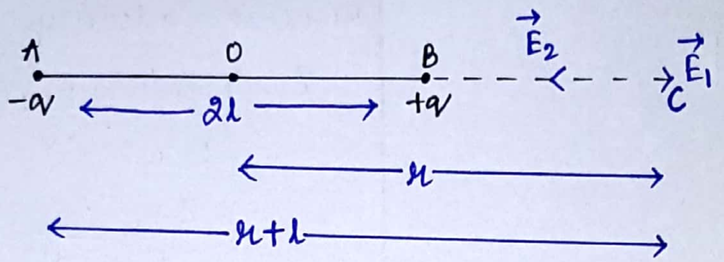
$$|P| = q(2l)$$

- vector quantity
- direction is always from negative charge to positive charge.
- Dimension – [ATL]
- SI unit – Cm

IDEAL DIPOLE / POINT DIPOLE :-

Suppose, $q \to \infty$, $2l \to 0$ such that p is finite. Such a dipole of negligibly small size is called as ideal dipole or point dipole.

ELECTRIC FIELD INTENSITY DUE TO DIPOLE AT THE AXIAL POSITION/END ON POSITION :-



C is any point on the axial line at a distance r from the centre of the dipole

Due to +q,

$$\vec{E}_1 = \frac{Kq}{BC^2}\hat{i}$$

$$= \frac{Kq}{(r-l)^2}\hat{i}$$

Due to −q,

$$\vec{E}_2 = \frac{Kq}{AC^2}(-\hat{i})$$

$$= \frac{Kq}{(r+l)^2}(-\hat{i})$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \frac{Kq}{(r-l)^2}\hat{i} + \frac{Kq}{(r+l)^2}(-\hat{i})$$

$$= Kq\left[\frac{(r+l)^2-(r-l)^2}{(r-l)^2\,(r+l)^2}\right]\hat{i}$$

$$= Kq\left[\frac{4rl}{(r^2-l^2)^2}\right]\hat{i}$$

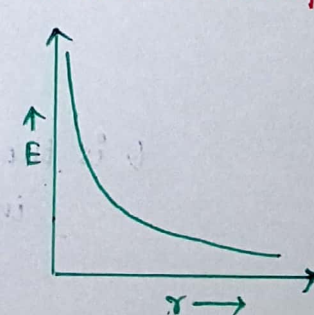
$$= Kq\,\frac{2r\cdot 2l}{(r^2-l^2)^2}\hat{i}$$

$$\boxed{\vec{E} = \frac{2Kpr}{(r^2-l^2)^2}\hat{i}}$$

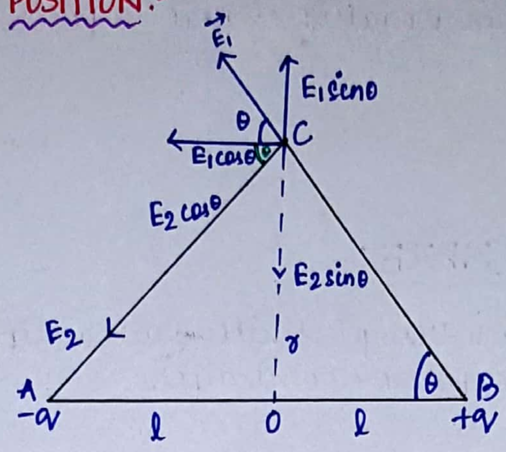
For ideal dipole, $l <<<< r$, so l^2 can be neglected

$$\vec{E} = \frac{2Kpr}{r^4}\hat{i}$$

$$\boxed{\vec{E} = \frac{2Kp}{r^3}\hat{i}}$$



ELECTRIC FIELD INTENSITY DUE TO DIPOLE AT AN EQUITORIAL POSITION OR BROAD SIDE ON POSITION:-



C is any point on the equitorial line at a distance r from the centre of the dipole.

Due to +q charge,

$$E_1 = \frac{Kq}{BC^2} = \frac{Kq}{r^2+l^2}$$

Due to -q charge,

$$E_2 = \frac{Kq}{AC^2} = \frac{Kq}{r^2+l^2}$$

$E_1 = E_2$

$E_1 \sin\theta$ and $E_2 \sin\theta$ cancel each other

Resultant Intensity,

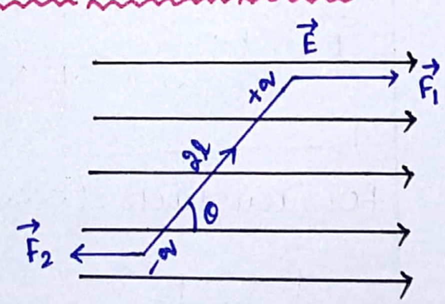
$$\vec{E} = (E_1\cos\theta + E_2\cos\theta)(-\hat{i})$$
$$= 2E_1\cos\theta(-\hat{i})$$
$$= 2\frac{Kq}{r^2+l^2}\frac{l}{\sqrt{r^2+l^2}}(-\hat{i})$$
$$= \frac{2Kql}{(r^2+l^2)^{3/2}}(-\hat{i})$$

$$\boxed{\vec{E} = \frac{KP}{(r^2+l^2)^{3/2}}(-\hat{i})}$$

for ideal dipole, $l \lll r$, l^2 can be neglected

$$\boxed{\vec{E} = \frac{KP}{r^3}(-\hat{i})}$$

DIPOLE IN UNIFORM ELECTRIC FIELD:-



θ is the angle between dipole moment and intensity.

Force on $+q$.

$$\vec{F_1} = q\vec{E}$$

Force on $-q$,

$$\vec{F_2} = -q\vec{E}$$

$$\vec{F} = \vec{F_1} + \vec{F_2} = 0$$

So no translatory motion

As two forces are not in same line of action, so they constitute a couple due to which dipole rotate.

$$\tau = (2l) F \sin\theta$$
$$= 2lqE \sin\theta$$
$$= PE \sin\theta$$

$$\boxed{\tau = PE \sin\theta}$$

In vector form,

$$\boxed{\vec{\tau} = \vec{P} \times \vec{E}}$$

$\vec{\tau} \perp \vec{P}$ and $\vec{\tau} \perp \vec{E}$

These are two pairs of perpendicular vector.

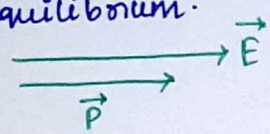
The direction of torque is $\perp^r$ to the plane inward according to figure.

Case-1

when $\theta = 0$

$\boxed{\tau = 0}$

It is a condition of stable equilibrium.

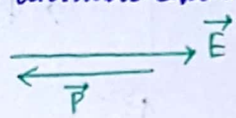


Case-2

when $\theta = 180^\circ$

$\boxed{\tau = 0}$

It is a condition of unstable equilibrium



Case-3

when $\theta = 90^\circ$

$\boxed{\tau = PE}$

Maximum torque

NEUTRAL POINT :-

It is a point in an electric field where when any charge is placed experience no force.

CASE-1

- for 2 like charges, neutral point lies between them. $+q_1$ N $+q_2$
- when similar charge is placed at neutral point, it is in unstable equilibrium along Y-axis and stable equilibrium along X-axis
- when dissimilar charge is placed at neutral point, it is in stable equilibrium along Y-axis and unstable equilibrium along X-axis.

CASE-2

- for two unlike charges, neutral point lies at the side of less magnitude charge.
- If $q_1 = q_2$, then neutral point is not possible.