Centre of Mass

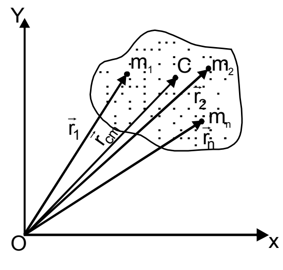
Section A - **Calculation of COM of system of N-particles, COM of continuous distributed system, Combination of structure and Cavity problems**

1. CENTRE OF MASS :

Every physical system has associated with it a certain point whose motion characterises the motion of the whole system. When the system moves under some external forces, then this point moves as if the entire mass of the system is concentrated at this point and also the external force is applied at this point for translational motion. This point is called the centre of mass of the system.

1.1 Centre of Mass of a System of 'N' Discrete Particles :

Consider a system of N point masses m_1, m_2, m_3, ... m_n whose position vectors from origin O are given by $\vec{r}_1, \vec{r}_2, \vec{r}_3$ .... $\vec{r}_n$ respectively. Then the position vector of the centre of mass C of the system is given by.



$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + ...+m_n\vec{r}_n}{m_1 + m_2 + ...+m_n} \;;$$

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i\vec{r}_i}{\sum_{i=1}^{n} m_i} \qquad \vec{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i\vec{r}_i$$

where, $m_i\vec{r}_i$ is called the **moment of mass** of particle with respect to origin.

$M = \left(\sum_{i=1}^{n} m_i\right)$ is the total mass of the system.

Further, $\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$ and

$$\vec{r}_{COM} = x_{COM}\hat{i} + y_{COM}\hat{j} + z_{COM}\hat{k}$$

So, the cartesian co-ordinates of the COM will be

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + ...+m_nx_n}{m_1 + m_2 + ...m_n} = \frac{\sum_{i=1}^{n} m_ix_i}{\sum_{i=1}^{n} m_i}$$

or $\quad x_{COM} = \dfrac{\sum_{i=1}^{n} m_ix_i}{M}$

Similarly, $\quad y_{COM} = \dfrac{\sum_{i=1}^{n} m_iy_i}{M}$

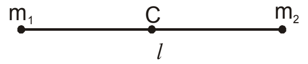
and $\quad z_{COM} = \dfrac{\sum_{i=1}^{n} m_iz_i}{M}$

Note :

- If the origin is taken at the centre of mass then $\sum_{i=1}^{n} m_i\vec{r}_i = 0$. hence, the COM is the point about which the sum of "mass moments" of the system is zero.
- If we change the origin then $\vec{r}_1, \vec{r}_2, \vec{r}_3$....... changes. So $\vec{r}_{cm}$ also changes but exact location of center of mass does not change.

1.2 Position of COM of two particles : -

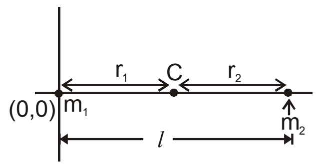
Consider two particles of masses m_1 and m_2 separated by a distance l as shown in figure.



Let us assume that m_1 is placed at origin and m_2 is placed at position (l, 0) and the distance of centre of mass from m_1 & m_2 is r_1 & r_2 respectively.

So $x_{COM} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

$r_1 = \dfrac{0 + m_2 l}{m_1 + m_2} = \dfrac{m_2 l}{m_1 + m_2}$ ...(1)



$r_2 = l - \dfrac{m_2 l}{m_1 + m_2} = \dfrac{m_1 l}{m_1 + m_2}$ ...(2)

From the above discussion, we see that

$r_1 = r_2 = \dfrac{l}{2}$ if $m_1 = m_2$, i.e., COM lies midway between the two particles of equal masses.

Similarly, $r_1 > r_2$ if $m_1 < m_2$ and $r_1 < r_2$ if $m_2 < m_1$ i.e., COM is nearer to the particle having larger mass.

From equation (1) & (2)

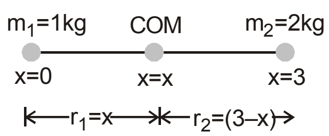
$m_1r_1 = m_2r_2$ ...(3)

Centre of mass of two particle system lie on the line joining the centre of mass of two particle system.

EXAMPLE 1

Two particle of mass 1 kg and 2 kg are located at x = 0 and x = 3 m. Find the position of their centre of mass.

Sol.



Since, both the particles lie on x-axis, the COM will also lie on x-axis. Let the COM is located at x = x, then

r_1 = distance of COM from the particle of mass 1 kg = x and r_2 = distance of COM from the particle of mass 2 kg = (3 – x)

Using $\dfrac{r_1}{r_2} = \dfrac{m_2}{m_1}$

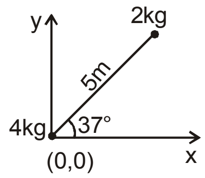
or $\dfrac{x}{3 - x} = \dfrac{2}{1}$

or $x = 2$ m

thus, the COM of the two particles is located at x = 2m.

EXAMPLE 2

Two particle of mass 4 kg & 2kg are located as shown in figure then find out the position of centre of mass.



Sol. First find out the position of 2 kg mass

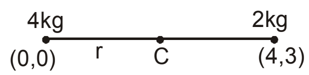
$x_{2kg} = 5 \cos 37° = 4$ m

$y_{2kg} = 5 \sin 37° = 3$ m

So these system is like two particle system of mass 4 kg and 2kg are located (0, 0) and (4, 3) respectively. then

$x_{com} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} = \dfrac{0 + 2 \times 4}{4 + 2} = \dfrac{8}{6} = \dfrac{4}{3}$

$y_{com} = \dfrac{m_1y_1 + m_2y_2}{m_1 + m_2} = \dfrac{0 + 3 \times 2}{4 + 2} = 1$ m

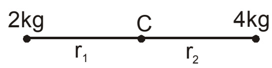


So position of C.O.M is $\left(\dfrac{4}{3}, 1\right)$

EXAMPLE 3

Two particles of mass 2 kg and 4 kg lie on the same line. If 4 kg is displaced rightwards by 5m then by what distance 2 kg should be move for which centre of mass will remain at the same position.

Sol. Let us assume that C.O.M. lie at point C and the distance of C from 2kg and 4kg particles are r_1 & r_2 respectively. Then from relation



$m_1r_1 = m_2r_2$

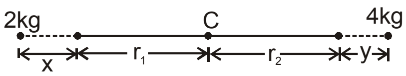
$2r_1 = 4r_2$ ...(i)

Now 4kg is displaced rightwards by 5m then assume 2kg is displaced leftwards by x distance to keep the C.O.M. at rest.

from relation $m_1r_1' = m_2r_2'$

$\Rightarrow \quad m_1(r_1 + x) = m_2 (r_2 + y)$

$2(r_1 + x) = 4(r_2 + 5)$ ...(ii)



$2x = 20$

$x = 10$ m

To keep the C.O.M at rest 2 kg displaced 10 m left wards

Aliter : If centre of mass is at rest then we can write

$m_1x = m_2y$

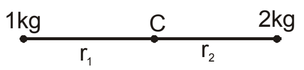
$2 \times x = 4 \times 5$

$x = 10$ m

EXAMPLE 4

Two particles of mass 1 kg and 2 kg lie on the same line. If 2kg is displaced 10m rightwards then by what distance 1kg should displaced so that centre of mass will displaced 2m right wards.

Sol. Initially let us assume that C.O.M is at point C which is r_1 & r_2 distance apart from mass m_1 & m_2 respectively as shown in figure.

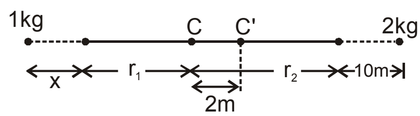


from relation $m_1\, r_1 = m_2\, r_2$

$\Rightarrow$ (1) $r_1 = 2r_2$

Now 2kg is displaced 10 m rightwards then we assume that 1 kg is displaced x m leftward to move the C.O.M 2m rightwards.

So from relation $m_1r_1' = m_2r_2'$



$\Rightarrow$ 1 $(x + r_1 + 2) = 2\,(10 + r_2 - 2)$

$\Rightarrow \quad x + r_1 + 2 = 20 + 2r_2 - 4$ ...(ii)

from eq. (i) & (ii) x = 14m (leftwards)

EXAMPLE 5

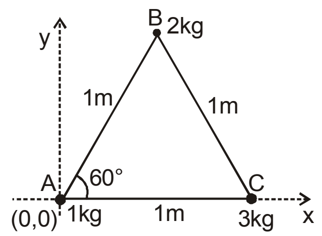
Three particles of mass 1 kg, 2 kg, and 3 kg are placed at the corners A, B and C respectively of an equilateral triangle ABC of edge 1m. Find the distance of their centre of mass from A.

Sol. Assume that 1kg mass is placed at origin as shown in figure.

co-ordinate of A = (0, 0)

co-ordinate of B = $(1\cos 60°, 1\sin 60°) = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

co-ordinate of C = (1, 0)



Let us assume that position of C.O.M is given by

$$\vec{r}_{com} = x_{com}\,\hat{i} + y_{com}\,\hat{j}$$

Now $\quad x_{com} = \dfrac{m_Ax_A + m_Bx_B + m_Cx_C}{m_A + m_B + m_C}$

$$= \frac{1(0) + 2\left(\frac{1}{2}\right) + 3(1)}{1 + 2 + 3} = \frac{4}{6} = \frac{2}{3}$$

$$y_{com} = \frac{1(0) + 2\left(\frac{\sqrt{3}}{2}\right) + 3(0)}{1 + 2 + 3} = \frac{\sqrt{3}}{6}$$

Position of centre of mass $= \left(\frac{2}{3}, \frac{\sqrt{3}}{6}\right)$

distance of C.O.M from point A $= \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{\sqrt{3}}{6}\right)^2}$

$$= \frac{\sqrt{19}}{6}\text{m}$$

1.3 Centre of Mass of a Continuous Mass Distribution

For continuous mass distribution the centre of mass can be located by replacing summation sign with an integral sign. Proper limits for the integral are chosen according to the situation

$$x_{cm} = \frac{\int x\,dm}{\int dm},\; y_{cm} = \frac{\int y\,dm}{\int dm},\; z_{cm} = \frac{\int z\,dm}{\int dm} \quad ...(i)$$

$\int dm$ = M (mass of the body)

here x,y,z in the numerator of the eq. (i) is the coordinate of the centre of mass of the dm mass.

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$

Note :

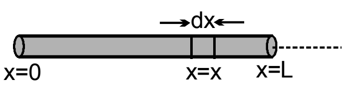
- If an object has symmetric mass distribution about x axis then y coordinate of COM is zero and vice-versa

(a) Centre of Mass of a Uniform Rod

Suppose a rod of mass M and length L is lying along the x-axis with its one end at x = 0 and the other at x = L. Mass per unit length of the rod $\lambda = \frac{M}{L}$

Hence, dm, (the mass of the element dx situated at x = x is) = λ dx

The coordinates of the element dx are (x, 0, 0). Therefore, x-coordinate of COM of the rod will be



$$x_{COM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L (x)(\lambda dx)}{\int_0^L \lambda dx} = \frac{1}{L}\int_0^L x\,dx = \frac{L}{2}$$

The y-coordinate of COM is

$$y_{COM} = \frac{\int y\,dm}{\int dm} = 0$$

Similarly, $z_{COM} = 0$

i.e., the coordinates of COM of the rod are $\left(\frac{L}{2}, 0, 0\right)$, i.e, it lies at the centre of the rod.

EXAMPLE 6

A rod of length L is placed along the x-axis between x = 0 and x = L. The linear density (mass/length) λ of the rod varies with the distance x from the origin as λ = Rx. Here, R is a positive constant. Find the position of centre of mass of this rod.

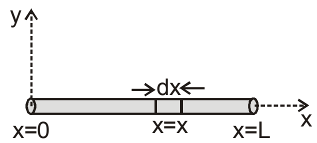
Sol. Mass of element dx situated at x = x is

$$dm = \lambda\,dx = R\,x\,dx$$

The COM of the element has coordinates (x, 0, 0). Therefore, x-coordinates of COM of the rod will be

$$x_{COM} = \frac{\int_0^L x\,dm}{\int dm} = \frac{\int_0^L (x)(Rx)dx}{\int_0^L (Rx)dx}$$

$$= \frac{R\int_0^L x^2 dx}{R\int_0^L x\,dx} = \frac{\left[\frac{x^3}{3}\right]_0^L}{\left[\frac{x^2}{2}\right]_0^L} = \frac{2L}{3}$$



The y-coordinates of COM of the rod is

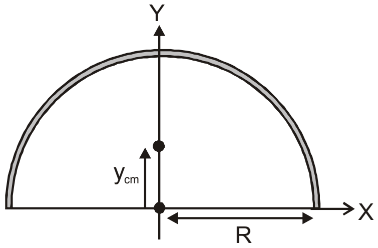
$$y_{COM} = \frac{\int y\,dm}{\int dm} = 0 \qquad \text{(as y = 0)}$$

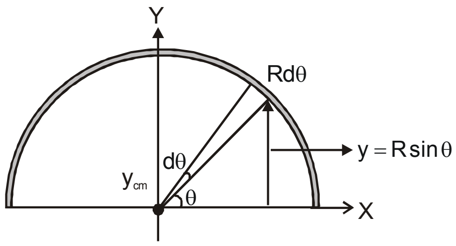
Similarly, $z_{COM} = 0$

Hence, the centre of mass of the rod lies at $\left[\frac{2L}{3}, 0, 0\right]$

(b) Centre of mass of a Semicircular Ring :

Figure shows the object (semi circular ring). By observation we can say that the x-coordinate of the centre of mass of the ring is zero as the half ring is symmetrical about y-axis on both sides of the origin. Only we are required to find the y-coordinate of the centre of mass.





To find y_{cm} we use $\quad y_{cm} = \dfrac{\int (dm)y}{\int dm}$ ...(i)

Here y is the position of C.O.M. of dm mass
Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width $d\theta$. If radius of the ring is R then its y coordinate-will be R $\sin\theta$, here dm is given as

$$dm = \lambda R d\theta$$

where λ = mass density of semi circular ring.
So from equation ----(i), we have

$$y_{cm} = \frac{\int_0^{\pi} \lambda R d\theta (R\sin\theta)}{\int_0^{\pi} \lambda R d\theta} = \frac{R}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$y_{cm} = \frac{2R}{\pi} \qquad \text{...(ii)}$$

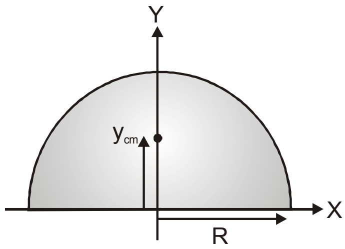
(c) Centre of mass of Semicircular Disc :

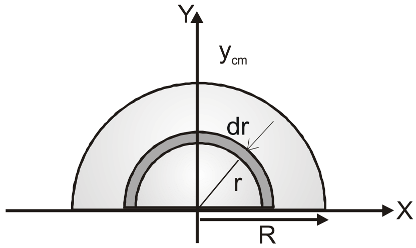
Figure shows the half disc of mass M and radius R. Here, we are only required to find the y-coordinate of the centre of mass of this disc as centre of mass will be located on its half vertical diameter. Here to find y_{cm}, we consider a small elemental ring of mass dm of radius r on the disc (disc can be considered to be made up such thin rings of increasing radii) which will be integrated from 0 to R. Here dm is given as

$$dm = \sigma \pi r dr$$

where σ is the mass density of the semi circular disc.

$$\sigma = \frac{M}{\pi R^2/2} = \frac{2M}{\pi R^2}$$





Now the y-coordinate of the element is taken as $\frac{2r}{\pi}$, (as in previous section, we have derived that the centre of mass of a semi circular ring is concentrated at $\frac{2R}{\pi}$)

$$y_{cm} = \frac{\int_0^R dm \,.\, y}{\int_0^R dm}$$

Here y is the position COM of dm mass.

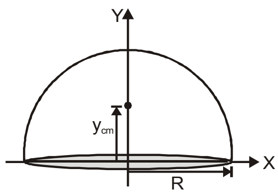
Here y_{cm} is given as $\quad y_{cm} = \dfrac{\int_0^R dm \frac{2r}{\pi}}{\int_0^R \sigma \pi r dr} = \int_0^R \frac{4}{\pi R^2} r^2 dr$

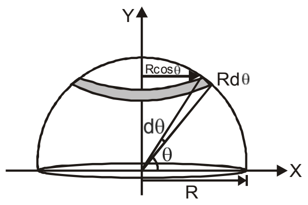
$$\Rightarrow \quad y_{cm} = \frac{4R}{3\pi}$$

(d) Centre of mass of a Hollow Hemisphere :

A hollow hemisphere of mass M and radius R. Now we consider an elemental circular strip of angular width $d\theta$ at an angular distance θ from the base of the hemisphere. This strip will have an area.

$$dS = 2\pi R \cos\theta \, R d\theta$$





Its mass dm is given as

$dm = \sigma 2\pi R\cos\theta R d\theta$

Here σ is the mass density of a hollow hemisphere

$$\sigma = \frac{M}{2\pi R^2}$$

Here y-coordinate of this strip of mass dm can be taken as R $\sin\theta$. Now we can obtain the centre of mass of the system as.

$$y_{cm} = \frac{\int_0^{\frac{\pi}{2}} dm R \sin\theta}{\int_0^{\pi/2} dm} = \frac{\int_0^{\frac{\pi}{2}} \left(\sigma 2\pi R^2 \cos\theta d\theta\right) R \sin\theta}{\int_0^{\pi/2} \left(\sigma 2\pi R^2 \cos\theta d\theta\right)}$$

$$= R \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta \, d\theta$$

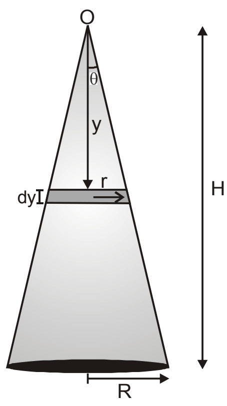
$$y_{cm} = \frac{R}{2}$$

(e) ***Centre of mass of a Solid Cone :***

A solid cone has mass M, height H and base radius R. Obviously the centre of mass of this cone will lie somewhere on its axis, at a height less than H/2. To locate the centre of mass we consider an elemental disc of width dy and radius r, at a distance y from the apex of the cone. Let the mass of this disc be dm, which can be given as

$dm = \rho \times \pi r^2 \, dy$

Here ρ is the mass density of the solid cone



here y_{cm} can be given as

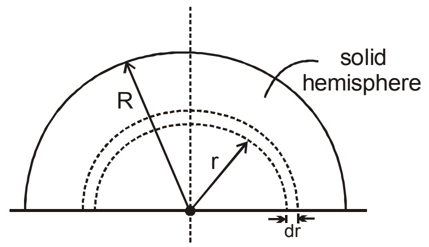
$$y_{cm} = \frac{1}{M} \int_0^H y \, dm$$

$$= \frac{1}{M} \int_0^H \left(\frac{3M}{\pi R^2 H} \pi \left(\frac{Ry}{H}\right)^2 dy \right) y$$

$$= \frac{3}{H^3} \int_0^H y^3 \, dy = \frac{3H}{4}$$

(f) ***C.O.M of a solid Hemisphere : -***

A hemisphere is of mass density ρ and radius R To find its centre of mass (only y co-ordinate) we consider an elemental hollow hemispshere of radius r on the solid hemisphere (solid hemisphere can be considered to be made up such hollow hemisphere of increasing radii) which will be integrate from O to R.



Here y Co-ordinate of centre of mass of elemental hollow hemisphere is (0, r/2, 0)

$dm = \rho \, 2\pi r^2 \, dr$

$$y_{CM} = \frac{\int_0^R dm.y}{\int_0^R dm} \; ; \; y_{CM} = \frac{\int_0^R \rho(2\pi r^2)dr\,(r/2)}{\int_0^R \rho.2\pi r^2.dr} \; ; \; y_{CM}$$

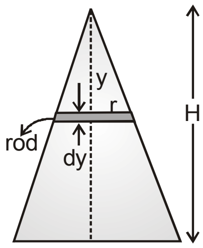
$$= \frac{3R}{8}$$

(g) ***Centre of mass of Triangular Plate :***

A triangular plate has mass density σ height H and base is 2R. Obviously the centre of mass of this plate will lie some where on its axis at a height less than H/2. To locate the centre of mass we consider an elemental rod of width dy and length 2r at a distance y from the apex of the plate. Let the mass of this rod be dm which can be gives as

$dm = \sigma \, (2r) \, dy$

from the theorem of triangle



$$\frac{H}{R} = \frac{y}{r}$$

$\Rightarrow \quad r = \dfrac{Ry}{H}$

Here Y_{CM} can be given as

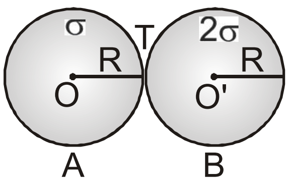
$$y_{CM} = \frac{\int_0^H y\,dm}{\int_0^H dm} \quad ; \quad y_{CM} = \frac{\int_0^H \sigma(2r)dy.y}{\int_0^H \sigma(2r)\,dy}$$

$$y_{CM} = \frac{\int_0^H \sigma(\frac{2Ry}{H})ydy}{\int_0^H \sigma(\frac{2Ry}{H})\,dy} \quad ; \quad y_{CM} = \frac{2H}{3}$$

2. COMBINATION OF STRUCTURE :

EXAMPLE 7

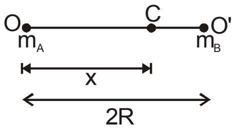
Two circular disc having radius R and mass density σ and 2σ respectively are placed as shown in figure. Then find out the position of COM of the system.



Sol. Mass of disc A $m_A = \sigma\pi R^2$

Mass of disc B $m_B = 2\sigma\pi R^2$

Due to symmetry the COM of disc A lie at point O and COM of disc B lie at point O′. So we realize the above problem in a following way



Centre of mass due to both the disc lie at point C (assume),

having distance x from m_A

$\Rightarrow \quad x = \dfrac{m_B(2R)}{m_A + m_B} \quad ; \quad x = \dfrac{2\sigma\pi R^2(2R)}{\sigma(\pi R^2 + 2\pi R^2)} \quad ;$

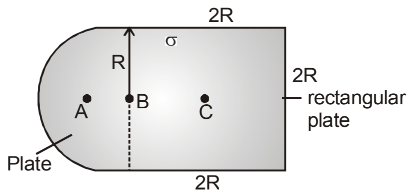
$x = \dfrac{4R}{3}$

So the centre of mass lie in the disc B having distance $\dfrac{4R}{3}$ from O.

EXAMPLE 8

Find out the position of centre of mass of the figure shown below.

Sol.



We divide the above problem in two parts

(i) First find out position of centre of mass of both semicircular plate and rectangular plate separately.

(ii) Then find the position of centre of mass of given structure .

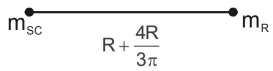
Centre of mass of semicircular disc lie at $\dfrac{4R}{3\pi}$

$\Rightarrow \quad AB = \dfrac{4R}{3\pi}$

Centre of mass of rectangular plate lie at the centre of plate at point C

$\Rightarrow \quad BC = R$

$\Rightarrow$



$m_{SC} = \dfrac{\sigma\pi R^2}{2} \quad ; \quad m_R = \sigma 4R^2$

$\Rightarrow$

$\Rightarrow$ Let us assume COM is at r_1 distance from m_R

$$\Rightarrow \quad r_1 = \frac{\sigma.\frac{\pi R^2}{2}\left(R + \frac{4R}{3\pi}\right)}{\sigma.\frac{\pi R^2}{2} + \sigma 4R^2}$$

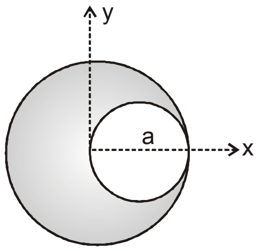
$$\Rightarrow \quad r_1 = \frac{\pi R(3\pi + 4)}{3(\pi + 8)}$$ **Ans.**

3. CAVITY PROBLEMS :

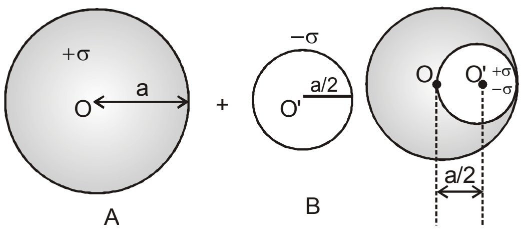
If some mass or area is removed from a rigid body then the position of centre of mass of the remaining portion is obtained by assuming that in a remaining part +m & – m mass is there. Further steps are explained by following example.

EXAMPLE 9

Find the position of centre of mass of the uniform lamina shown in figure. If the mass density of the lamina is σ.



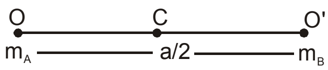
Sol. We assume that in remaining portion a disc of radius a/2 having mass density $+\sigma$ is there then we also include one disc of a/2 radius having $-\sigma$ mass density. So now the problem change in following form



So the centre of mass of both disc A & B lie in their respective centre such as O & O'.

Now

$\Rightarrow$ C.O.M. of the lamina $\Rightarrow \dfrac{m_A\, a/2}{m_A + m_B}$



$m_A = \sigma\,(\pi\, a^2)$

$m_B = -\,\sigma\,(\pi)\,(a/2)^2 = -\,\sigma\,\pi\dfrac{a^2}{4}$

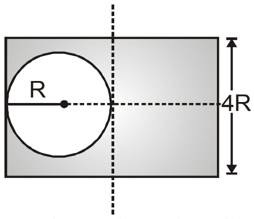
$$\Rightarrow \quad c = \frac{\sigma\pi a^2 . a/2}{\sigma\pi a^2 - \sigma\pi\dfrac{a^2}{4}} = \frac{a^3/2}{3a^2/4};$$

$$\frac{a^3}{2} \times \frac{4}{3a^2} = \frac{2a}{3}$$

i.e., C.O.M lie on leftward side from point O.

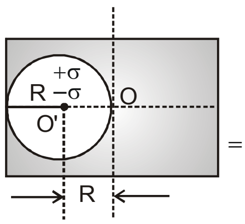
EXAMPLE 10

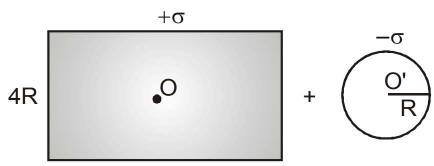
Find out the position of centre of mass of the uniform lamina as shown in figure.



Sol. We assume that a disc of radius R having mass density $\pm\,\sigma$ is in the removed section.

Now the problem change in following form





When disc of mass density $+\sigma$ and radius R is include than a complete rectangular plate is make having centre of mass at point O. When consider only disc having mass density $-\sigma$ and radius R then C.O.M of this disc lie at point O′

Then the position of C.O.M

$$= \frac{\sigma(4R)^2 . R}{-\sigma\pi R^2 + \sigma(4R^2)} = \frac{\sigma 16R^3}{\sigma R^2(16-\pi)} = \frac{16R}{16-\pi}$$

i.e., centre of mass lie in the rightwards side from the cavity.

EXAMPLE 11

The centre of mass of rigid body always lie inside the body. Is this statement true or false?

Sol. False.

EXAMPLE 12

The centre of mass always lie on the axis of symmetry if it exists. Is this statement true of false?

Sol. True

EXAMPLE 13

If all the particles of a system lie in y-z plane, the x-coordinate of the centre of mass will be zero. Is this statement true or not?

Sol. True

Note

The student can now attempt section A from exercise.

Section B - Motion of COM, Conservation of Momentum, Trolley problems

4. MOTION OF CENTRE OF MASS AND CONSERVATION OF MOMENTUM: -

The position of centre of mass is given by

$$\vec{r}_{COM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...} \quad ....(1)$$

Here m_1, m_2, m_3 ..... are the mass in the system and $\vec{r}_1, \vec{r}_2, \vec{r}_3$......... is the corresponding position vector of m_1, m_2, m_3 respectively

4.1 Velocity of C.O.M of system :

To find the velocity of centre of mass we differentiate equation (1) with respect to time

$$\frac{d\vec{r}_{com}}{dt} = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...}$$

$$\Rightarrow \vec{V}_{com} = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...}$$

$$\vec{V}_{com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...}{m_1 + m_2 + m_3 + ...} \quad ...(2)$$

4.2 Acceleration of centre of mass of the system : -

To find the acceleration of C.O.M we differentiate equation (2)

$$\Rightarrow \frac{d\vec{V}_{com}}{dt} = \frac{m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + m_3\frac{d\vec{v}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...}$$

$$\vec{a}_{com} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...}{m_1 + m_2 + m_3 + ...} \quad ...(3)$$

Now $(m_1 + m_2 + m_3)$

$$\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$$

$$\vec{F}_{net(system)} = \vec{F}_{1net} + \vec{F}_{2net} + \vec{F}_{3net} + ...$$

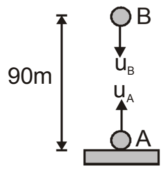
The internal forces which the particles exert on one another play absolutely no role in the motion of the centre of mass.

EXAMPLE 14

Two particles A and B of mass 1 kg and 2 kg respectively are projected in the directions shown in figure with speeds u_A = 200 m/s and u_B = 50 m/s. Initially they were 90 m apart. Find the maximum height attained by the centre of mass of the particles. Assume acceleration due to gravity to be constant. ($g = 10\ m/s^2$)

Sol. Using $m_A r_A = m_B r_B$

or $(1)(r_A) = (2)(r_B)$



or $r_A = 2r_B$ ...(i)

and $r_A + r_B = 90$ m ...(ii)

Solving these two equations, we get

$r_A = 60$ m and $r_B = 30$ m

i.e., COM is at height 60 m from the ground at time t = 0.

Further, $\vec{a}_{COM} = \dfrac{m_A\vec{a}_A + m_B\vec{a}_B}{m_A + m_B}$

$= g = 10\ m/s^2$ (downwards)

as $\vec{a}_A = \vec{a}_B = g$ (downwards)

$$\vec{u}_{COM} = \frac{m_A\vec{u}_A + m_B\vec{u}_B}{m_A + m_B}$$

$$= \frac{(1)(200)-(2)(50)}{1+2} = \frac{100}{3} \text{m/s} \text{ (upwards)}$$

Let, h be the height attained by COM beyond 60 m. Using,

$$v_{COM}^2 = u_{COM}^2 + 2a_{COM}h$$

or $$0 = \left(\frac{100}{3}\right)^2 - (2)(10)h$$

or $$h = \frac{(100)^2}{180} = 55.55 \text{ m}$$

Therefore, maximum height attained by the centre of mass is

$$H = 60 + 55.55 = 115.55 \text{ m}$$ **Ans.**

Case I : If $F_{net} = 0$ then we conclude :

(a) The acceleration of centre of mass is zero ($\vec{a}_{com} = 0$)

If a_1, a_2, a_3.... is acceleration of m_1, m_2, m_3 mass in the system then a_1, a_2, a_3 may or may not be zero.

(b) K.E. of the system is not constant it may change due to internal force.

(c) Velocity of centre of mass is constant ($\vec{v}_{com} = \text{constant}$) but v_1, v_2, v_3 may or may not constant. It may be change due to internal force.

from eq (2)

$$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ... = \text{constant}$$

This is called momentum conservation.

"If resultant external force is zero on the system, then the net momentum of the system must remain constant".

Case II : When centre of mass is at rest.

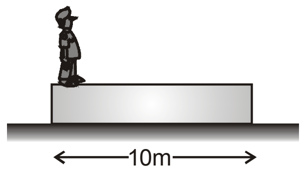
(a) $\vec{V}_{com} = 0$ then

$$\frac{d\vec{r}_{com}}{dt} = 0 \quad \Rightarrow \vec{r}_{com} = \text{constant.}$$

i.e. $\vec{r}_1, \vec{r}_2, \vec{r}_3$ .... may or may not change

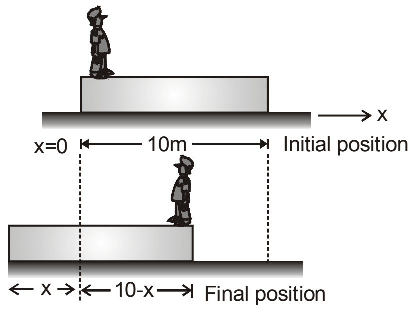
EXAMPLE 15

A wooden plank of mass 20 kg is resting on a smooth horizontal floor. A man of mass 60 kg starts moving from one end of the plank to the other end. The length of the plank is 10 m. Find the displacement of the plank over the floor when the man reaches the other end of the plank.



Sol. Here, the system is man +plank. Net force on this system is horizontal direction is zero and initially the centre of mass of the system is at rest. Therefore, the centre of mass does not move in horizontal direction.

Let x be the displacement of the Plank. Assuming the origin, i.e., x = 0 at the position shown in figure.



As we said earlier also, the centre of mass will not move in horizontal direction (x-axis). Therefore, for centre of mass to remain stationary,

$$x_i = x_f$$

$$20x = 60 \times (10 - x)$$

or $x = \frac{30}{4}$ m or x = 7.5 m **Ans.**

EXAMPLE 16

Mr. Verma (50 kg) and Mr. Mathur (60 kg) are sitting at the two extremes of a 4 m long boat (40 kg) standing still in water. To discuss a mechanics problem, they come to the middle of the boat. Neglecting friction with water, how far does the boat move on the water during the process?

Sol. Here the system is Mr. Verma + Mr. Mathur + boat. Net force on this system is in horizontal direction is zero and initially the centre of mass of the system is at rest. Therefore the C.O.M does not move in horizontal direction. Let x be the displacement of the boat. Then We can use the concept

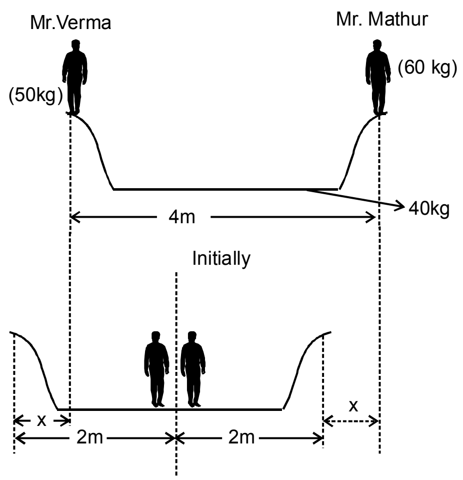
$$m_1x_1 = m_v x_v + m_M x_M$$

$40 \times x = 50 \times (2 - x) - 60(2 + x)$

$40x = 100 - 50x - 120 - 60x$

$150x = -20$

$x = -\frac{2}{15} m$



$x \approx 13$ cm (right wards)

Case III : When net force is zero only in one direction.

Let us assume that F_{net} in x direction is zero then we conclude

(i) Acceleration of the system in x direction is zero ($a_x = 0$)

(ii) $v_{(com)x}$ = constant

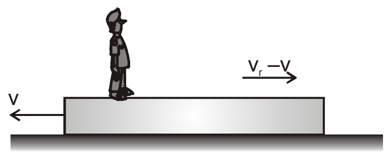
$\Rightarrow m_1v_{1x} + m_2v_{2x} + m_3v_{3x}$ = constant.

i.e., momentum is conserved only in x direction

EXAMPLE 17

A man of mass m_1 is standing on a platform of mass m_2 kept on a smooth horizontal surface. The man starts moving on the platform with a velocity v_r relative to the platform. Find the recoil velocity of platform.

Sol. Absolute velocity of man = $v_r - v$ where v = recoil velocity of platform. Taking the platform and the man a system, net external force on the system in horizontal direction is zero. The linear momentum of the system remains constant. Initially both the man and the platform were at rest.



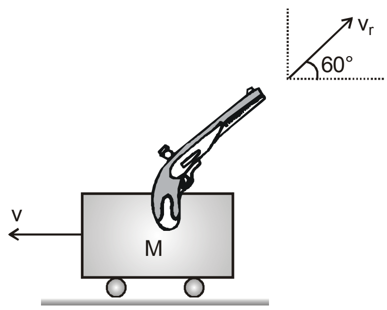
Hence, $0 = m_1(v_r - v) - m_2v$

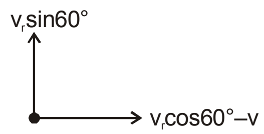
$\therefore \quad v = \frac{m_1v_r}{m_1 + m_2}$ **Ans.**

EXAMPLE 18

A gun (mass = M) fires a bullet (mass = m) with speed v_r relative to barrel of the gun which is inclined at an angle of 60° with horizontal. The gun is placed over a smooth horizontal surface. Find the recoil speed of gun.

Sol. Let the recoil speed of gun is v. Taking gun + bullet as the system. Net external force on the





Components of velocity of bullet relative to ground

system in horizontal direction is zero. Initially the system was at rest. Therefore, applying the principle of conservation of linear momentum in horizontal direction, we get

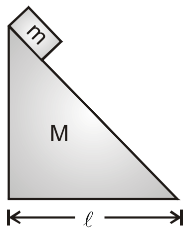
$Mv - m(v_r \cos 60° - v) = 0$

$\therefore \quad v = \dfrac{mv_r \cos 60°}{M+m}$

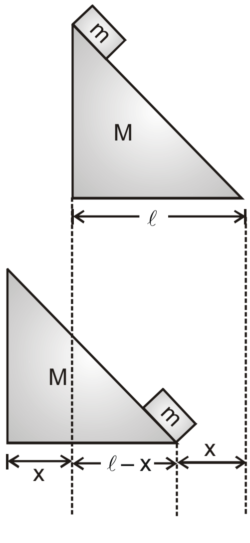
or $\quad v = \dfrac{mv_r}{2(M+m)}$ **Ans.**

EXAMPLE 19

A particle of mass m is placed at rest on the top of a smooth wedge of mass M, which in turn is placed at rest on a smooth horizontal surface as shown in figure. Then the distance moved by the wedge as the particle reaches the foot of the wedge is :



Sol. There is no external force in horizontal direction on the wedge block system, So the x-coordinate of the C.O.M of the wedge block system is at rest.



Let us assume that wedge move x when block reaches the ground. We can use the following relation when

x - coordinate of C.O.M is at rest

$m_1x_1 = m_2x_2$

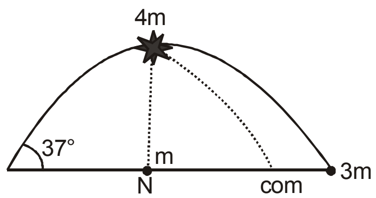
$Mx = m(\ell - x)$

$x = \dfrac{m\ell}{m+M}$

EXAMPLE 20

A projectile is fired at a speed of 100 m/s at an angle of 37° above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio 1 : 3, the lighter piece coming to rest. Find the distance from the launching point to the point where the heavier piece lands.

Sol. Internal force do not effect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is,



$$x_{COM} = \frac{2u^2 \sin\theta \cos\theta}{g} = \frac{2 \times 10^4 \times \frac{3}{5} \times \frac{4}{5}}{10} m$$

$$= 960 \text{ m}$$

The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range, i.e., at x = 480 m. If the heavier block hits the ground at x_2, then

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \Rightarrow 960$$

$$= \frac{(m)(480) + (3m)(x_2)}{(m + 3m)} \quad x_2 = 1120 \text{ m}$$

Ans.

EXAMPLE 21

A shell is fired from a cannon with a speed of 100 m/s at an angle 60° with the horizontal (positive x-direction). At the highest point of its trajectory, the shell explodes into two equal

fragments. One of the fragments moves along the negative x-direction with a speed of 50 m/s. What is the speed of the other fragment at the time of explosion.

Sol. As we know in absence of external force the motion of centre of mass of a body remains unaffected. Thus, here the centre of mass of the two fragments will continue to follow the original projectile path. The velocity of the shell at the highest point of trajectory is $v_M = u \cos\theta = 100 \times \cos 60° = 50$ m/s

Let v_1 be the speed of the fragment which moves along the negative x-direction and the other fragment has speed v_2,. which must be along positive x-direction. Now from momentum conservation, we have

$$mv = \frac{-m}{2}v_1 + \frac{m}{2}v_2$$

or $2v = v_2 - v_1$ or $v_2 = 2v + v_1$

$= (2 \times 50) + 50 = 150$ m/s

EXAMPLE 22

A particle of mass 2 m is projected at an angle of 45° with horizontal with a velocity of $20\sqrt{2}$ m/s. ***After 1 s explosion takes place and the particle is broken into two equal pieces. As a result of explosion one part comes to rest. Find the maximum height attained by the other part.***

(Take g = 10 m/s²)

Sol. Applying conservation of linear momentum at the time of collision, or at t = 1 s,

$$m\vec{v} + m(0) = 2m(20\hat{i} + 10\hat{j})$$

$\therefore$ $\vec{v} = 40\hat{j} + 20\hat{j}$

At 1 sec, masses will be at height :

$$h_1 = u_y t + \frac{1}{2}v_y t^2 = (20)(1) + \frac{1}{2}(-10)(1)^2 = 15\text{m}$$

After explosion other mass will further rise to a height :

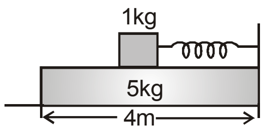
$$h_2 = \frac{u_y^2}{2g} = \frac{(20)^2}{2 \times 10} = 20 \text{ m}$$

$u_y = 20$ m/s just after collision.

$\therefore$ Total height $h = h_1 + h_2 = 35$ m

EXAMPLE 23

A plank of mass 5 kg placed on a frictionless horizontal plane. Further a block of mass 1 kg is placed over the plank. A massless spring of natural length 2m is fixed to the plank by its one end. The other end of spring is compressed by the block by half of spring's natural length. They system is now released from the rest. What is the velocity of the plank when block leaves the plank? (The stiffness contant of spring is 100 N/m)



Sol. Let the velocity of the block and the plank, when the block leaves the spring be u and v respectively.

By conservation of energy $\frac{1}{2}kx^2 = \frac{1}{2}mu^2 + \frac{1}{2}Mv^2$ [M = mass of the plank, m = mass of the block]

$\Rightarrow$ $100 = u^2 + 5v^2$ ...(i)

By conservation of momentum $mu + Mv = 0$

$\Rightarrow$ $u = -5v$ ...(ii)

Solving Eqs(i) and (ii)

$$30v^2 = 100 \Rightarrow v = \sqrt{\frac{10}{3}} \text{ m/s}$$

From this moment until block falls, both plank and block keep their velocity constant.

Thus, when block falls, velocity of plank = $\sqrt{\frac{10}{3}}$ m/s

Ans.

EXAMPLE 24

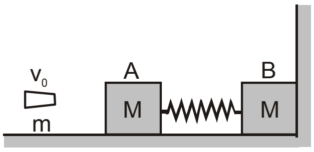
Two identical blocks each of mass M = 9 kg are placed on a rough horizontal surface of frictional coefficient μ = 0.1. The two blocks are joined by a light spring and block B is in contact with a vertical fixed wall as shown in figure. A bullet

of mass m = 1kg and v_0 = 10 m/s hits block A and gets embedded in it. Find the maximum compression of spring. (Spring constant = 240 N/m, g = 10 m/s²)

Sol. For the collision

$1 \times 10 = 10 \times v \Rightarrow \quad v = 1$ m/s

If x be the maximum compression



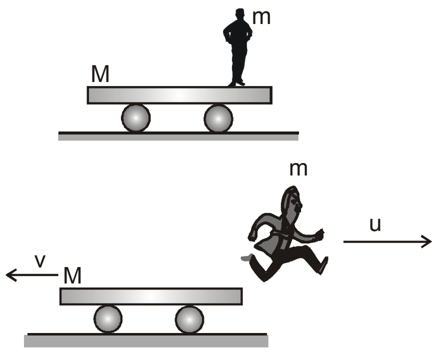
$$\frac{1}{2} \times 10 \times 1^2 = \mu(m + M)gx + \frac{1}{2}kx^2$$

$$5 = 10x + 120\,x^2 \quad \Rightarrow x = \frac{1}{6}\text{m} \qquad \textbf{Ans.}$$

EXAMPLE 25

A flat car of mass M is at rest on a frictionless floor with a child of mass m standing at its edge. If child jumps off from the car towards right with an initial velocity u, with respect to the car, find the velocity of the car after its jump.

Sol. Let car attains a velocity v, and the net velocity of the child with respect to earth will be u – v, as u is its velocity with respect to car.



Initially, the system was at rest, thus according to momentum conservation, momentum after jump must be zero, as

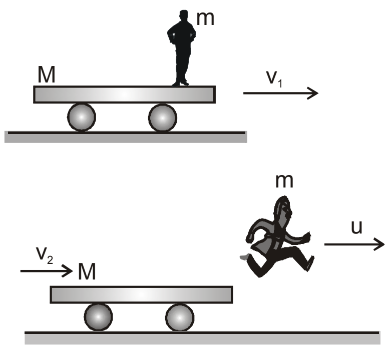
$$m(u - v) = M\,v$$

$$v = \frac{mu}{m + M}$$

EXAMPLE 26

A flat car of mass M with a child of mass m is moving with a velocity v_1 on a friction less surface. The child jumps in the direction of motion of car with a velocity u with respect to car. Find the final velocities of the child and that of the car after jump.

Sol. This case is similar to the previous example, except now the car is moving before jump. Here also no external force is acting on the system in horizontal direction, hence momentum remains conserved in this direction. After jump car attains a velocity v_2 in the same direction, which is less than v_1, due to backward push of the child for jumping. After jump child attains a velocity u + v_2 in the direction of motion of car, with respect to ground.



According to momentum conservation

$$(M + m)\,v_1 = Mv_2 + m\,(u + v_2)$$

Velocity of car after jump is

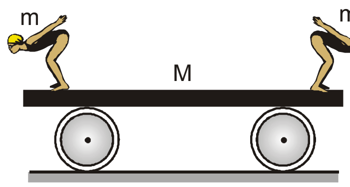
$$v_2 = \frac{(M + m)v_1 - mu}{M + m}$$

Velocity of child after jump is $u + v_2 = \dfrac{(M + m)v_1 + (M)u}{M + m}$

EXAMPLE 27

Two persons A and B, each of mass m are standing at the two ends of rail-road car of mass M. The person A jumps to the left with a horizontal speed u with respect to the car. There after, the person B jumps to the right, again with

the same horizontal speed u with respect to the car. Find the velocity of the car after both the persons have jumped off.



Sol. Let car attain the velocity v in right ward and velocity of man A with respect to ground is v′ then

$$v' = v - u$$

from momentum conservation

$$0 = mv' + (M + m)v$$

$\Rightarrow$ $m(v - u) + (M + m)v = 0$

$\Rightarrow$ $v = \dfrac{mu}{(M+2m)}$

After wards mass B jumps to the right with the same horizontal speed u with respect to car, than car attain v" velocity from linear momentum conservation.

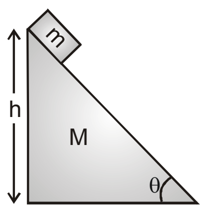
$$(M+m)v = m(u + v") + Mv"$$

$$(M+m)\left[\frac{mu}{M+2m}\right] = mu + (m+M)v"$$

Now $v" = \dfrac{m^2u}{(M+2m)(M+m)}$

EXAMPLE 28

A block of mass m is placed on a triangular block of mass M, which in turn is placed on a horizontal surface as shown in figure. Assuming frictionless surfaces find the velocity of the triangular block when the smaller reaches the bottom end.



Sol. Let us assume that wedge move leftward with velocity v and block move down ward with velocity u with respect to wedge.

$\therefore$ Net force is horizontal direction is zero so momentum is conserved in x direction.

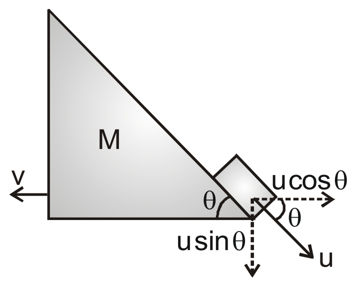
Now velocity of block with respect to ground is

$$\vec{v}_m = \vec{v}_{mw} + \vec{v}_w$$

$$\vec{V}_m = \vec{u} + \vec{v}$$

$$\vec{V}_m = (u\cos\theta - v)\hat{i} - u\sin\theta\hat{j}$$

Now from momentum conservation in x direction



$0 = - Mv + mV_{mx}$

$\Rightarrow$ $Mv = m(u\cos\theta - v)$ ...(1)

From energy conservation

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mv_m^2$$

$\Rightarrow$ $mgh = \dfrac{1}{2}mv^2 + \dfrac{1}{2}m(u^2 + v^2 - 2uv\cos\theta)$ ...(2)

from eq. (1) & (2)

$$v = \left[\frac{2m^2gh\cos\theta}{(M+m)(M+m\sin^2\theta)}\right]^{1/2}$$

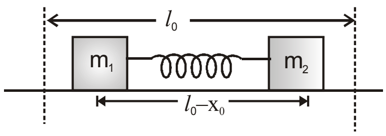
Note

The student can now attempt section B from exercise.

Section C - Spring block system

5. SPRING BLOCK SYSTEM :

A light spring of spring constant k and natural length l_0 attached in a compressed condition between two blocks of mass m_1 & m_2 on a smooth horizontal surface as shown in the figure. The spring is initially compressed by a distance x_0.



When system is released the block acquire velocities in opposite direction. Let us assume that the velocities of block m_1 & m_2 is v_1 & v_2 respectively at natural length of the spring and since no external force acts on this system in horizontal direction. Hence the linear momentum remains constant. Then from momentum conservation.

$$0 = m_2v_2 - m_1v_1$$

$$m_2v_2 = m_1v_1 \qquad ..(1)$$

From mechanical energy conservation

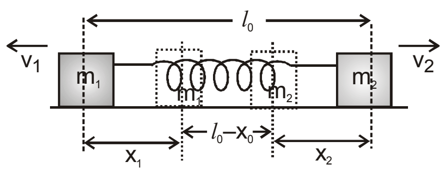
$$K_i + U_i = K_f + U_f$$

$$\Rightarrow \; 0 + \frac{1}{2}kx_0^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + 0$$

$$\Rightarrow \; \frac{1}{2}kx_0^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad ...(2)$$

In initial condition there is no external force on the system and both the block is at stationary condition. Therefore centre of mass of the system is at rest. So we can write.

$$m_1x_1 = m_2x_2 \qquad ...(3)$$



from above figure we can conclude

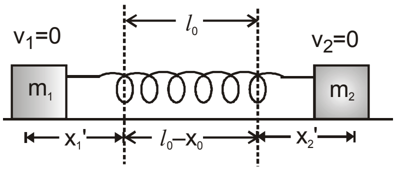
$$l_0 - x_0 + x_1 + x_2 = l_0$$

$$\Rightarrow \qquad x_0 = x_1 + x_2 \qquad (4)$$

Due to inertia both the block move further from the position of the natural length of the spring.

Maximum extension occur when both the blocks come to rest. Let us assume that x_1' & x_2' are the extension in the spring from the initial position due to block m_1 & m_2 from natural length

So at maximum extension $v_1 = v_2 = 0$



$\therefore$ Centre of mass is at rest Therefore we can write

$$m_1x_1' = m_2x_2'$$

$$x_1' + x_2' \qquad + l_0 - x_0 = l_0 + x_0$$

$$x_1' + x_2' \qquad = 2x_0$$

EXAMPLE 29

A light spring of spring constant k is kept compressed between two blocks of masses m and M on a smooth horizontal surface. When released, the blocks aquirse velocities in opposite directions. The spring loses contact with the blocks when it acquires natural length. If the spring was initially compressed through a distance x, find the final speeds of the two blocks.

Sol. Consider the two blocks plus the spring to be the system. No external force acts on this system in horizontal direction. Hence, the linear momentum will remain constant. Suppose, the block of mass M moves with a speed v_1 and the other block with a speed v_2 after losing contact with the spring. From conservation of linear momentum in horizontal direction we have

$$Mv_1 - mv_2 = 0 \text{ or } v_1 = \frac{m}{M}v_2 \qquad ...(i)$$

Initially, the energy of the system = $\frac{1}{2}kx^2$

Finally, the energy of the system

$$= \frac{1}{2}mv_2^{\,2} + \frac{1}{2}Mv_1^{\,2}$$

As there is no friction, mechanical energy will remain conserved.

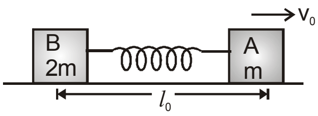
Therefore, $\frac{1}{2}mv_2^{\,2} + \frac{1}{2}Mv_1^{\,2} = \frac{1}{2}kx^2 \qquad ..(ii)$

Solving Eqs. (i) and (ii), we get

or, $$v_2 = \left[\frac{kM}{m(M+m)}\right]^{1/2} x$$

and $$v_1 = \left[\frac{kM}{m(M+m)}\right]^{1/2} x \qquad \textbf{Ans.}$$

IInd Format : Figure shows two blocks of masses 2m and m are placed on a frictionless surface and connected with a spring. An external kick gives a velocity v_0 m/s to the m mass towards right



Now velocity of centre of mass is

$$\vec{V}_{com} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

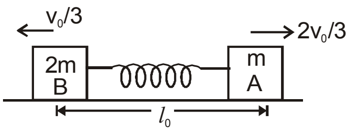
$$\Rightarrow \quad V_{com} = \frac{mv_0 + 0}{2m + m} = \frac{v_0}{3} m / sec$$

Due to kick on m mass block is starts moving with a velocity v_0 towards right immediately but due to inertia 2m block remain at rest at that moment. Thus velocity of block A & B with respect to the centre of mass is

$v_A = v_0 - \frac{v_0}{3} = \frac{2v_0}{3}$ m/sec. (towards right)

$v_B = 0 - \frac{v_0}{3} = - \frac{v_0}{3} = \frac{v_0}{3}$ (towards left)

Now the following figure shown the condition when centre of mass is rest.



If the maximum extension of the spring is x_0 then at this position both the block come to rest condition with respect to COM so from mechanical energy conservation

$\Rightarrow K_i + U_i = K_f + U_f$ ...(1)

$$K_i = \frac{1}{2}m\left(\frac{2v_0}{3}\right)^2 + \frac{1}{2}2m\left(\frac{v_0}{3}\right)^2$$

$U_i = 0$ (spring is in natural length)

$K_f = 0$ $(V_A = V_B = 0)$

$$U_f = \frac{1}{2}Kx_0^2$$

Put the above value is equation 1

$$\Rightarrow \frac{1}{2}m\left(\frac{2v_0}{3}\right)^2 + \frac{1}{2}2m\left(\frac{v_0}{3}\right)^2 + 0 = \frac{1}{2}kx_0^2$$

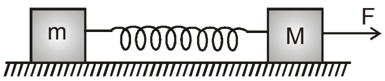
$$\Rightarrow kx_0^2 = \frac{2}{3}mv_0^2$$

Maximum extension $x_0 = v_0\sqrt{\frac{2}{3k}m}$

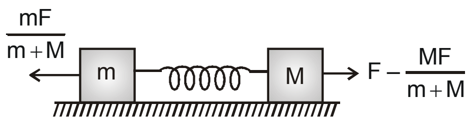
IIIrd format :

Example

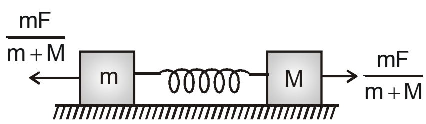
A block of mass m is connected to another block of mass M by a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane and are at rest. The spring is unstretched when a constant force F starts acting on the block of mass M to pull it. Find the maximum extension of the spring.



We solve the situation in the reference frame of centre of mass. As only F is the external force acting on the system, due to this force, the acceleration of the centre of mass is F/(M+m). Thus with respect to centre of mass there is a Pseudo force on the two masses in opposite direction, the free body diagram of m and M with respect to centre of mass (taking centre of mass at rest) is shown in figure.



Taking centre of mass at rest, if m moves maximum by a distance x_1 and M moves maximum by a distance x_2, then the work done by external forces (including Pseudo force) will be



$$W = \frac{mF}{m+M}.x_1 + \left(F - \frac{MF}{m+M}\right).x_2$$

$$= \frac{mF}{m+M}.(x_1 + x_2)$$

This work is stored in the form of potential energy of the spring as

$$U = \frac{1}{2}k(x_1 + x_2)^2$$

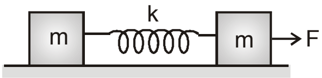
Thus on equating we get the maximum extension in the spring, as after this instant the spring starts contracting.

$$\frac{1}{2}k(x_1 + x_2)^2 = \frac{mF}{m+M}.(x_1 + x_2)$$

$$x_{max} = x_1 + x_2 = \frac{2mF}{k(m+M)}$$

EXAMPLE 30

Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on frictionless horizontal surface. A constant force F is applied on one of the blocks pulling it away from the other as shown in figure.



(a) Find the displacement of the centre of mass at time t

(b) If the extension of the spring is x_0 at time t, find the displacement of the two blocks at this instant.

Sol. (a) The acceleration of the centre of mass is

$$a_{COM} = \frac{F}{2m}$$

The displacement of the centre of mass at time t will be

$$x = \frac{1}{2}a_{COM}t^2 = \frac{Ft^2}{4m} \quad \textbf{Ans.}$$

(b) Suppose the displacement of the first block is x_1 and that of the second is x_2. Then,

$$x = \frac{mx_1 + mx_2}{2m}$$

or, $$\frac{Ft^2}{4m} = \frac{x_1 + x_2}{2}$$

or, $$x_1 + x_2 = \frac{Ft^2}{2m} \qquad \text{...(i)}$$

Further, the extension of the spring is $x_1 - x_2$. Therefore,

$$x_1 - x_2 = x_0 \qquad \text{...(ii)}$$

From Eqs. (i) and (ii),

$$x_1 = \frac{1}{2}\left(\frac{Ft^2}{2m} + x_0\right) \text{ and } x_2$$

$$= \frac{1}{2}\left(\frac{Ft^2}{2m} - x_0\right) \quad \textbf{Ans.}$$

Note

The student can now attempt section C from exercise.

Section D - Impulse

6. IMPULSE :

Impulse of a force $\vec{F}$ acting on a body for the time interval $t = t_1$ to $t = t_2$ is defined as

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}\,dt$$

$$\vec{I} = \int \vec{F}\,dt$$

$$= \int m\frac{d\vec{v}}{dt}dt = \int m\,d\vec{v}$$

$\vec{I} = m(\vec{v}_2 - \vec{v}_1) = \Delta\vec{P}$ = change in momentum due to force $\vec{F}$

Also $\vec{I}_{Re} = \int_{t_1}^{t_2} \vec{F}_{Res}dt = \Delta\vec{P}$

(impulse - momentum theorem)

Note :

* Impulse applied to an object in a given time interval can also be calculated from the area under force time (F-t) graph in the same time interval.

6.1 Instantaneous Impulse :

There are many cases when a force acts for such a short time that the effect is instantaneous, e.g., a bat striking a ball. In such cases, although the magnitude of the force and the time for which it acts may each be unknown but the value of their product (i.e., impulse) can be known by measuring the initial and final momentum. Thus, we can write.

$$\vec{I} = \int \vec{F}\,dt = \Delta\vec{P} = \vec{P}_f - \vec{P}_i$$

Important Points :

(1) It is a vector quantity.

(2) Dimensions = $[MLT^{-1}]$

(3) SI unit = kg m/s

(4) Direction is along change in momentum.

(5) Magnitude is equal to area under the F-t. graph.

(6) $\vec{I} = \int \vec{F}\,dt = \vec{F}_{av}\int dt = \vec{F}_{av}\,\Delta t$

(7) It is not a property of a particle, but it is a measure of the degree to which an external force changes the momentum of the particle.

EXAMPLE 31

The hero of a stunt film fires 50 g bullets from a machine gun, each at a speed of 1.0 km/s. If he fires 20 bullets in 4 seconds, what average force does he exert against the machine gun during this period.

Sol. The momentum of each bullet

= (0.050 kg) (1000 m/s)

= 50 kg-m/s.

The gun has been imparted this much amount of momentum by each bullet fired. Thus, the rate of change of momentum of the gun

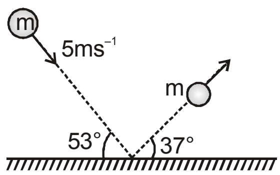
$$= \frac{(50\text{kg} - \text{m/s}) \times 20}{4\text{s}}$$

= 250 N

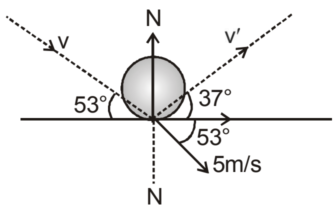
In order to hold the gun, the hero must exert a force of 250 N against the gun.

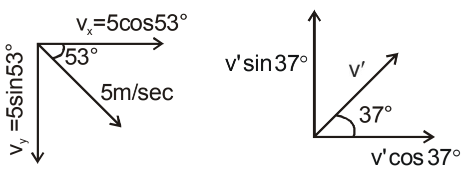
EXAMPLE 32

A ball of mass m = 1kg strikes smooth horizontal floor shown in figure. Find out impulse exerted on the floor is :



Sol. As the ball strike on the surface on impulsive normal force is exerted on the ball as shown in figure.





This normal force can change only the component v_y. So in x direction momentum is conserved. ($F_{net\,x} = 0$)

$\Rightarrow$ $v' \cos 37° = 5 \cos 53°$

$$v' = \frac{5 \times 3 \times 5}{5 \times 4} = \frac{15}{4}\text{m/sec}$$

So, $v'_y = v' \sin 37° = \frac{15}{4} \times \frac{3}{5} = \frac{9}{4}\text{m/sec}$

Impulse = change in linear momentum in y direction

$$I = \int N.dt = m(v_y - (-v'_y)) = 1\left(4 + \frac{9}{4}\right)$$

= 6.25 N-sec

6.2 Impulsive force :

A force, of relatively higher magnitude and acting for relatively shorter time, is called impulsive force. An impulsive force can change the momentum of a body in a finite magnitude in a very short time interval. Impulsive force is a relative term. There is no clear boundary between an impulsive and Non-Impulsive force.

Note :

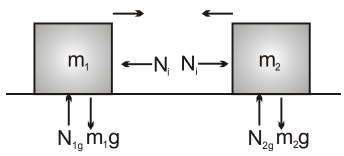
* Usually colliding forces are impulsive in nature. Since, the application time is very small, hence, very little motion of the particle takes place.

Important points :

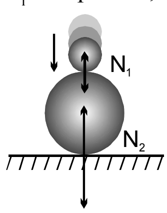
1. Gravitational force and spring force are always non-Impulsive.
2. Normal, tension and friction are case dependent.
3. An impulsive force can only be balanced by another impulsive force.

1. Impulsive Normal :

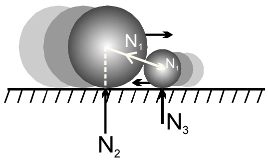
In case of collision, normal forces at the surface of collision are always impulsive **e.g.**



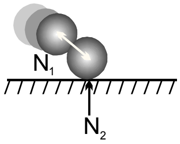
N_i = Impulsive; N_g = Non-impulsive



Both normals are Impulsive



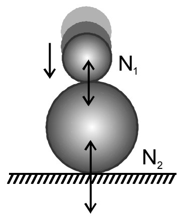
N_1, N_3 = Impulsive; N_2 = non-impulsive



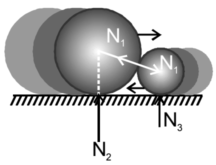
Both normals are Impulsive

2. Impulsive Friction :

If the normal between the two objects is impulsive, then the friction between the two will also be impulsive



Friction at both surfaces is impulsive



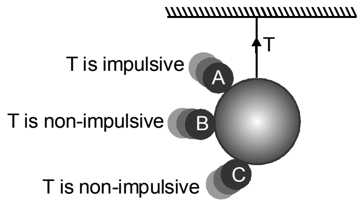
Friction due to N_2 is non-impulsive and due to N_3 and N_1 are impulsive

3. Impulsive Tensions :

When a string jerks, equal and opposite tension act suddenly at each end. Consequently equal and opposite impulses act on the bodies attached with the string in the direction of the string. There are two cases to be considered.

- **One end of the string is fixed :**
  The impulse which acts at the fixed end of the string cannot change the momentum of the fixed object. The object attached to the free end however will undergo a change in momentum in the direction of the string. The momentum remains unchanged in a direction perpendicular to the string where no impulsive forces act.

- **Both ends of the string attached to movable objects :**
  In this case equal and opposite impulses act on the two objects, producing equal and opposite changes in momentum. The total momentum of the system therefore remains constant, although the momentum of each individual object is changed in the direction of the string. Perpendicular to the string however, no impulse acts and the momentum of each particle in this direction is unchanged.



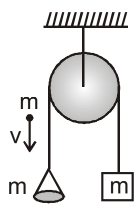
All normal are impulsive but tension
T is impulsive only for the ball A

For this example :

In case of rod, tension is always impulsive and in case of spring, tension is always non-impulsive.

EXAMPLE 33

A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley. Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v, find the speed with which the system moves just after the collision.



Sol. Let the required speed is V.

Further, let J_1 = impulse between particle and pan

and J_2 = impulse imparted to the block and the pan by the string

Using, Impulse = change in momentum

For particle $J_1 = mv - mV$ ...(i)

For pan $J_1 - J_2 = mV$ ...(ii)

For block $J_2 = mV$ ...(iii)

Solving, these three equation, we get $V = \frac{v}{3}$

Ans.

Alternative solution :

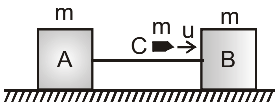
Applying conservation of linear momentum along the string ;

$mv = 3mV$

we get, $V = \frac{v}{3}$ **Ans.**

EXAMPLE 34

Two identical block A and B, connected by a massless string are placed on a frictionless horizontal plane. A bullet having same mass, moving with speed u strikes block B from behind as shown. If the bullet gets embedded into the block B then find :



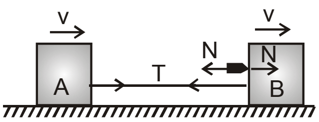
(A) The velocity of A, B, C after collision

(B) Impulse on A due to tension in the string

(C) Impulse on C due to normal force of collision.

(D) Impulse on B due to normal force of collision.

Sol. Let us assume that all the three are move with velocity v



Take rightward direction is +ve. Then first we write impulse equation on bullet

$$-\int N.dt = mv - mu \qquad ...(1)$$

Now impulse equation on block B

$$\int (N - T)dt = mv \qquad ...(2)$$

Impulse equation on block A

$$\int T.dt = mv \qquad ...(3)$$

(a) Add eq. (1), (2), (3) then

$$0 = 3mv - mu \Rightarrow \quad v = \frac{u}{3}$$

(b) Impulse on A due to Tension in the string from

eq. (3) $\int T.dt = \frac{mu}{3}$

(c) Impulse on C due to normal force of collision

from eq. (1) $\int N.dt = m\left(\frac{u}{3} - u\right) = -\frac{2mu}{3}$

(d) Impulse on B due to normal force of collision from eq. (2)

$$\int (N - T)dt = \frac{mu}{3}$$

$$\int N.dt = \frac{mu}{3} + \int T.dt = \frac{mu}{3} + \frac{mu}{3} = \frac{2mu}{3}$$

7. COEFFICIENT OF RESTITUTION (e)

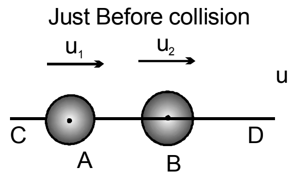
The coefficient of restitution is defined as the ratio of the impulses of reformation and deformation of either body.

$$e = \frac{\text{Impulse of reformation}}{\text{Impulse of deformation}} = \frac{\int F_r dt}{\int F_d dt}$$

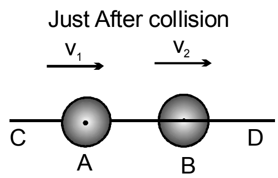
$$e = \frac{\text{Velocity of separation of point of contact}}{\text{Velocity of approach of point of contact}}$$

Example for calculation of e :

Two smooth balls A and B approaching each other such that their centres are moving along line CD in absence of external impulsive force. The velocities of A and B just before collision be u_1 and u_2 respectively. The velocities of A and B just after collision be v_1 and v_2 respectively.



$u_1 > u_2$



$v_2 > v_1$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

Note : Coefficient of restitution is a factor between two colliding bodies which is depends on the material of the body but independent of shape.

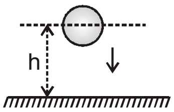
We can say e is a factor which relates deformation and reformation of the body.

$$0 \le e \le 1$$

EXAMPLE 35

If a body falls normally on a surface from height h, what will be the height regained after collision if coefficient of restitution is e?

Sol.



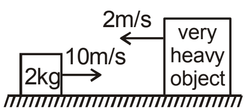
If a body falls from height h, from equations of motion we know that it will hit the ground with a velocity say $u = \sqrt{2gh}$ which is also the velocity of approach here.

Now if after collision it regains a height h_1 then again by equations of motion $v = \sqrt{2gh_1}$ which is also the velocity of separation. So, by definition of e,

$$e = \sqrt{\frac{2gh_1}{2gh}} \quad \text{or } h_1 = e^2 h$$

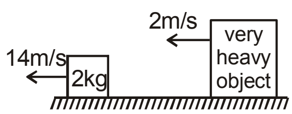
EXAMPLE 36

A block of mass 2 kg is pushed towards a very heavy object moving with 2 m/s closer to the block (as shown). Assuming elastic collision and frictionless surface, find the final velocities of the blocks.



Sol. Let v_1 and v_2 be the final velocities of 2kg block and heavy object respectively then,

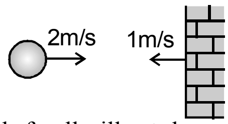
$$v_1 = u_1 + 1\,(u_1 - u_2) = 2u_1 - u_2$$
$$= -\,14 \text{ m/s}$$



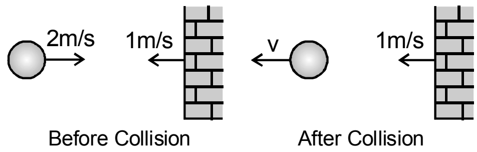
$v_2 = -\,2\text{m/s}$

EXAMPLE 37

A ball is moving with velocity 2 m/s towards a heavy wall moving towards the ball with speed 1 m/s as shown in fig. Assuming collision to be elastic, find the velocity of the ball immediately after the collision.



Sol. The speed of wall will not change after the collision. So, let v be the velocity of the ball after collision in the direction shown in figure. Since collision is elastic (e = 1).



separation speed = approach speed

or $\quad v - 1 = 2 + 1$

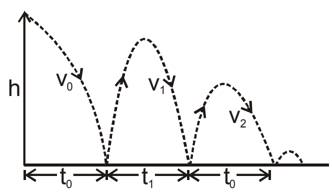
or $\quad v = 4$ m/s $\qquad$ **Ans.**

EXAMPLE 38

A ball is dropped from a height h on to a floor. If in each collision its speed becomes e times of its striking value (a) find the time taken by ball to stop rebounding (b) find the total change in momentum in this time (c) find the average force exerted by the ball on the floor using results of part (a) and (b).

Sol. (a) When the ball is dropped from a height h, time taken by it to reach the ground will be

$$t_0 = \sqrt{\frac{2h}{g}} \text{ and its speed } v_0 = \sqrt{2gh}$$



Now after collision its speed will becomes e times, i.e., $v_1 = ev_0 = e\sqrt{2gh}$ and so, it will take time to go up till its speed becomes zero = (v_1/g). The same time it will take to come down. So total time between I and II collision will be $t_1 = 2v_1/g$. Similarly, total time between II and III collision $t_2 = 2v_2/g$.

So total time of motion

$$\Delta T = t_0 + t_1 + t_2 + ...$$

or $\quad \Delta T = t_0 + \dfrac{2v_1}{g} + \dfrac{2v_2}{g}$ ....

or $\quad \Delta T = t_0 + \dfrac{2ev_0}{g} + \dfrac{2e^2v_0}{g}$ ....

[as $v_2 = ev_1 = e^2v_0$]

or $\quad \Delta T = \sqrt{\frac{2h}{g}}[1+2e(1+e+e^2+....)]$

$$= \sqrt{\frac{2h}{g}}\left[1+2e\left(\frac{1}{1-e}\right)\right] = \sqrt{\frac{2h}{g}}\left[\frac{1+e}{1-e}\right]$$

(b) Change in momentum in I collision

$$= mv_1 - (-mv_0) = m(v_1 + v_0)$$

Change in momentum in II collision = $m(v_2 + v_1)$

Change in momentum in nth collision = $m(v_n + v_{n-1})$

Adding these all total change in momentum

$$\Delta p = m[v_0 + 2v_1 + ...+ 2v_{n-1} + v_n]$$

or $\quad \Delta p = mv_0[1 + 2e + e^2 + ...]$

or $\quad \Delta p = mv_0\left[1+2e\left(\frac{1}{1-e}\right)\right] = m\sqrt{2gh}\left[\frac{1+e}{1-e}\right]$ ...(2)

(C) Now as $\vec{F} = \frac{d\vec{p}}{dt}$ so, $F_{av} = \frac{\Delta p}{\Delta T}$

Substituting the value of ΔT and Δp from Eqns. (1) and (2)

$$F_{av} = m\sqrt{2gh}\left[\frac{1+e}{1-e}\right] \times \sqrt{\frac{g}{2h}}\left[\frac{1-e}{1+e}\right] = mg \quad ...(3)$$

7.1 Line of Motion

The line passing through the centre of the body along the direction of resultant velocity.

7.2 Line of Impact

The line passing through the common normal to the surfaces in contact during impact is called line of impact. The force during collision acts along this line on both the bodies.

Direction of Line of impact can be determined by :

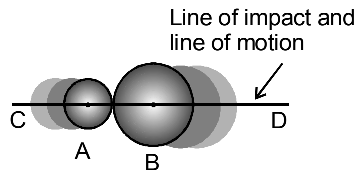
(a) Geometry of colliding objects like spheres, discs, wedge etc.

(b) Direction of change of momentum.

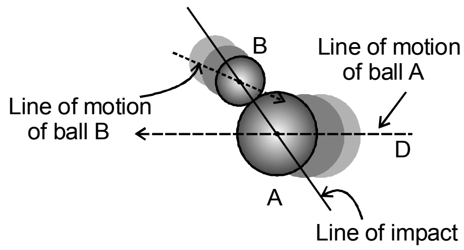
If one particle is stationary before the collision then the line of impact will be along its motion after collision.

Examples of line of impact

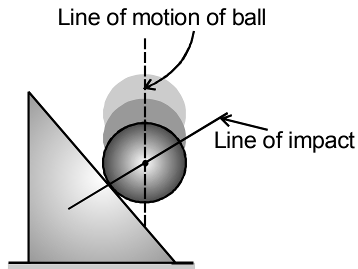
(i) Two balls A and B are approaching each other such that their centres are moving along line CD.



(ii) Two balls A and B are approaching each other such that their centre are moving along dotted lines as shown in figure.



(iii) Ball is falling on a stationary wedge.



Note

In previous discussed examples line of motion is same as line of impact. But in problems in which line of impact and line of motion is different then e will be

$$e = \frac{\text{velocity of seperation along line of impact}}{\text{velocity of approach along line of impact}}$$

EXAMPLE 40

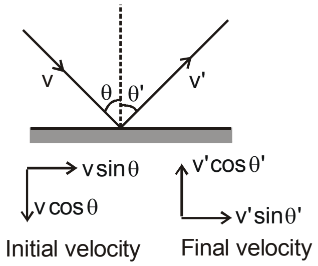
A ball of mass m hits a floor with a speed v making an angle of incident θ with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection of the ball.

Sol. Suppose the angle of reflection is θ' and the speed after the collision is v' (shown figure) The floor exerts a force on the ball along the normal during the collision. There is no force parallel to the surface. Thus, the parallel component of the velocity of the ball remains unchanged. This gives

$$v' \sin \theta' = v \sin \theta \qquad ...(i)$$

For the components normal to the floor, the velocity of separation is $v' \cos \theta'$ and the velocity of approach is $v \cos \theta$.

Hence, $v' \cos \theta' = ev \cos \theta$



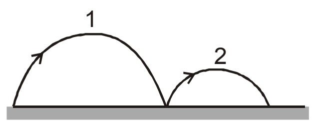
From (i) and (ii), $v' = v\sqrt{\sin^2\theta + e^2\cos^2\theta}$

Hence, $\tan \theta' = \dfrac{\tan\theta}{e}$

For elastic collision, e = 1, so that $\theta' = \theta$ and $v' = v$.

EXAMPLE 41

A ball is projected from the ground at some angle with horizontal. Coefficient of restitution between the ball and the ground is e. Let a, b and c be the ratio of times of flight, horizontal range and maximum height in two successive paths. Find a, b and c in terms of e?



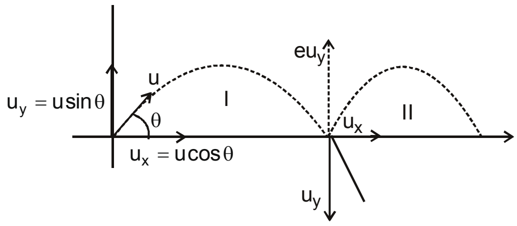
Sol. Let us assume that ball is projected with speed u at an angle θ with the horizontal. Then

Before first collision with the ground.

Time fo flight $T = \dfrac{2u_y}{g}$

Horizontal range $R = \dfrac{2u_x u_y}{g}$

Maximum Height $H_{max} = \dfrac{u_y^2}{2g} \qquad ...(1)$



After striking the ground the component u_y is change into $e\,u_y$, so

Time of flight $T' = \dfrac{2eu_y}{g}$, $R' = \dfrac{2u_x(eu_y)}{g}$

$$H'_{max} = \frac{(eu_y)^2}{2g} \qquad ...(2)$$

from eq (1) & (2)

Now $\dfrac{T}{T'} = a = \dfrac{1}{e}$

$\dfrac{R}{R'} = b = \dfrac{1}{e}$; $\dfrac{H_{max}}{H'_{max}} = \dfrac{1}{e^2} = c$

EXAMPLE 42

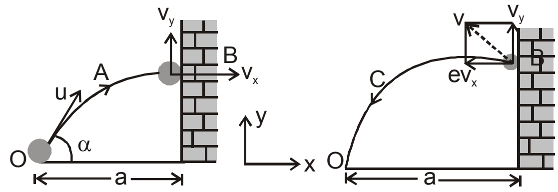
A ball is projected from the ground with speed u at an angle α with horizontal. It collides with a wall at a distance a from the point of projection and returns to its original position. Find the coefficient of restitution between the ball and the wall.

Sol. A ball is a projected with speed u at an angle α with horizontal. It collides at a distance a with a wall parallel to y-axis as shown in figure.

Let v_x and v_y be the components of its velocity along x and y-directions at the time of impact with wall. Coefficient of restitution between the ball and the wall is e.

Component of its velocity along y-direction (common tangent) v_y will remain unchanged while component of its velocity along x-direction (common normal) v_x will becomes ev_x is opposite direction.

*Further, since v_y does not change due to collision, the time of flight (time taken by the ball to return to the same level) and maximum height attained by the ball remain same as it would had been in the absence of collision with the wall. Thus,



From O A B, R = a = u cos α . t_{OAB}

from BCO, R = a = eucosα. t_{BCO}

$T = t_{OAB} + t_{BCO}$

or $$\frac{2u\sin\alpha}{g} = \frac{a}{u\cos\alpha} + \frac{a}{eu\cos\alpha}$$

or $$\frac{a}{eu\cos\alpha} = \frac{2u\sin\alpha}{g} - \frac{a}{u\cos\alpha}$$

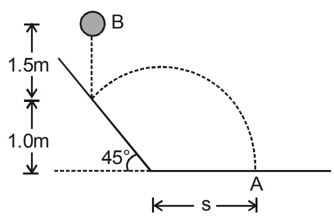
or $$\frac{a}{eu\cos\alpha} = \frac{2u^2\sin\alpha\cos\alpha - ag}{gu\cos\alpha}$$

$$e = \frac{ag}{2u^2\sin\alpha\cos\alpha - ag}$$

or $$e = \frac{1}{\left(\frac{u^2\sin 2\alpha}{ag} - 1\right)}$$ **Ans.**

EXAMPLE 43

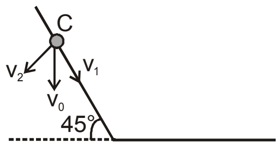
To test the manufactured properties of 10 N steel balls, each ball is released from rest as shown and strikes a 45° inclined surface. If the coefficient of restitution is to be e = 0.8. determine the distance s to where the ball must strike the horizontal plane at A. At what speed does the ball strike at A? ($g = 9.8\ m/s^2$)

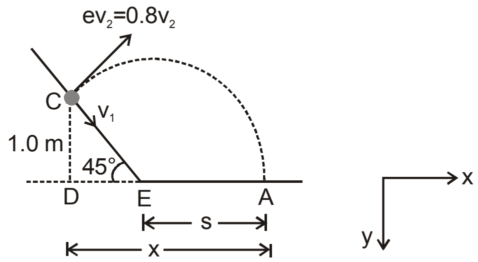


Sol. $v_0 = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 1.5} = 5.42$ m/s

Component of velocity parallel and perpendicular to plane at the time of collision.

$$v_1 = v_2 = \frac{v_0}{\sqrt{2}} = 3.83 \text{ m/sec.}$$





Component parallel to plane (v_1) remains unchanged, while component perpendicular to plane becomes ev_2, where

$$ev_2 = 0.8 \times 3.83 = 3.0 \text{ m/s}$$

∴ Component of velocity in horizontal direction after collision

$$v_x = \frac{(v_1 + ev_2)}{\sqrt{2}} = \frac{(3.83 + 3.0)}{\sqrt{2}} = 4.83 \text{ m/s}$$

While component of velocity in vertical direction after collision.

$$v_y = \frac{v_1 - ev_2}{\sqrt{2}} = \frac{3.83 - 3.0}{\sqrt{2}} = 0.59 \text{ m/s}$$

Let *t* be the time, the particle takes from point C to A, then

$$1.0 = 0.59\,t + \frac{1}{2} \times 9.8 \times t^2; \qquad t = 0.4$$ sec

Solving this we get,

∴ DA = $v_x t$ = (4.83)(0.4) = 1.93 m

∴ S = DA – DE = 1.93 – 1.0

S = 0.93 m

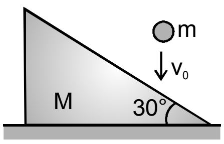
$v_{yA} = v_{yc} + gt = (0.59) + (9.8)(0.4) = 4.51$ m/s

$v_{xA} = v_{xC} = 4.83$ m/s

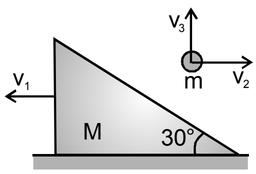
∴ $v_A = \sqrt{(v_{xA})^2 + (v_{yA})^2} = 6.6$ m/s

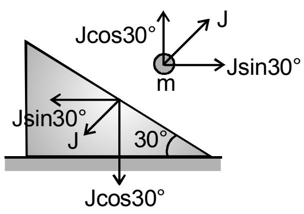
EXAMPLE 44

A ball of mass m = 1 kg falling vertically with a velocity v_0 = 2m/s strikes a wedge of mass M = 2kg kept on a smooth, horizontal surface as shown in figure. The coefficient of restitution between the ball and the wedge is e = $\frac{1}{2}$. Find the velocity of the wedge and the ball immediately after collision.



Sol. Given M = 2kg and m = 1kg





Let, J be the impulse between ball and wedge during collision and v_1, v_2 and v_3 be the components of velocity of the wedge and the ball in horizontal and vertical directions respectively.

Applying impulse = change in momentum

we get $J \sin 30° = Mv_1 = mv_2$

or $\frac{J}{2} = 2v_1 = v_2$

$J \cos 30° = m(v_3 + v_0)$ ...(i)

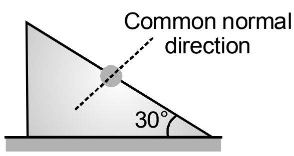
or $\frac{\sqrt{3}}{2}J = (v_3 + 2)$ ...(ii)

Applying, relative speed of separation = e (relative speed of approach) in common normal direction, we get

$(v_1 + v_2) \sin 30° + v_3 \cos 30° = \frac{1}{2}(v_0 \cos 30°)$

or $v_1 + v_2 + \sqrt{3}v_3 = \frac{\sqrt{3}}{2}$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we get

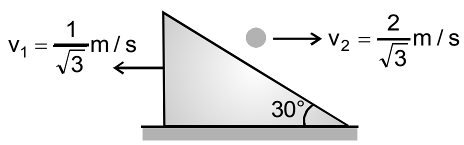


$v_1 = \frac{1}{\sqrt{3}} m/s$

$v_2 = \frac{2}{\sqrt{3}} m/s$ and $v_3 = 0$

Thus, velocities of wedge

and ball are $v_1 = \frac{1}{\sqrt{3}}$ m/s



and $v_2 = \frac{2}{\sqrt{3}} m/s$ in horizontal direction as shown in figure.

Note

The student can now attempt section D from exercise.

Section E - Collision in 1D, Oblique collision

8. COLLISION OR IMPACT

Collision is an event in which an impulsive force acts between two or more bodies for a short time, which results in change of their velocities.

Note :

- In a collision, particles may or may not come in physical contact.
- The duration of collision, Δt is negligible as compared to the usual time intervals of observation of motion.
- In a collision the effect of external non impulsive forces such as gravity are not taken into account as due to small duration of collision (Δt) average impulsive force responsible for collision is much larger than external forces acting on the system.

The collision is in fact a redistribution of total momentum of the particle :

Thus law of conservation of linear momentum is indepensible in dealing with the phenomenon of collision between particles. Consider a situation shown in figure.

Two balls of masses m_1 and m_2 are moving with velocities v_1 and v_2 ($<v_1$) along the same straight line in a smooth horizontal surface. Now let us see what happens during the collision between two particles.

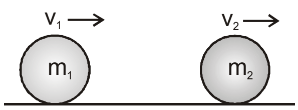
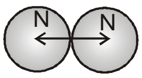
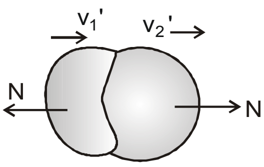


figure (a)



figure(b)



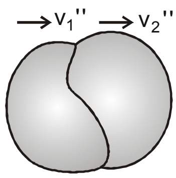
figure(c)

figure (a) : Balls of mass m_1 is behind m_2. Since $v_1 > v_2$, the balls will collide after some time.

figure (b) : During collision both the balls are a little bit deformed. Due to deformation two equal and opposite normal forces act on both the balls. These forces decreases the velocity of m_1 and increase the velocity of m_2

figure (c): Now velocity of ball m_1 is decrease from v_1 to v_1' and velocity of ball m_2 is increase from v_2 to v_2'. But still $v_1' > v_2'$ so both the ball are continuously deformed.

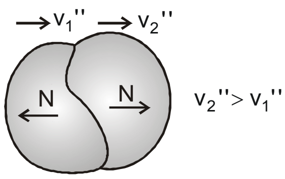
figure(d) : Contact surface of both the balls are deformed till the velocity of both the balls become equal. So at maximum deformation velocities of both the blocks are equal



figure(d)

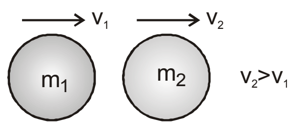
at maximum deformation $v_1'' = v_2''$

figure(e) : Normal force is still in the direction shown in figure i.e. velocity of m_1 is further decreased and that of m_2 increased. Now both the balls starts to regain their original shape and size.



figure(e)

figure (f) : These two forces redistributes their linear momentum in such a manner that both the blocks are separated from one another, Velocity of ball m_2 becomes more than the velocity of block m_1 i . e . , $v_2 > v_1$



figure(f)

The collision is said to be elastic if both the blocks regain their original form, The collision is said to be inelastic. If the deformation is permanent, and the blocks move together with same velocity after the collision, the collision is said to be perfectly inelastic.

8.1 Classification of collisions

(a) ***On the basis of line of impact***

(i) Head-on collision : If the velocities of the colliding particles are along the same line before and after the collision.

(ii) Oblique collision : If the velocities of the colliding particles are along different lines before and after the collision.

(b) ***On the basis of energy :***

(i) Elastic collision :

(a) In an elastic collision, the colliding particles regain their shape and size completely after collision. i.e., no fraction of mechanical energy remains stored as deformation potential energy in the bodies.

(b) Thus, kinetic energy of system after collision is equal to kinetic energy of system before collision.

(c) $e = 1$

(d) Due to F_{net} on the system is zero linear momentum remains conserved.

(ii) Inelastic collision :

(a) In an inelastic collision, the colliding particles do not regain their shape and size completely after collision.

(b) Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of the particles no longer remains conserved.

(c) However, in the absence of external forces, law of conservation of linear momentum still holds good.

(d) $(\text{Energy loss})_{\text{Perfectly Inelastic}} > (\text{Energy loss})_{\text{Partial Inelastic}}$

(e) $0 < e < 1$

(iii) Perfectly Inelastic collision :

(i) In this the colliding bodies do not return to their original shape and size after collision i.e. both the particles stick together after collision and moving with same velocity

(ii) But due to F_{net} of the system is zero linear momentum remains conserved.

(iii) Total energy is conserved.

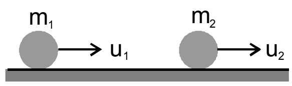
(iv) Initial kinetic energy > Final K.E. Energy

(v) Loss in kinetic energy goes to the deformation potential energy

(vi) $e = 0$

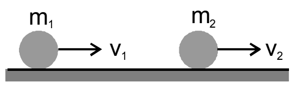
8.2 Value of Velocities after collision :

Let us now find the velocities of two particles after collision if they collide directly and the coefficient of restitution between them is given as e.



(a)
Before Collision

$u_1 > u_2$



(b)
After Collision

$v_2 > v_1$

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$\Rightarrow$ $(u_1 - u_2)e = (v_2 - v_1)$ ...(i)

By momentum conservation

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(ii)

$v_2 = v_1 + e(u_1 - u_2)$ ...(iii)

from above equation

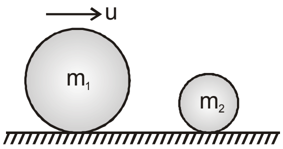
$$v_1 = \frac{m_1u_1 + m_2u_2 + m_2e(u_2 - u_1)}{m_1 + m_2} \quad \text{...(iii)}$$

$$v_2 = \frac{m_1u_1 + m_2u_2 + m_1e(u_1 - u_2)}{m_1 + m_2} \quad \text{...(iv)}$$

Special cases :

1. If $m_1 >> m_2$ and $u_2 = 0$ and $u_1 = u$

and $e = 1$



$m_1 = m_2$

from eq. (iii) & (iv)

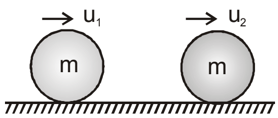
$$v_1 = \frac{m_1u - m_2u}{m_1 + m_2} = \frac{u(m_1 - m_2)}{m_1 + m_2}$$

$$v_1 \simeq u$$

$$v_2 = \frac{m_1u + m_2u}{m_1 + m_2} = \frac{2m_1u}{m_1 + m_2} \; ; \; v_2 = 2u$$

2. If $m_1 = m_2 = m$ and $e = 1$ then

from eq. (iii) & (iv)



$$v_1 = \frac{m(u_1 + u_2) + m(u_2 - u_1)}{2m}$$

$v_1 = u_2$

In this way $v_2 = u_1$

i.e when two particles of equal mass collide elastically and the collision is head on, they exchange their velocities.

8.3 Collision in two dimension (oblique) :

1. A pair of equal and opposite impulses act along common normal direction. Hence, linear momentum of individual particles change along common normal direction. If mass of the colliding particles remain constant during collision, then we can say that linear velocity of the individual particles change during collision in this direction.

2. No component of impulse act along common tangent direction. Hence, linear momentum or linear velocity of individual particles (if mass is constant) remain unchanged along this direction.

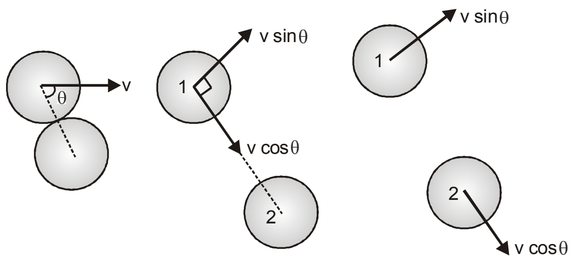
3. Net impulse on both the particles is zero during collision. Hence, net momentum of both the particles remain conserved before and after collision in any direction.

4. Definition of coefficient of restitution can be applied along common normal direction, i.e., along common normal direction we can apply Relative speed of separation = e (relative speed of approach)

EXAMPLE 45

A ball of mass m makes an elastic collision with another identical ball at rest. Show that if the collision is oblique, the bodies go at right angles to each other after collision.

Sol. In head on elastic collision between two particles, they exchange their velocities. In this case, the component of ball 1 along common normal direction, $v \cos \theta$ becomes zero after collision, while



Before collision After collision

that of 2 becomes $v \cos \theta$. While the components along common tangent direction of both the particles remain unchanged. Thus, the components along common tangent and common normal direction of both the balls in tabular form are given a head :

Ball	Component along common tangent direction		Component along common normal direction	
	Before collision	After collision	Before collision	After collision
1	$v \sin\theta$	$v \sin\theta$	$v \cos\theta$	0
2	0	0	0	$v \cos\theta$

From the above table and figure, we see that both the balls move at right angles after collision with velocities $v \sin \theta$ and $v \cos \theta$.

Note

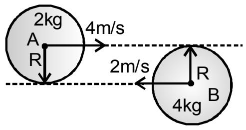
When two identical bodies have an oblique elastic collision, with one body at rest before collision, then the two bodies will go in $\perp$ directions.

EXAMPLE 46

Two spheres are moving towards each other. Both have same radius but their masses are 2kg and 4kg. If the velocities are 4m/s and 2m/s respectively and coefficient of restitution is e = 1/3, find.

(a) The common velocity along the line of impact.

(b) Final velocities along line of impact.



(c) Impulse of deformation.

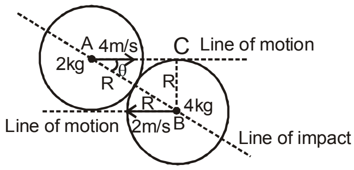
(d) impulse of reformation

(e) Maximum potential energy of deformation

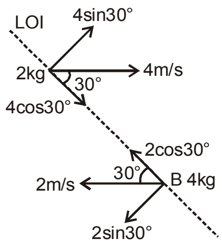
(f) Loss in kinetic energy due to collision.

Sol. In ΔABC $\sin\theta = \dfrac{BC}{AB} = \dfrac{R}{2R} = \dfrac{1}{2}$

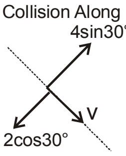
or $\theta = 30°$



(a) By conservation of momentum along line of impact.



Just Before Collision Along LOI

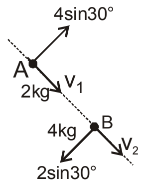


Maximum Deformed State

$$2(4\cos 30°) - 4(2\cos 30°) = (2 + 4)v$$

or $v = 0$ (common velocity along LOI)

(b) Let v_1 and v_2 be the final velocity of A and B respectively then, by conservation of momentum along line of impact, $2(4\cos 30°) - 4(2\cos30°) = 2(v_1) + 4(v_2)$



Just After Collision Along LOI

or $0 = v_1 + 2v_2$ ....(1)

By coefficient of restitution,

$$e = \frac{\text{velocity of separation along LOI}}{\text{velcoity of approach along LOI}}$$

or $\dfrac{1}{3} = \dfrac{v_2 - v_1}{4\cos 30° + 2\cos 30°}$ or

$v_2 - v_1 = \sqrt{3}$ ...(2)

from the above two equations,

$v_1 = \dfrac{-2}{\sqrt{3}} m/s$ and $v_2 = \dfrac{1}{\sqrt{3}} m/s$

(c) $J_0 = m_1(v - u_1) = 2(0 - 4\cos 30°) = -4\sqrt{3}$ N-s

(d) $J_R = eJ_0 = \dfrac{1}{3}(-4\sqrt{3}) = -\dfrac{4}{\sqrt{3}} N-s$

(e) Maximum potential energy of deformation is equal to loss in kinetic energy during deformation upto maximum deformed state,

$$U = \frac{1}{2}m_1(u_1\cos\theta)^2 + \frac{1}{2}m_2(u_2\cos\theta)^2 - \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}2(4\cos 30°)^2 + \frac{1}{2}4(-2\cos 30°)^2 - \frac{1}{2}(2+4)(0)$$

or U = 18 Joule

(f) Loss in kinetic energy

$$\Delta KE = \frac{1}{2}m_1(u_1\cos\theta)^2 + \frac{1}{2}m_2(u_2\cos\theta)^2 - \left(\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2\right)$$

$$= \frac{1}{2}\,2\,(4\cos 30°) + \frac{1}{2}\,4\,(-2\cos 30°) - \left(\frac{1}{2}2\left(\frac{2}{\sqrt{3}}\right)^2 + \frac{1}{2}4\left(\frac{1}{\sqrt{3}}\right)^2\right)$$

ΔKE = 16 Joule

Note

The student can now attempt section E from exercise.

Section F - Variable Mass system, Rocket propulsion

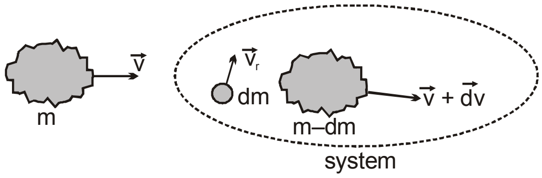
9. VARIABLE MASS

In our discussion of the conservation linear momentum, we have so far dealt with systems whose system whose mass remains constant. We now consider those mass is variable, i.e., those in which mass enters or leaves the system. A typical case is that of the rocket from which hot gases keep on escaping thereby continuously decreasing its mass.

In such problem you have nothing to do but apply a thrust force $(\vec{F}_t)$ to the main mass in addition to the all other force acting on it. This thrust force is given by,

$$\vec{F}_t = \vec{v}_{rel}\left(\frac{dm}{dt}\right)$$

Here $\vec{v}_{rel}$ is the velocity of the mass gained or mass ejected relative to the main mass. In case of rocket this is sometimes called the exhaust velocity of the gases. $\frac{dm}{dt}$ is the rate at which mass is increasing or decreasing.



The expression for the thrust force can be derived from the conservation of linear momentum in the absence of any external forces on a system as follows :

Suppose at some moment t = t mass of a body is m and its velocity is $\vec{v}$. After some time at t = t + dt its mass becomes (m – dm) and velocity becomes $\vec{v} + \vec{dv}$. The mass dm is ejected with relative velocity $\vec{v}_r$. Absolute velocity of mass 'dm' is therefore $(\vec{v}_r + \vec{v} + \vec{dv})$. If no external forces are acting on the system, the linear momentum of the system will remain conserved,

or $\quad \vec{P}_i = \vec{P}_f$

or $\quad m\vec{v} = (m - dm)(\vec{v} + \vec{dv}) + dm(\vec{v}_r + \vec{v} + \vec{dv})$

or

$$m\vec{v} = m\vec{v} + m\vec{dv} - dm\vec{v} - (dm)(\vec{dv}) + dm\vec{v} + \vec{v}_r dm + (dm)(\vec{dv})$$

$\therefore \quad m\vec{dv} = -\vec{v}_r dm$

or $\quad m\left(\frac{\vec{dv}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$

Here, $m\left(\frac{\vec{dv}}{dt}\right)$ = thrust force $(\vec{F}_1)$ and

$-\frac{dm}{dt}$ = rate at which mass is ejecting

Problems related to variable mass can be solved in following three steps

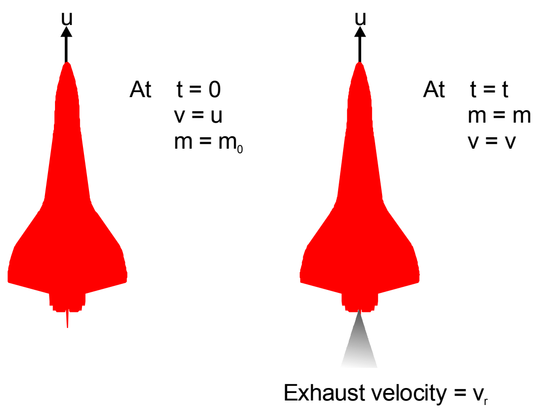
1. Make a list of all the forces acting on the main mass and apply them on it.
2. Apply an additional thrust force $\vec{F}_t$ on the mass, the magnitude of which is $\left|\vec{v}_r\left(\pm\frac{dm}{dt}\right)\right|$ and direction is given by the direction of $\vec{v}_r$ in case the mass is increasing and otherwise the direction of $-\vec{v}_r$ if it is decreasing.
3. Find net force on the mass and apply

$$\vec{F}_{net} = m\frac{\vec{dv}}{dt}$$

(m = mass at that particular instant)

9.1 Rocket Propulsion

Let m_0 be the mass of the rocket at time t = 0. m its mass at any time t and v its velocity at that moment. Initially let us suppose that the velocity of the rocket is u.



Further, let $\left(\frac{-dm}{dt}\right)$ be the mass of the gas ejected per unit time and v_r the exhaust velocity of the gases. Usually $\left(\frac{-dm}{dt}\right)$ and v_r are kept constant throughout the journey of the rocket. Now, let us write few equations which can be used in the problems of rocket propulsion. At time t = t

1. Thrust force on the rocket

$$F_t = v_r\left(-\frac{dm}{dt}\right) \qquad \text{(upwards)}$$

2. Weight of the rocket

$$W = mg \qquad \text{(downwards)}$$

3. Net force on the rocket

$$F_{net} = F_t - W \qquad \text{(upwards)}$$

or $$F_{net} = v_r\left(\frac{-dm}{dt}\right) - mg$$

4. Net acceleration of the rocket $a = \frac{F}{m}$

or $$\frac{dv}{dt} = \frac{v_r}{m}\left(\frac{-dm}{dt}\right) - g$$

or $$dv = v_r\left(\frac{-dm}{m}\right) - g\,dt$$

or $$\int_u^v dv = v_r \int_{m_0}^{m} \frac{-dm}{m} - g\int_0^t dt$$

or $$v - u = v_r \ln\left(\frac{m_0}{m}\right) - gt$$

Thus, $$v = u - gt + v_r \ln\left(\frac{m_0}{m}\right) \quad ...(i)$$

Note : 1. $F_t = v_r\left(-\frac{dm}{dt}\right)$ is upwards, as v_r is downwards and $\frac{dm}{dt}$ is negative.

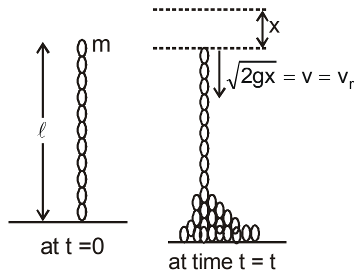
2. If gravity is ignored and initial velocity of the rocket u = 0, Eq. (i) reduces to $v = v_r \ln\left(\frac{m_0}{m}\right)$.

EXAMPLE 47

A uniform chain of length ℓ begins to fall with a velocity v on the table. Find the thrust force exerted by the chain on the table.

Sol. Let us assume that the mass of the chain is m and length ℓ.

We assume that after time t, x length of the chain has fallen on the table. Then the speed of the upper part of the chain is $\sqrt{2gx}$ as shown in figure.



Now its time t + dt, length of chain has fallen on the table is v dt. Then the mass of chain has fallen on the table is

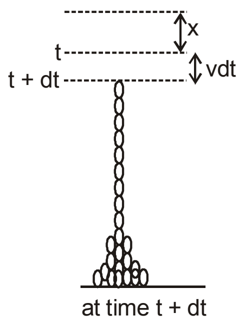
$$dm = \frac{m}{\ell}.vdt$$

Now the rate of increase of mass

$$\frac{dm}{dt} = \frac{m}{\ell}v = \frac{m}{\ell}\sqrt{2gx}$$

Here v is downward and mass is increasing so thrust force act in down ward direction and is given by

$$f_t = v_r\frac{dm}{dt}$$



$$= \sqrt{2gx}\,\frac{m}{\ell}(\sqrt{2gx})$$

$$f_t = \ell\, v^2$$

Note

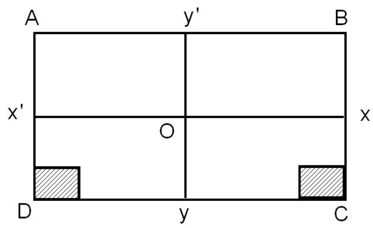
The student can now attempt section F from exercise.

Exercise - 1 Objective Problems | JEE Main

Section A - Calculation of COM of system of N-particles, COM of continuous distributed system, Combination of structure and Cavity problems

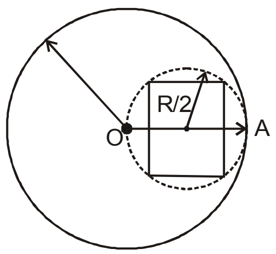
1. The centre of mass of two particles lies
(A) on the line perpendicular to the line joining the particles
(B) on a point outside the line joining the particles
(C) on the line joining the particles.
(D) none of the above

2. A uniform square plate ABCD has a mass of 10kg. If two point masses of 3 kg each are placed at the corners C and D as shown in the adjoining figure, then the centre of mass shifts to the point which is lie on -



(A) OC (B) OD
(C) OY (D) OX

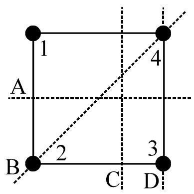
3. There is a thin uniform disc of radius R and mass per unit area σ, in which a hole of radius R/2 has been cut out as shown in the figure. Inside the hole a square plate of same mass per unit area σ is inserted so that its corners touch the periphery of the hole. Find centre of mass of the system.



(A) $\frac{R[2+\pi]}{2[3\pi+2]}$ (B) $\frac{R[2-\pi]}{2[3\pi-2]}$

(C) $\frac{R[2-\pi]}{2[3\pi+2]}$ (D) $\frac{R[2-\pi]}{[3\pi+2]}$

4. In the adjacent diagram, objects 1 and 2 each have mass *m* while objects 3 and 4 each have mass 2*m*. Note four lines A, B, C and D. The center of mass of the system is most likely to be at the intersection of lines :



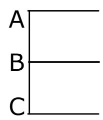
(A) A and B (B) B and C
(C) A and D (D) A and C

5. For particles of mass 5,3,2,4 kg are at the points (1,6), (-1,5), (2,-3), (-1,-4). Find the coordinates of their centre of mass.

(A) $\left(\frac{2}{7},\frac{23}{4}\right)$ (B) $\left(\frac{1}{7},\frac{23}{4}\right)$

(C) $\left(\frac{2}{7},\frac{11}{4}\right)$ (D) $\left(\frac{1}{7},\frac{23}{3}\right)$

6. As shown in diagram there are five identical rods. Length of each rod is ℓ and mass m. Find out distance of C.O.M. of system From (C)



(A) $\ell\sqrt{\frac{109}{50}}$ (B) $\ell\sqrt{\frac{190}{50}}$

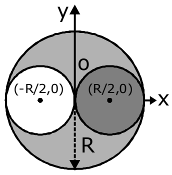
(C) $\ell\sqrt{\frac{190}{100}}$ (D) $\ell\sqrt{\frac{109}{100}}$

7. The position vectors of three particles of mass m_1 = 1 kg, m_2 = 2 kg and m_3 = 3 kg are $\vec{r}_1=\left(\hat{i}+4\hat{j}+\hat{k}\right), \vec{r}_2=\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\vec{r}_3=\left(2\hat{i}-\hat{j}-2\hat{k}\right)$ respectively. Find the position vector of their centre of mass.

(A) $1.5\hat{i}+0.5\hat{j}-0.5\hat{k}$ (B) $2.5\hat{i}+\hat{j}-\hat{k}$

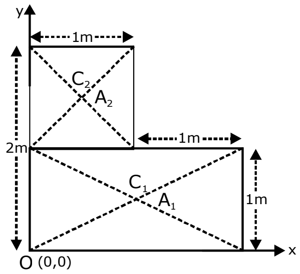
(C) $2.0\hat{i}+\hat{j}-\hat{k}$ (D) $3.5\hat{i}+\hat{j}-\hat{k}$

8. In the figure a uniform disc of radius R, from which a hole of radius R/2 has been cut out from left of the centre and is placed on right of the centre of disc. Find the centre of mass of the resulting disc.



(A) $\frac{R}{3},0$ (B) $\frac{R}{2},0$ (C) $\frac{R}{4},0$ (D) R, 0

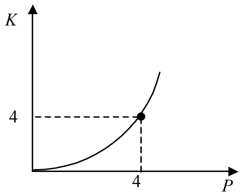
9. Find the centre of mass of a uniform L-shaped lamina (a thin flat plate) origine with dimensions as shown in figure. The mass of lamina is 3 kg.



(A) $\left(\frac{5}{6},\frac{5}{6}\right)$ (B) $\left(\frac{5}{6},\frac{7}{6}\right)$

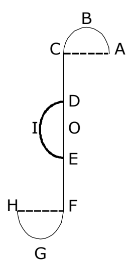
(C) $\left(\frac{5}{3},\frac{5}{6}\right)$ (D) $\left(\frac{5}{6},\frac{5}{7}\right)$

10. A graph between kinetic energy and momentum of a particle is plotted as shown in the figure. The mass of the moving particle is :



(A) 1 kg (B) 2 kg
(C) 3 kg (D) 4 kg

11. A Wireframe is made of a wire or uniform cross-section. which is shown in figure ABC. HGF and DIE are semicircular arcs of redius r. CD=DO=OE=EF=r and 'O' is the centre of circle. Then centre of mass of frame is:



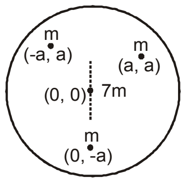
(A) At distance $\left(\frac{2r}{3\pi+4}\right)$ towards left of O

(B) At distance $\left(\frac{2r}{3\pi+4}\right)$ towards right of O

(C) At distance $\left(\frac{4r}{3\pi+4}\right)$ towards left of O

(D) At distance $\left(\frac{4r}{3\pi+4}\right)$ towards right of O

12. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink unsed to draw the outer circle is 6m. The coordinates of the centres of the different parts are outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0,–a). The y-coordinate of the centre of mass of the ink in this drawing is:



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

Section B - Motion of COM, Conservation of Momentum, Trolley problems

13. A man of mass M stands at one end of a plank of length L which lies at rest on a frictionless surface. The man walks to other end of the plank. If the mass of the plank is $\frac{M}{3}$, then the distance that the man moves relative to ground is :

(A) $\frac{3L}{4}$ (B) $\frac{L}{4}$ (C) $\frac{4L}{5}$ (D) $\frac{L}{3}$

14. Two balls A and B of masses 100gm and 250 gm respectively are connected by a stretched spring of negligible mass and placed on a smooth table. When the balls are released simultaneously the initial acceleration of B is 10 cm/sec^2 west ward. What is the magnitude and direction of initial acceleration of the ball A-
(A) 25 cm/sec^2 Eastward
(B) 25 cm/sec^2 North ward
(C) 25 cm/sec^2 West ward
(D) 25 cm/sec^2 South ward

15. A particle of mass 3m is projected from the ground at some angle with horizontal. The horizontal range is R. At the highest point of its path it breaks into two pieces m and 2m. The smaller mass comes to rest and larger mass finally falls at a distance x from the point of projection where x is equal to

(A) $\frac{3R}{4}$ (B) $\frac{3R}{2}$ (C) $\frac{5R}{4}$ (D) 3R

16. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time?

(A) 11.2 m (B) 13.8 m
(C) 14.3 m (D) 15.4 m

17. Two particles having mass ratio n : 1 are interconnected by a light inextensible string that passes over a smooth pulley. If the system is released, then the acceleration of the centre of mass of the system is :

(A) $(n-1)^2$ g (B) $\left(\frac{n+1}{n-1}\right)^2 g$

(C) $\left(\frac{n-1}{n+1}\right)^2 g$ (D) $\left(\frac{n+1}{n-1}\right) g$

18. Internal forces can change

(A) the linear momentum but not the kinetic energy of the system.
(B) the kinetic energy but not the linear momentum of the system.
(C) linear momentum as well as kinetic energy of the system.
(D) neither the linear momentum nor the kinetic energy of the system.

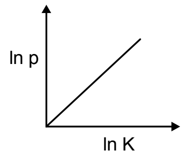
19. A small sphere is moving at a constant speed in a vertical circle. Below is a list of quantities that could be used to describe some aspect of the motion of the sphere

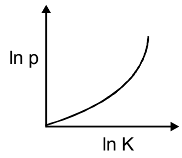
I - kinetic energy
II - gravitational potential energy
III - momentum

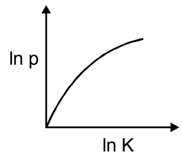
Which of these quantities will change as this sphere moves around the circle ?

(A) I and II only (B) I and III only
(C) III only (D) II and III only

20. Which of the following graphs represents the graphical relation between momentum (p) and kinetic energy (K) for a body in motion ?

(A) 

(B) 

(C) 

(D) None

21. Two balls are thrown in air. The acceleration of the centre of mass of the two balls while in air (neglect air resistance)

(A) depends on the direction of the motion of the balls
(B) depends on the masses of the two balls
(C) depends on the speeds of the two balls
(D) is equal to g

22. Conservation of linear momentum is equivalent to-

(A) Newton's second law of motion
(B) Newton's first law of motion
(C) Newton's third law of motion
(D) Conservation of angular momentum.

23. A body of mass m collides against a wall with the velocity υ and rebounds with the same speed. Its change of momentum is-

(A) 2 mυ (B) mυ
(C) $-$ mυ (D) 0

24. A bomb initially at rest explodes by it self into three equal mass fragments. The velocities of two fragments are ($3\hat{i} + 2\hat{j}$) m/s and ($-\hat{i} - 4\hat{j}$) m/s. The velocity of the third fragment is (in m/s)-

(A) $2\hat{i} + 2\hat{j}$ (B) $2\hat{i} - 2\hat{j}$
(C) $-2\hat{i} + 2\hat{j}$ (D) $-2\hat{i} - 2\hat{j}$

25. A stone of mass m_1 moving with a uniform speed v suddenly explodes on its own into two fragments. If the fragment of mass m_2 is at rest, the speed of the other fragment is-

(A) $\frac{m_1 v}{(m_1 - m_2)}$ (B) $\frac{m_2 v}{(m_1 - m_2)}$

(C) $\frac{m_1 v}{(m_1 + m_2)}$ (D) $\frac{m_1 v}{m_2}$

26. A nucleus of mass number A originally at rest emits α-particle with speed v. The recoil speed of daughter nucleus is :

(A) $\frac{4v}{A-4}$ (B) $\frac{4v}{A+4}$

(C) $\frac{v}{A-4}$ (D) $\frac{v}{A+4}$

27. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the compartment plus passengers system is C_2. If the passengers move about inside the compartment along the track.

(A) both C_1 and C_2 will move with respect to the ground

(B) neither C_1 nor C_2 will move with respect to the ground

(C) C_1 will move but C_2 will be stationary with respect to the ground

(D) C_2 will move but C_1 will be stationary with respect to the ground

28. A system of N particles is free from any external forces

(a) Which of the following is true for the magnitude of the total momentum of the system ?

(A) It must be zero

(B) It could be non-zero, but it must be constant

(C) It could be non-zero, and it might not be constant

(D) It could be zero, even if the magnitude of the total momentum is not zero.

(b) Which of the following must be true for the sum of the magnitudes of the momenta of the individual particles in the system ?

(A) It must be zero

(B) It could be non-zero, but it must be constant

(C) It could be non-zero, and it might not be constant

(D) The answer depends on the nature of the internal forces in the system

29. An isolated rail car of mass M is moving along a straight, frictionless track at an initial speed v_0. The car is passing under a bridge when a crate filled with N bowling balls, each of mass m, is dropped from the bridge into the bed of the rail car. The crate splits open and the bowling balls bounce around inside the rail car, but none of them fall out. Is the momentum of the rail car + bowling balls system conserved in this collision ?

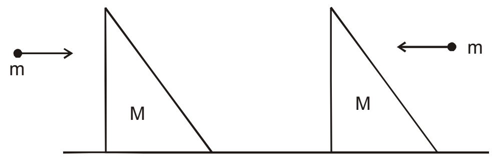
(A) Yes, the momentum is completely conserved

(B) Only the momentum component in the vertical direction is conserved

(C) Only the momentum component parallel to the track is conserved

(D) No components are conserved

30. Surface is perfectly smooth



Choose the correct Statement

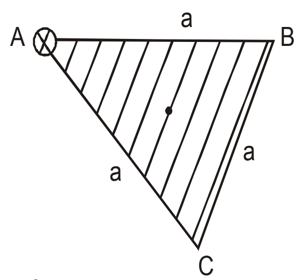
(A) Linear Momentum of system is conserved in case A only

(B) Linear Momentum will be conserved in case B only.

(C) Linear Momentum will be conserved in both cases.

(D) Linear Momentum can not be conserved.

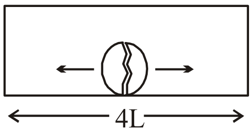
31. A uniform triangular plate ABC of moment of inertia I (about an axis passing through A and perpendicular to plane of the plate) can rotate freely in the vertical plane about point 'A' as shown in figure. The plate is released from the position shown in the figure. Line AB is horizontal. The acceleration of centre of mass just after the release of plate is :



(A) $\frac{mga^2}{\sqrt{3}I}$ (B) $\frac{mga^2}{4I}$

(C) $\frac{mga^2}{2\sqrt{3}I}$ (D) $\frac{mga^2}{3I}$

32. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4L at it's centre. It explodes in two parts of mass m & 2m. The two parts move in opposite direction and stick to the apposite side of the walls of box. Box is kept on a smooth horizontal surface.



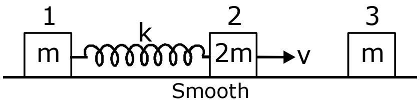
What is the distance moved by the box during this time interval.

(A) 0 (B) $\frac{L}{6}$

(C) $\frac{L}{12}$ (D) $\frac{L}{3}$

Section C - Spring block system

33. Two blocks 1 and 2 of masses m and 2m respectively are connected by a spring of force constant k. The masses are moving to the right with uniform velocity v each, the heavier mass, leading the lighter one. The spring is of natural length in the motion. Block 2 collides head on with a third block 3 of mass m, at rest, the collision being completely inelastic. Determine the velocity of blocks at the instant of maximum compression of the spring.



(A) $\frac{5v}{4} m/sec$ (B) $\frac{5v}{2} m/sec$

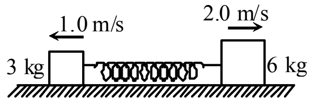
(C) $\frac{5v}{3} m/sec$ (D) $\frac{3v}{4} m/sec$

34. Two blocks A and B of mass m and 2m are connected by a massless spring of force constant k. They are placed on a smooth horizontal plane. Spring is stretched by an amount x and then released. The relative velocity of the blocks when the spring comes to its natural length is –

(A) $\left(\sqrt{\frac{3k}{2m}}\right)x$ (B) $\left(\sqrt{\frac{2k}{3m}}\right)x$

(C) $\sqrt{\frac{2kx}{m}}$ (D) $\sqrt{\frac{3km}{2x}}$

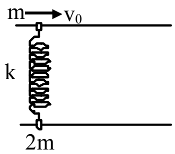
35. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be –



(A) 30 cm (B) 25 cm

(C) 20 cm (D) 15 cm

36. Two ring of mass m and 2 m are connected with a mass less spring and can slips over two frictionless parallel horizontal rails as shown in figure. Ring of mass m is given velocity 'v_0' in the direction shown. Maximum stretch in spring will be -



(A) $\sqrt{\frac{m}{k}}v_0$ (B) $\sqrt{\frac{3m}{k}}v_0$

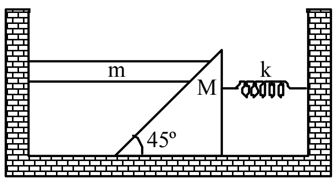
(C) $\sqrt{\frac{2m}{3k}}v_0$ (D) $\sqrt{\frac{2m}{k}}v_0$

37. A block is hanged from spring in a cage. Elongation in spring is 'x_1' and 'x_2' when cage moves up and down respectively with same acceleration. The expansion in spring when the cage move horizontally with same acceleration -

(A) $\frac{x_1 + x_2}{2}$ (B) $\sqrt{\frac{x_1^2 - x_2^2}{2}}$

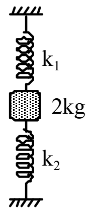
(C) $\sqrt{\frac{x_1^2 + x_2^2}{2}}$ (D) $\sqrt{x_1 x_2}$

38. All surfaces shown in figure are smooth. System is released with the spring unstretched. In equilibrium, compression in the spring wiil be –



(A) $\frac{mg}{\sqrt{2}k}$ (B) $\frac{2mg}{k}$ (C) $\frac{(M+m)g}{\sqrt{2}k}$ (D) $\frac{mg}{k}$

39. A 2 kg block is connected with two springs of force constants $k_1 = 100$ N/m and $k_2 = 300$ N/m as shown in figure. The block is released from rest with the springs unstretched. The acceleration of the block in its lowest position is : ($g = 10$ m/s^2) –

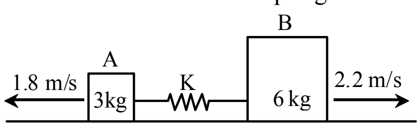


(A) zero (B) 10 m/s^2 upwards
(C) 10 m/s^2 downwards (D) 5 m/s^2 upwards

40. Two masses m and 2m are attached to two ends of an ideal spring and the spring is in the compressed state. The energy of spring is 60 joule. If the spring is released, then-

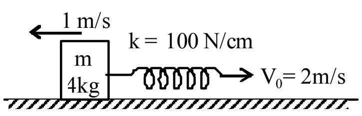
(A) the energy of both bodies will be same
(B) energy of smaller body will be 10J
(C) energy of smaller body will be 20J
(D) energy of smaller body will be 40 J

41. Two blocks A(3kg) and B(6kg) are connected by a spring of stiffness 512 N/m and placed on a smooth horizontal surface. Initially the spring has its equilibrium length. Velocities 1.8m/s and 2.2 m/s are imparted to A and B in opposite direction. The maximum extension in the spring will be –



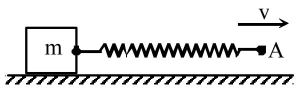
(A) 25 cm (B) 10 cm
(C) 12 cm (D) 2.5 cm

42. The spring block system lies on a smooth horizontal surface. The free end of the spring is being pulled towards right with constant speed $V_0 = 2$m/s. At t = 0 sec, the spring of constant k = 100 N/cm is unstreched and the block has a speed 1 m/s to left. The maximum extension of the spring is –



(A) 2cm (B) 4 cm
(C) 6 cm (D) 8 cm

43. A block of mass 'm' is attached to a spring in natural length of spring constant 'k'. The other end 'A' of the spring is moved with a constant velocity 'v' away from the block. Find the maximum extension in the spring -



(A) $\frac{1}{4}\sqrt{\frac{mv^2}{k}}$ (B) $\sqrt{\frac{mv^2}{k}}$

(C) $\frac{1}{2}\sqrt{\frac{mv^2}{k}}$ (D) $2\sqrt{\frac{mv^2}{k}}$

Section D - Impulse

44. A super-ball is to bounce elastically back and forth between two rigid walls at a distance d from each other. Neglecting gravity and assuming the velocity of super-ball to be v_0 horizontally, the average force being exerted by the super-ball on each wall is :

(A) $\frac{1}{2}\frac{mv_0^2}{d}$ (B) $\frac{mv_0^2}{d}$

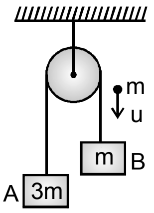
(C) $\frac{2mv_0^2}{d}$ (D) $\frac{4mv_0^2}{d}$

45. A force exerts an impulse I on a particle changing its speed from u to 2u. The applied force and the initial velocity are oppositely directed along the same line. The work done by the force is

(A) $\frac{3}{2}Iu$ (B) $\frac{1}{2}Iu$
(C) Iu (D) 2 Iu

46. A boy hits a baseball with a bat and imparts an impulse J to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals.
(A) half the original impulse
(B) the original impulse
(C) twice the original impulse
(D) four times the original impulse

47. A system of two blocks A and B are connected by an inextensible massless strings as shown. The pulley is masselss and frictionless. Initially the system is at rest when, a bullet of mass 'm' moving with a velocity 'u' as shown hits the block 'B' and gets embedded into it. The impulse imparted by tension force to the block of mass 3m is :



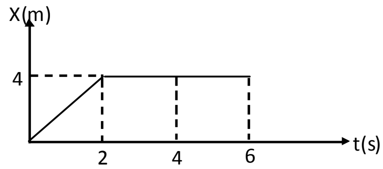
(A) $\frac{5mu}{4}$

(B) $\frac{4mu}{5}$

(C) $\frac{2mu}{5}$

(D) $\frac{3mu}{5}$

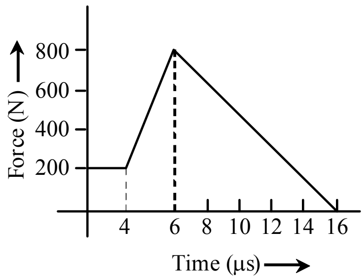
48. The position-time graph of a particle of mass 0.1 *kg* is shown. The impulse at $t = 2s$ is :



(A) $0.2\ kg\ ms^{-1}$ (B) $-0.2\ kg\ ms^{-1}$

(C) $0.1\ kg\ ms^{-1}$ (D) $-0.4\ kg\ ms^{-1}$

49. The magnitude of force (in Newtons) acting on a body varies with time (in micro second) as shown in the figure. The magnitude of total impulse of the force on the body from t = 4μs to t = 16μs is –



(A) 5×10^{-2} Ns (B) 5×10^{-3} Ns
(C) 5×10^{-4} Ns (D) 5×10^{-6} Ns

50. An impulse $\vec{I}$ changes the velocity of a particle from $\vec{V_1}$ to $\vec{V_2}$. Kinetic energy gained by the particle is–

(A) $(1/2)\vec{I}\cdot(\vec{V_1}+\vec{V_2})$ (B) $(1/2)\vec{I}\cdot(\vec{V_1}-\vec{V_2})$

(C) $\vec{I}\cdot(\vec{V_2}-\vec{V_1})$ (D) $\vec{I}\cdot(\vec{V_2}+\vec{V_1})$

51. Two blocks of masses 10 kg and 4 kg are connected by a spring of negligible mass and placed on a frictionless horizontal surface. An impulse gives a velocity of 14 m/s to the heavier block in the direction of the lighter block. The velocity of the centre of mass is -
(A) 30 m/s (B) 20 m/s
(C) 10 m/s (D) 5 m/s

Section E - Collision in 1D, Oblique collision

52. A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \le 1$).
(A) 45° (B) 30°
(C) 53° (D) 60°

53. In an inelastic collision-
(A) momentum is conserved but kinetic energy is not
(B) momentum is not conserved but kinetic energy is conserved
(C) neighter momentum nor kinetic energy is conserved
(D) both the momentum and kinetic energy are conserved

54. Two perfectly elastic balls of same mass m are moving with velocities u_1 and u_2. They collide elastically n times. The kinetic energy of the system finally is:

(A) $\frac{1}{2}\frac{m}{u}u_1^2$ (B) $\frac{1}{2}\frac{m}{u}(u_1^2+u_2^2)$

(C) $\frac{1}{2}m(u_1^2+u_2^2)$ (D) $\frac{1}{2}mn(u_1^2+u_2^2)$

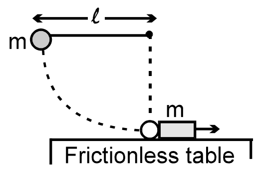
55. When two bodies collide elastically, then
(A) kinetic energy of the system alone is conserved
(B) only momentum is conserved
(C) both energy and momentum are conserved
(D) neighter energy nor momentum is conserved

56. A ball hits the floor and rebounds after an inelastic collision . In this case-
(A) the momentum of the ball just after the collision is the same as that just before the collision
(B) the mechanical energy of the ball remains the same in the collision
(C) the total momentum of the ball and the earth is conserved.
(D) the total energy of the ball and the earth is conserved

57. Six steel balls of identical size are lined up long a straight frictionless groove. Two similar balls moving with a speed V along the groove collide with this row on the extreme left hand then-

(A) all the balls will start moving to the right with speed 1/8 each
(B) all the six balls initially at rest will move on with speed V/6 each and two identical balls will come to rest
(C) two balls from the extreme right end will move on with speed V each and the remaining balls will remain at rest
(D) one ball from the right end will move on with speed 2V, the remaining balls will be at rest.

58. The bob of a simple pendulum of length l dropped from a horizontal position strikes a block of the same mass, placed on a horizontal table (frictionless) as shown in the diagram, the block shall have kinetic energy-



(A) Zero (B) mgl.
(C) 1/2 mgl. (D) 2mgl.

59. Two balls A and B having masses 1 kg and 2 kg, moving with speeds 21 m/s and 4 m/s respectively in opposite direction, collide head on. After collision A moves with a speed of 1 m/s in the same direction, then the coefficient of restitution is
(A) 0.1 (B) 0.2
(C) 0.4 (D) None

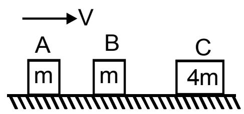
60. A truck moving on horizontal road east with velocity $20ms^{-1}$ collides elastically with a light ball moving with velocity 25 ms^{-1} along west. The velocity of the ball just after collision
(A) 65 ms^{-1} towards east
(B) 25 ms^{-1} towards west
(C) 65 ms^{-1} towards west
(D) 20 ms^{-1} towards east

61. A sphere of mass m moving with a constant velocity hits another stationary sphere of the same mass, if e is the coefficient of restitution, then ratio of speed of the first sphere to the speed of the second sphere after collision will be

(A) $\left(\frac{1-e}{1+e}\right)$ (B) $\left(\frac{1+e}{1-e}\right)$

(C) $\left(\frac{e+1}{e-1}\right)$ (D) $\left(\frac{e-1}{e+1}\right)$

62. Three blocks are initially placed as shown in the figure. Block A has mass m and initial velocity v to the right. Block B with mass m and block C with mass 4 m are both initially at rest. Neglect friction. All collisions are elastic. The final velocity of block A is



(A) 0.60 v to the left (B) 1.4 v to the left
(C) v to the left (D) 0.4 v to the left

63. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occured ?
(A) inelastic
(B) elastic
(C) completely inelastic
(D) Cannot be determined from the information given

64. A ball is dropped from a height h. As is bounces off the floor, its speed is 80 percent of what it was just before it hit the floor. The ball will then rise to a height of most nearly
(A) 0.80 h (B) 0.75 h
(C) 0.64 h (D) 0.50 h

65. A ball is thrown vertically downwards with velocity $\sqrt{2gh}$ from a height h. After colliding with the ground it just reaches the starting point. Coefficient of restitution is
(A) $1/\sqrt{2}$ (B) 1/2
(C) 1 (D) $\sqrt{2}$

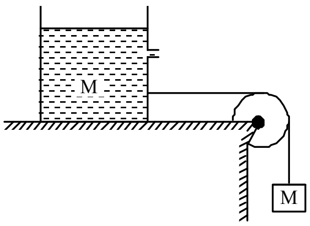
Section F - Variable Mass system, Rocket propulsion

66. A chain of mass M and length ℓ is held vertically such that its bottom end just touches the surface of a horizontal table. The chain is released from rest. Assume that the portion of chain on the table does not form a heap. The momentum of the portion of the chain above the table after the top end of the chain falls down by a distance $\frac{\ell}{8}$.

(A) $\frac{3}{14}M\sqrt{g\ell}$ (B) $\frac{3}{16}M\sqrt{g\ell}$

(C) $\frac{7}{16}M\sqrt{g\ell}$ (D) $\frac{9}{14}M\sqrt{g\ell}$

67. Container is massless. Mass of liquid filled in the container is M. There is a hole in the container as shown. There is no friction between container and ground. Then –



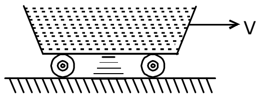
(A) Initial acceleration of system is less than g/2
(B) Initial acceleration of system is greater than g/2
(C) System moves with constant velocity
(D) System remains at rest

68. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. They collide at time t_o. Their velocities become $\vec{v}_1'$ and $\vec{v}_2'$ at time $2t_o$ while still moving in air. The value of $|(m_1\vec{v}_1' + m_2\vec{v}_2') - (m_1\vec{v}_1 + m_2\vec{v}_2)|$ is

(A) zero (B) $(m_1 + m_2)gt_o$

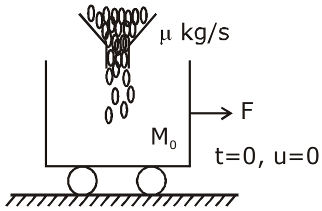
(C) $2(m_1 + m_2)gt_o$ (D) $\frac{1}{2}(m_1 + m_2)gt_o$

69. A trolley filled with sand moves on a smooth horizontal surface with an initial velocity v. A small hole is there at the base of trolley, from which sand is leaking out at constant rate. As the sand leaks out



(A) The velocity of the trolley increases.
(B) The velocity of the trolley remains unchanged.
(C) The momentum of the trolley is conserved
(D) The momentum of the total system (trolley + leaked sand) conserved.

70. Find velocity of wagon as function of time when it is being filled with stationary hopper at the rate μ kg/sec. and constant force F is applied horizontally on the wagon as shown in figure.
M_0 is the mass of the wagon and initial speed of the wagon u = 0 at time t = 0.



(A) $\frac{Ft}{M_0 - \mu t}$ (B) $\frac{2Ft}{M_0 + \mu t}$

(C) $\frac{Ft^2}{M_0 + \mu t}$ (D) $\frac{Ft}{M_0 + \mu t}$

71. A wagon fully loaded with sand moue on parallel rails with constant speed v, Now, bottom door of wagon s partially opened and sand starts falling at the rate m kg per second. Find ext. force req. (if any) to keep pull of wagon constant as function of time?

(A) 0 (B) $\frac{Ft}{M_0 + \mu t}$

(C) $\frac{2Ft}{M_0 + \mu t}$ (D) None of these

Section A - Calculation of COM of system of N-particles, COM of continuous distributed system, Combination of structure and Cavity problems

1. Consider following statements

(A) CM of a uniform semicircular disc of radius $R = 2R/\pi$ from the centre

(B) CM of a uniform semicircular ring of radius $R = 4R/3\pi$ from the centre

(C) CM of a solid hemisphere of radius $R = 4R/3\pi$ from the centre

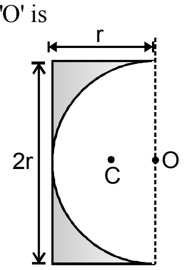
(D) CM of a hemisphere shell of radius $R = R/2$ from the centre

Which statements are correct?

(A) 1, 2, 4 (B) 1, 3, 4

(C) 4 only (D) 1, 2 only

2. A semicircular portion of radius 'r' is cut from a uniform rectangular plate as shown in figure. The distance of centre of mass 'C' of remaining plate, from point 'O' is



(A) $\frac{2r}{(3-\pi)}$ (B) $\frac{3r}{2(4-\pi)}$

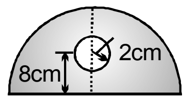
(C) $\frac{2r}{(4+\pi)}$ (D) $\frac{2r}{3(4-\pi)}$

3. From a circle of radius a, an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is

(A) $3(\pi - 1)a$ (B) $\frac{(\pi-1)a}{6}$

(C) $\frac{a}{3(\pi-1)}$ (D) $\frac{a}{3(\pi+1)}$

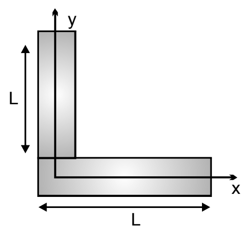
4. In the figure shown a hole of radius 2 cm is made in semicircular disc of radius 6π at a distance 8 cm from the centre C of the disc. The distance of the centre of mass of this system from point C is



(A) 4 cm (B) 8 cm

(C) 6 cm (D) 12 cm

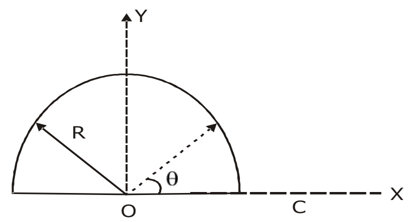
5. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown, can be, if the meeting point is the origin of co-ordinates



(A) (L/2, L/2) (B) (2L/3, L/2)

(C) (L/3, L/3) (D) (L/3, L/6)

6. A thin disc of non uniform mass distribution is shown in x-y plane. Surface mass density of disc varies according to relation $\sigma_o \sin\theta$, where σ_o is a positive constant and θ is the angle made by position vector of any point on disc with positive x-axis. The coordinates of centre of mass of disc are



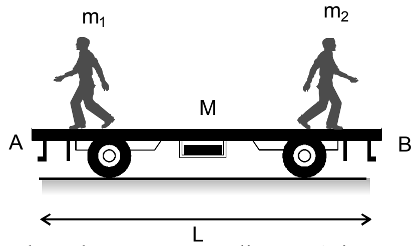
(A) $(0, 2R\pi)$ (B) $(0, \pi R/6)$

(C) $(0, 3\pi R/2)$ (D) none of these

Section B - Motion of COM, Conservation of Momentum, Trolley problems

Question No. 7 to 11

Two persons of mass m_1 and m_2 are standing at the two ends A and B respectively, of a trolley of mass M as shown.



7. When the person standing at A jumps from the trolley towards left with u_{rel} with respect to the trolley, then

(A) the trolley moves towards right

(B) the trolley rebounds with velocity $\frac{m_1u_{rel}}{m_1+m_2+M}$

(C) the centre of mass of the system remains stationary

(D) all the above

8. When only the person standing at B jumps from the trolley towards right while the person at A keeps standing, then

(A) the trolley moves towards left

(B) the trolley mones with velocity $\frac{m_2u_{rel}}{m_1+m_2+M}$

(C) the centre of mass of the system remains stationary

(D) all the above

9. When both the persons jump simultaneously with same speed then

(A) the centre of fmass of the systyem remains stationary

(B) the trolley remains stationary

(C) the trolley moves toward the end where the person with heavier mass is standing

(D) None of these

10. When both the persons jump simultaneously with u_{rel} with respect to the trolley, then the velocity of the trolley is

(A) $\frac{|m_1-m_2|u_{rel}}{m_1+m_2+M}$ (B) $\frac{|m_1-m_2|u_{rel}}{M}$

(C) $\left|\frac{m_1u_{rel}}{m_2+M}-\frac{m_2u_{rel}}{m_1+M}\right|$ (D) none of these

11. Choose the incorrect statement, if $m_1 = m_2 = m$ and both the persons jump one by one, then

(A) the centre of mass of the system remains stationary

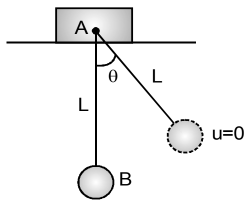
(B) the final velocity of the trolley is in the direction of the person who jumps first

(C) the final velocity of the trolley is $\left(\frac{mu_{rel}}{M+m}-\frac{mu_{rel}}{M+2m}\right)$

(D) none of these

Question No. 12 to 15

A small ball B of mass m is suspended with light inelastic string of length L from a block A of same mass m which can move on smooth horizontal surface as shown in the figure. The ball is displaced by angle θ from equilibrium position & then released.



12. The displacement of block when ball reaches the equilibrium position is

(A) $\frac{L\sin\theta}{2}$ (B) $L\sin\theta$

(C) L (D) none of these

13. Tension in string when it is vertical, is

(A) mg (B) $mg(2 - \cos\theta)$

(C) $mg(3 - 2\cos\theta)$ (D) none of these

14. Maximum velocity of block during subsequent motion of the system after release of ball is

(A) $[gl(1 - \cos\theta)]^{1/2}$

(B) $[2gl(1 - \cos\theta)]^{1/2}$

(C) $[g/\cos\theta]^{1/2}$

(D) informations are insufficient to decide

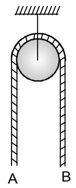
15. The displacement of centre of mass of A + B system till the string becomes vertical is

(A) zero (B) $\frac{L}{2}(1-\cos\theta)$

(C) $\frac{L}{2}(1-\sin\theta)$ (D) none of these

Question No. 16 to 17

A uniform chain of length 2L is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth & string is inextensible



16. The acceleration of end B when it has been displaced by distance x, is

(A) $\frac{x}{L}g$ (B) $\frac{2x}{L}g$

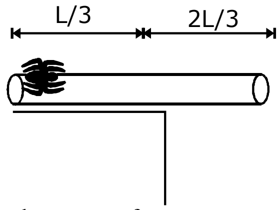
(C) $\frac{x}{2}g$ (D) g

17. The velocity v of the string when it slips out of the pulley (height of pulley from floor > 2L)

(A) $\sqrt{\frac{gL}{2}}$ (B) $\sqrt{2gL}$

(C) $\sqrt{gL}$ (D) none of these

18. A straw of length L, mass M lies over a smooth horizontal table with its $(2/3)^{rd}$ part hanging in air. There is an insect of mass m at the end of straw initially. Insects slowly moves on straw to other end such that straw never falls off the table.



(A) Displacement of straw w.r.t. ground (in terms of m,M,L) when the insect reaches the other end of straw is $\frac{mL}{M+m}$

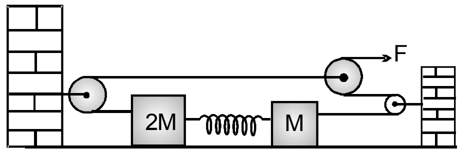
(B) Displacement of straw w.r.t. ground (in terms of m,M,L) when the insect reaches the other end of straw is $\frac{2mL}{M+m}$

(C) The minimum mass ratio $\left(\frac{m}{M}\right)$ of insect of straw is $\frac{1}{2}$

(D) The minimum mass ratio $\left(\frac{m}{M}\right)$ of insect of straw is $\frac{1}{4}$

Section C - Spring block system

19. In the diagram shown, no friction at any contact surface. Initially, the spring has no deformation. What will be the maximum deformation in the spring ? Consider all the strings to be sufficiency large. Consider the spring constant to be K



(A) 4F / 3K (B) 8F / 3K

(C) F / 3K (D) none

20. Two identical blocks each of mass 1kg are joined together with a compressed spring. When the system is released the two blocks appear to be moving with unequal speeds in the opposite directions as shown in fig.

Choose the correct statement -

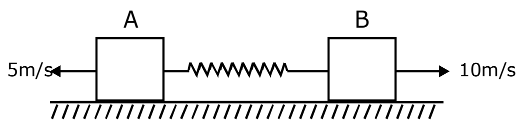
(A) It is not possible

(B) Whatever may be the speed of the blocks the centre of mass will remain stationary

(C) The centre of mass of the system is moving with a velocity of $2ms^{-1}$

(D) The centre of mass of the system is moving with a velocity of $1ms^{-1}$

21. The two blocks A and B of same mass connected to a spring and placed on a smooth surface. They are given velocities (as shown in the figure) when the spring is in its natural length :



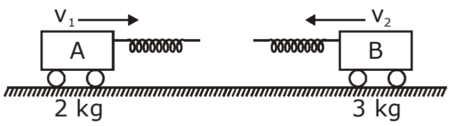
(A) The maximum velocity will be 10m/s

(B) The maximum velocity will be greater than 10 m/s

(C) The spring will have maximum extension when A and B both stop

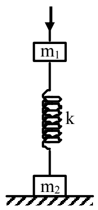
(D)The spring will have maximum extension when A and B both move towards left.

22. Two carts (A & B) having spring bumpers collides as shown in figure. Initially $v_1 \neq 0$ & $v_2 = 0$. When the separation between the carts is minimum.



(A) the cart B is still at rest.
(B) both carts have same momentum
(C) both carts have same KE
(D) The KE of the system is at a minimum.

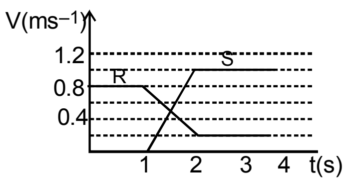
23. A system consists of two cubes of masses m_1 and m_2 respectively connected by a spring of force constant k. The force (F) that should be applied to the upper cube for which the lower one just lifts after the force is removed is-



(A) m_1g (B) $\frac{m_1m_2}{m_1+m_2}g$
(C) $(m_1 + m_2)$ g (D) m_2g

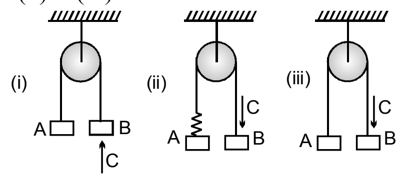
Section D - Impulse

24. The diagram shows the velocity - time graph for two masses R and S that collided elastically. Which of the following statements is true ?



I. R and S moved in the same direction after the collision.
II. The velocities of R and S were equal at the mid time of the collision.
III. The mass of R was greater than mass of S.
(A) I only (B) II only
(C) I and II only (D) I, II and III

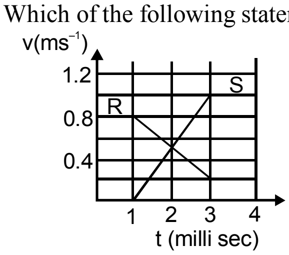
25. In the figure (i), (ii) & (iii) shown the objects A, B & C are of same mass. String, spring & pulley are massless. C strikes B with velocity 'u' in each case and sticks to it. The ratio of velocity of B in case (i) to (ii) to (iii) is



(A) 1 : 1 : 1 (B) 3 : 3 : 2
(C) 3 : 2 : 2 (D) none of these

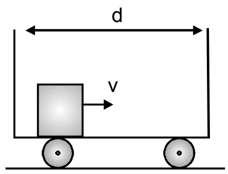
Section E - Collision in 1D, Oblique collision

26. The diagram to the right shown the velocity-time graph for two masses R and S that collided elastically. Which of the following statements is true?



(I) R and S moved in the same direction after the collision.
(II) Kinetic energy of the system (R & S) is minimum at t = 2 milli sec.
(III) The mass of R was greater than mass of S.
(A) I only (B) II only
(C) I and II only (D) I, II and III

27. In a smooth stationary cart of length d, a small block is projected along it's length with velocity v towards front. Coefficient of restitution for each collision is e. The cart rests on a smooth ground and can move freely. The time taken by block to come to rest w.r.t. cart is



(A) $\frac{ed}{(1-e)v}$ (B) $\frac{ed}{(1+e)v}$

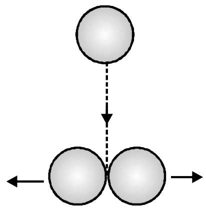
(C) $\frac{d}{e}$ (D) inifinite

28. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collision with a vertical wall and returns to ground. The total time of flight of the ball is

(A) $\frac{2v\sin\theta}{g}$ (B) $\frac{2v\cos\theta}{g}$

(C) $\frac{v\sin 2\theta}{g}$ (D) $\frac{v\cos\theta}{g}$

29. In the figure shown, the two identical balls of mass M and radius R each, are placed in contact with each other on the frictionless horizontal surface. The third ball of mass M and radius R/2, is coming down vertically and has a velocity = v_0 when it simultaneously hits the two balls and itself comes to rest. The each of the two bigger balls will move after collision with a speed equal to



(A) $4v_0/\sqrt{5}$ (B) $2v_0/\sqrt{5}$

(C) $v_0/\sqrt{5}$ (D) none

30. In the above, suppose that the smaller ball does not stop after collision, but continues to move downwards with a speed = $v_0/2$, after the collision. Then, the speed of each bigger ball after collision is

(A) $4v_0/\sqrt{5}$ (B) $2v_0/\sqrt{5}$

(C) $v_0/2\sqrt{5}$ (D) none

31. Which of the following does not hold when two particles of masses m_1 and m_2 undergo elastic collision?

(A) when $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of kinetic energy in head an collision

(B) when $m_1 = m_2$ and m_2 is stationary, there is maximum transfer of momentum in head on collision

(C) when $m_1 >> m_2$ and m_2 is stationary, after head on collision m_2 moves with twice the velocity of m_1.

(D) when the collision is oblique and $m_1=m_2$ with m_2 stationary, after the collision the particle move in opposite directions.

32. A metal ball hits a wall and does not rebound whereas a rubber ball of the same mass on hitting the wall the same velocity rebounds back. It can be concluded that–

(A) metal ball sufferes greater change in momentum

(B) rubber ball suffers greater change in momentum.

(C) the initial momentum of metal ball is greater than the initial momentum of rubber ball.

(D) both suffer same change in momentum.

33. Before a rubber ball bounces off from the floor the ball is in contact with the floor for a fraction of second. Which of the following statements are correct-

(A) conservation of energy is not valid during this period

(B) conservation of energy is valid during this period

(C) as ball compressed kinetic energy is converted compressed potential energy

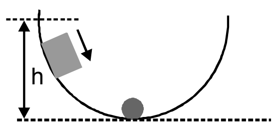
(D) B and C both

34. A ball of 0.1kg strikes a wall at right angle with a speed of 6 m/s and rebounds along its original path at 4 m/s. The change in momentum in Newton- sec is-

(A) 10^3 (B) 10^2

(C) 10 (D) 1

35. A block of mass m starts from rest and slides down a frictionless semi-circular track from a height h as shown. When it reaches the lowest point of the track, it collides with a stationary piece of putty also having mass m. If the block and the putty stick together and continue to slide, the maximum height that the block-putty system could reach is



(A) h/4 (B) h/2

(C) h (D) independent of h

36. Two billiard balls undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Initially, ball 1 moves with a speed v towards ball 2 which is at rest. Immediately after the collision, ball 1 travels at a speed of v/3 in the same direction. What type of collision has occured ?

(A) inelastic

(B) elastic

(C) completely inelastic

(D) Cannot be determined from the information given

Section F - Variable Mass system, Rocket propulsion

Paragraph Q. 37 to Q. 38

An empty wagon of mass m_0 is stationary & is filled with sand at rate of μ kg/sec. from a stationary hopper ? find -

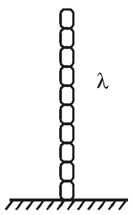
37. Force F as a function of time that has to be applied on wagon to maintain its all constant ?

(A) $aM_0 - 2a\mu t$ (B) $aM_0 - a\mu t$
(C) $aM_0 + 2a\mu t$ (D) $aM_0 + a\mu t$

38. Force, F as function of time that has to be applied to maintain its all constant equal to v.

(A) μv (B) $2\mu v$
(C) $\sqrt{3}\,\mu v$ (D) None of these

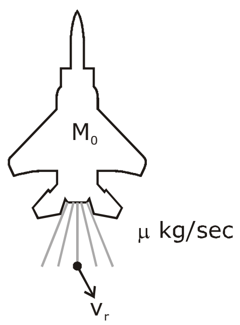
39. Find N of ground on chain when chain has fallen through distance x. Linear mass density of chain is λ.



(A) $3\lambda gx$ (B) λgx
(C) $2\lambda gx$ (D) $5\lambda gx$

Paragraph Q. 40 to Q. 41

If rocket engine ejects fuel at the rate μ kg/s with relative velocity v_r. At time t = 0, speed of rocket is u. Consider the value of acceleration due to gravity is constant and equal to g.



40. Find velocity of rocket as function of time.

(A) $u - v_r.\ln\left(\frac{M_0}{M_0 - \mu t}\right) + gt$

(B) $u + v_r.\ln\left(\frac{M_0}{M_0 - \mu t}\right) - gt$

(C) $u + v_r.\ln\left(\frac{M_0}{M_0 + \mu t}\right) + gt$

(D) $u + v_r.\ln\left(\frac{M_0}{M_0 + \mu t}\right) - gt$

41. If rocket is in gravity free space then find velocity of rocket as function of time.

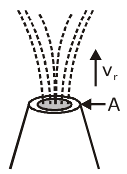
(A) $u - v_r.\ln\left(\frac{M_0}{M_0 - \mu t}\right)$

(B) $u + 2v_r.\ln\left(\frac{M_0}{M_0 - \mu t}\right)$

(C) $u + v_r.\ln\left(\frac{M_0}{M_0 + \mu t}\right)$

(D) $u + v_r.\ln\left(\frac{M_0}{M_0 - \mu t}\right)$

42. Find thrust force on hose pipe. Density of liquid is ρ.

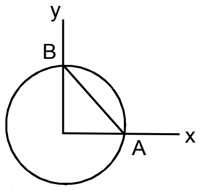


(A) $\rho A.\ gh$ (B) $\rho A.\ 3gh$
(C) $\rho A.\ 2gh$ (D) None

Section A - Calculation of COM of system of N-particles, COM of continuous distributed system, Combination of structure and Cavity problems

1. A body has its centre of mass at the origin. The x-coordinates of the particles
(A) may be all positive
(B) may be all negative
(C) may be all non-negative
(D) may be positive for some cases and negative in other cases

2. An object comprises of a uniform ring of radius R and its uniform chord AB (not necessarily made of the same material) as shown. Which of the following can not be the centre of mass of the object.



(A) (R/3, R/3) (B) (R/3, R/2)
(C) (R/4, R/4) (D) $(R/\sqrt{2}, R/\sqrt{2})$

3. In which of the following cases the centre of mass of a an rod is certainly not at its centre ?
(A) the density continuously increases from left to right
(B) the density continuously decreases from left to right
(C) the density decreases from left to right upto the centre and then increase
(D) the density increases from left to right upto the centre and then decreases

4. If the external forces acting on a system have zero resultant, the centre of mass
(A) must not move
(B) must not accelerate
(C) may move
(D) may accelerate

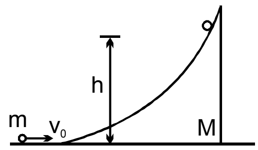
5. A body has its centre of mass at the origin. The x-coordinates of the particles -
(A) may be all positive
(B) may be all negative
(C) may be all non-negative
(D) may be positive for some particles and negative in other particles

6. In which of the following cases the centre of mass of a rod is certainly not at its centre ?
(A) the density continuously increases from left to right
(B) the density continuously decreases from left to right
(C) the density decreases from left to right upto the centre and then increases
(D) the density increases from left to right upto the centre and then decreases

Section B - Motion of COM, Conservation of Momentum, Trolley problems

Question No. 7 to 13

A particle of mass m moving horizontal with v_0 strikes a smooth wedge of mass M, as shown in figure. After collision, the ball starts moving up the inclined face of the wedge and rises to a height h.



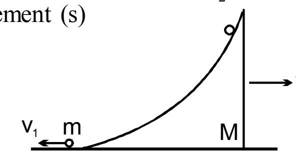
7. The final velocity of the wedge v_2 is
(A) $\frac{mv_0}{M}$ (B) $\frac{mv_0}{M+m}$
(C) v_0 (D) insufficient data

8. When the particle has risen to a height h on the wedge, then choose the correct alternative(s)
(A) The particle is stationary with respect to ground
(B) Both are stationary with respect to the centre of mass
(C) The kinetic energy of the centre of mass remains constant
(D) The kinetic energy with respect to centre of mass is converted into potential energy

9. The maximum height h attained by the particle is
(A) $\left(\frac{m}{m+M}\right)\frac{v_0^2}{2g}$ (B) $\left(\frac{m}{M}\right)\frac{v_0^2}{2g}$
(C) $\left(\frac{M}{m+M}\right)\frac{v_0^2}{2g}$ (D) none of these.

10. Identify the correct statement(s) related to the situation when the particle starts moving downward.
(A) The centre of mass of the system remains stationary
(B) Both the particle and the wedge remain stationary with respect to centre of mass
(C) When the particle reaches the horizontal surface its velocity relative to the wedge is v_0
(D) None of these

11. Suppose the particle when reaches the horizontal surfaces, its velocity with respect to ground is v_1 and that of wedge is v_2. Choose the correct statement (s)



(A) $mv_1 = Mv_2$ (B) $Mv_2 - mv_1 = mv_0$
(C) $v_1 + v_2 = v_0$ (D) $v_1 + v_2 < v_0$

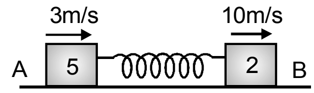
12. Choose the correct statement(s) related to particle m
(A) Its kinetic energy is $K_f = \left(\frac{mM}{m+M}\right)gh$
(B) $v_1 = v_0\left(\frac{M-m}{M+m}\right)$
(C) The ratio of its final kinetic energy to its initial kinetic energy is $\frac{K_f}{K_i} = \left(\frac{M}{m+M}\right)^2$
(D) It moves opposite to its initial direction of motion

13. Choose the correct statement related to the wedge M
(A) Its kinetic energy is $K_f = \left(\frac{4m^2}{m+M}\right)gh$
(B) $v_2 = \left(\frac{2m}{m+M}\right)v_0$
(C) Its gain in kinetic energy is $\Delta K = \left(\frac{4mM}{(m+M)^2}\right)\left(\frac{1}{2}mv_0^2\right)$
(D) Its velocity is more that the velocity of centre of mass

14. Two identical balls are interconnected with a massless and inextensible thread. The system is in gravity free space with the thread just taut. Each ball is imparted a velocity v, one towards the other ball and the other perpendicular to the first, at t = 0. Then,
(A) the thread will become taut at t = (L/v)
(B) the thread will become taut at some time t < (L/v).
(C) the thread will always remain taut for t > (L/v)
(D) the kinetic energy of the system will always remain mv^2.

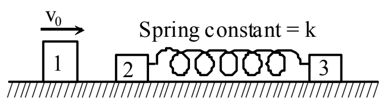
Section C - Spring block system

15. Two blocks A (5kg) and B(2kg) attached to the ends of a spring constant 1120 N/m are placed on a smooth horizontal plane with the spring undeformed. Simultaneously velocities of 3m/s and 10m/s along the line of the spring in the same direction are imparted to A and B then



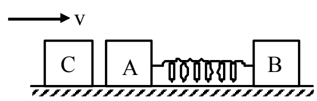
(A) when the extension of the spring is maximum the velocities of A and B are zero.
(B) the maximum extension of the spring is 25 cm
(C) maximum extension and maximum compression occur alternately.
(D) the maximum compression occur for the first time after $\frac{\pi}{56}$ sec.

16. Figure shows three identical blocks 1,2 & 3 each of mass m kept on frictionless surface. Block 1 is given initial velocity v_0 in the direction shown. Block 1 hits block 2 and stops



(A) Coefficient of restitution between block 1&2 is e=1/2
(B) Coefficient of restitution between block 1&2 is e=1.
(C) Maximum compression in spring is $\sqrt{\frac{m}{2k}}\,v_0$
(D) Maximum compression in spring is $\sqrt{\frac{m}{k}}\,v_0$

17. Two blocks A and B each of mass m are connected by a light spring of natural length L and spring constant k. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown. A third identical block C, also of mass m, moves on the floor with a speed v along the line joining A to B and collides with A. Collision is elastic and head on. Then -



(A) the kinetic energy of the A-B system at maximum compression of the spring is zero
(B) the kinetic energy of the A-B system at maximum compression of the spring is $\left(\frac{mv^2}{4}\right)$
(C) the maximum compression of the spring is $\left(v\sqrt{\frac{m}{k}}\right)$
(D) the maximum compression of the spring is $\left(v\sqrt{\frac{m}{2k}}\right)$

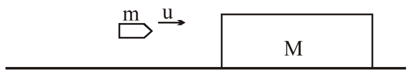
18. Two blocks of mass m_1 and m_2, resting on a frictionless table, are connected by a stretched spring and then released -
(A) ratio of speed of blocks is m_2/m_1
(B) ratio of kinetic energy of blocks is m_2/m_1
(C) centre of mass will move towards heavier block
(D) mechanical energy will remain conserve in this process

19. For a two-body system in absence of external forces, the kinetic energy as measured from ground frame is K_o and from center of mass frame is K_{cm}. Pick up the RIGHT statement
(A) The kinetic energy as measured from center of mass frame is least
(B) Only the portion of energy K_{cm} can be transformed from one form to another due to internal changes in the system.
(C) The system always retains at least K_o -K_{cm} amount of kinetic energy as measured from ground frame irrespective of any kind of internal changes in the system.
(D) The system always retains at least K_{cm} amount of kinetic energy as measured from ground frame irrespective of any kind of internal changes in the system

Section D - Impulse

20. Two balls A and B having masses 1 kg and 2 kg, moving with speeds 21 m/s and 4 m/s respectively in opposite direction, collide head on. After collision A moves with a speed of 1 m/s in the same direction, then correct statements is :
(A) The velocity of B after collision is 6 m/s opposite to its direction of motion before collision.
(B) The coefficient of restitution is 0.2.
(C) The loss of kinetic energy due to collision is 200 J.
(D) The impulse of the force between the two balls is 40 Ns.

21. A bullet of mass m travelling horizontally with speed u strikes a wooden block of mass 'M' placed on a smooth horizontal plane. The penetration is assumed uniform and the bullet comes to rest after penetrating a distance 'd' into the block. Then choose the correct statement :

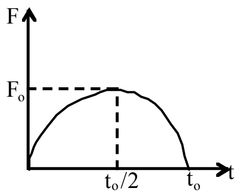


(A) Ultimately the total loss of kinetic energy is $\frac{mMu^2}{2(M+m)}$
(B) The value of the resistance force (assumed constant) offered by the wood is $F = \frac{mMu^2}{2d(M+m)}$
(C) The distance covered by the bullet w.r.t. the ground before it comes to rest w.r.t. block is $\frac{dm}{M+m}$
(D) The block moves greater distance than the bullet wrt ground.

22. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A, then -
(A) The total momentum of the 'A plus B' system is p before and after the impact and (p – J) during the impact
(B) During the impact A gives impulse J to B
(C) The coefficient of restitution is $\frac{2J}{p} - 1$
(D) The coefficient of restitution is $\frac{J}{p} + 1$

23. Two identical spheres A and B are free to move and to rotate about their centers. They are given the same impulse J. The lines of action of the impulses pass through the centre of A, and away from the centre of B -
(A) A and B will have the same speed
(B) B will have greater kinetic energy than A
(C) They will have the same kinetic energy, but the linear kinetic energy of B will be less than that of A
(D) The kinetic energy of B will depend on the point of impact of the impulse on B

24. A variable force acts on a body of mass m (initially at rest) from t = 0 to t = t_0. The curve of F plotted versus t is semicircle as shown, then -



(A) Impulse imparted to particle is infinite

(B) Impulse imparted to particle is $\frac{1}{4}\pi F_o t_o$

(C) The velocity acquired by the particle is $\frac{\pi F_o t_o}{4m}$

(D) The gain in momentum is $\frac{\pi F_o t_o}{4}$

25. A thin uniform rod of mass m and length ℓ is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Then immediately after impact -

(A) the angular momentum of the rod is $J\ell$.

(B) the angular velocity of the rod is $\frac{3J}{m\ell}$

(C) the kinetic energy of the rod is $\frac{3J^2}{2m}$

(D) the linear velocity of the midpoint of rod is $\frac{3J}{2m}$

Section E - Collision in 1D, Oblique collision

26. A ball moving with a velocity v hits a massive wall moving towards the ball with a velocity u. An elastic impact lasts for a time Δt.

(A) The average elastic force acting on the ball is $\frac{m(u+v)}{\Delta t}$

(B) The average elastic force acting on the ball is $\frac{2m(u+v)}{\Delta t}$

(C) The kinetic energy of the ball increases by 2mu (u + v)

(D) The kinetic energy of the ball remains the same after the collision.

27. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle when has twice its mass during the impact.

(A) The minimum kinetic energy of the system is 1 joule

(B) The maximum elastic potential energy of the system is 2 joule.

(C) Momentum and total kinetic energy of the system are conserved at every instant.

(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

28. In an inelastic collision,

(A) the velocity of both the particles may be same after collision.

(B) kinetic energy is not conserved

(C) linear momentum of the system is conserved.

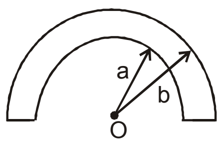
(D) velocity of separation will be less than velocity of approach.

Exercise - 3 | Level-I Subjective | JEE Advanced

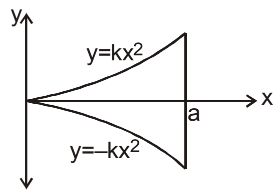
Section A - Calculation of COM of system of N-particles, COM of continuous distributed system, Combination of structure and Cavity problems

1. The mass of an uniform ladder of length l increases uniformly from one end A to the other end B,
(a) Form an expression for linear mass density as a function of distance x from end A where linear mass density λ_0. The density at one end being twice that of the other end.
(b) find the position of the centre of mass from end A.

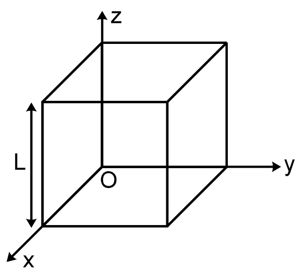
2. Find the distance of centre of mass of a uniform plate having semicircular inner and other boundaries of radii a and b from the centre O.



3. A thin sheet of metal of uniform thickness is cut into the shape bounded by the line x = a and y = ± k x^2, as shown.
Find the coordinates of the centre of mass.

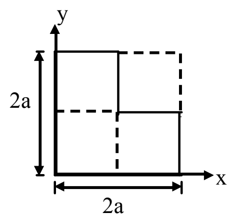


4. Figure shows a cubical box that has been constructed from uniform metal plate of negligible thickness. The box is open at the top and has edge length 40 cm. The z co-ordinate of the centre of mass of the box is x cm, then x is :

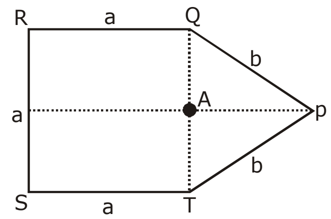


5. A uniform disc of radius R is put over another uniform disc of radius 2R of the same thickness and density. The peripheries of the two discs touch each other. Locate the centre of mass of the system from the centre of large disc.

6. Find the position of centre of mass of the uniform planner sheet shown in figure with respect to the origin (O) -



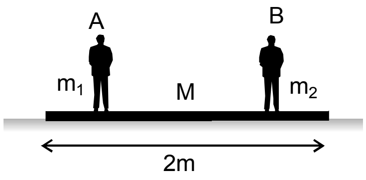
7. A homogeneous plate PQRST is as shown in figure. The centre of mass of plate lies at midpoint A of segment QT. Then the ratio of $\frac{b}{a}$ is
(PQ = PT = b; QR = RS = ST = a)



Section B - Motion of COM, Conservation of Momentum, Trolley problems

8. Two balls of equal masses are projected upwards simultaneously, one from the ground with speed 50 m/s and other from a 40m high tower with initial speed 30 m/s. Find the maximum height attained by their centre of mass.

9. In the figure shown, when the persons A and B exchange their positions, then

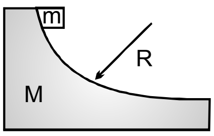


m_1=50kg, m_2 = 70kg, M = 80 kg

(i) the distance moved by the centre of mass of the system is__________.
(ii) the plank moves towards_________
(iii) the distance moved by the plank is _____.
(iv) the distance moved by A with respect to ground is ____________
(v) the distance moved by B with respect to ground is ____________.

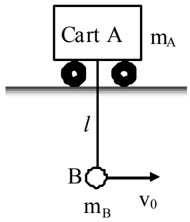
10. In the arrangement, m_A = 2 kg and m_B = 1 kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.

11. A small cube of mass m slides down a circular path of radius R cut into a large block of mass M. M rests on a table and both blocks move without friction. The blocks initially are at rest and m starts from the top of the path. Find the velocity v of the cube as it leaves the block.

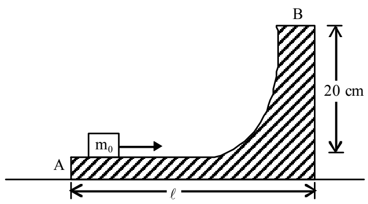


12. A (trolley + child) of total mass 200 kg is moving with a uniform speed of 36 km/h on a frictionless track. The child of mass 20 kg starts running on the trolley from one end to the other (10 m away) with a speed of 10 ms^{-1} relative to the trolley in the direction of the trolley's motion and jumps out of the trolley with the same relative velocity. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run?

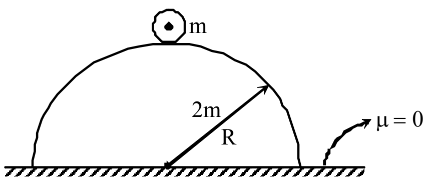
13. A ball B is suspended from a string of length l attached to a cart A, which may roll on a frictionless surface. Initially the cart is at rest and the ball is given a horizontal velocity v_0. Determine (a) the velocity of B as it reaches the maximum height; (b) the maximum height reached by the ball.



14. A wedge of mass M = 2 m_0 rests on a smooth horizontal plane. A small block of mass m_0 rests over it at left end A as shown in figure. A sharp impulse is applied on the block, due to which it starts moving to the right with velocity v_0 = 6 m/s. At highest point of its trajectory, the block collides with a particle of same mass m_0 moving vertically downwards with velocity v = 2 m/s and gets stuck with it. If the combined mass lands at the end point A of the body of mass M, calculate length ℓ in cm. Neglect friction, take g = 10 m/s^2.

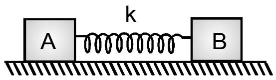


15. A small sphere of radius r = 2cm and mass 'm' is placed on a big sphere of mass 2m and radius R = 10 cm, which is kept on a smooth horizontal surface as shown in figure. The displacement (in cm) of hemisphere when line joining between centre of small sphere and hemisphere make angle 30° with vertical is (Assuming that small spheres does not break-off the surface of hemisphere) -



Section C - Spring block system

16. In the figure shown the spring is compressed by 'x_0' and released. Two blocks 'A' and 'B' of masses 'm' and '2m' respectively are attached at the ends of the spring. Blocks are kept on a smooth horizontal surface and released.



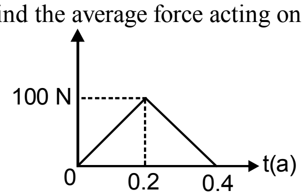
(a) Find the speed of block A by the time compression of the spring is reduced to $x_0/2$.
(b) Find the work done by the spring on 'A' by the time compression of the spring reduced to $x_0/2$.

17. 2 mass of block 2 kg & 4 kg are attached to a stiffness spring 100 N/w. 2 kg mass is given velocity 2 m/s & 4 kg to 4 m/s in same direction. Find maximum elongation in the spring during the motion of two blocks.

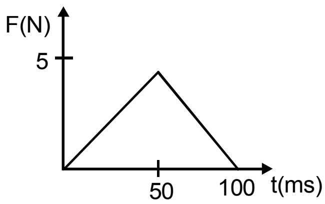
Section D - Impulse

18. The figure showns the force versus time graph for a particle.
(i) Find the change in momentum Δp of the particle

(ii) Find the average force acting on the particle

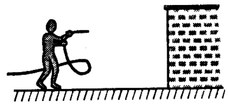


19. A force F acts on an object (mass = 1kg) which is initially at rest as shown in the figure. Draw the graph showing the momentum of the object varying during the time for which the force acts.



Section E - Collision in 1D, Oblique collision

20. A man hosing down his driveway hits the wall by mistake. Knowing that the velocity of the stream is 25 m/s and the cross-sectional area of the stream is 300 mm^2, determine the force exerted on the wall. Assume that streamstrikes wall horizontally and after striking the wall, stream comes to rest. Also find the pressure exerted on the wall by stream.



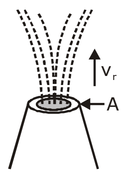
21. A bullet of mass m strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from u to v, find the magnitude of the impulse acting on the bullet.

22. A neutron intially at rest, decays into a proton, an electron and an antineutrio. The ejected electron has a momentum of $p_1 = 1.4 \times 10^{-28}$ kg-m/s and the antineutrino $p_2 = 6.5 \times 10^{-27}$ kg-m/s. Find the recoil speed of the proton (a) if the electron and the antineutrino are ejected along the the same direection and (b) if they are ejected along perpendicular direction. Mass of the proton $m_p = 1.67 \times 10^{-27}$ kg.

23. A steel ball of mass 0.5 kg is dropped from a height of 4 m on to a horizontal heavy steel slab. The collision is elastic and the ball rebounds to its original height.
(a) Calculate the impulse delivered to the ball during impact.
(b) If the ball is in contact with the slab for 0.002 s, find the average reaction force on the ball during impact.

Section F - Variable Mass system, Rocket propulsion

24. Find thrust force on hose pipe if density of water is 10^3 kg/m^3, Area of cross–section of the hose is 10^{-6} m^2 and maximum height achieved by stream 10 meters is



Section A - Calculation of COM of system of N-particles, COM of continuous distributed system, Combination of structure and Cavity problems

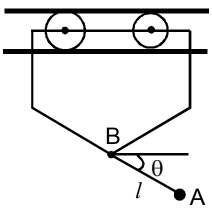
1. Mass is non-uniformly distributed on the circumference of a ring of radius a and centre at origin. Let b be the distance of the centre of mass of the ring from origin. Then -

2. The linear mass density of a ladder of length l increases uniformly from one end A to the other end B,
(a) Form an expression for linear mass density as a function of distance x from end A where linear mass density λ_0. The density at one end being twice that of the other end.
(b) find the position of the centre of mass from end A.

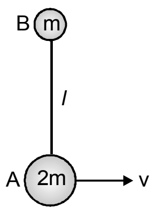
Section B - Motion of COM, Conservation of Momentum, Trolley problems

3. A 24-kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.
(a) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)
(b) How much energy was released during the explosion?

4. The simple pendulum A of mass m_A and length l is suspended from the trolley B of mass m_B. If the system is released from rest at $\theta = 0$, determine the velocity v_B of the trolley and tension in the string when $\theta = 90°$. Friction is negligible.

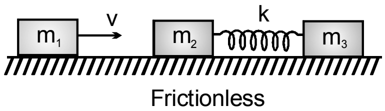


5. Two masses A and B connected with an inextensible string of length l lie on a smooth horizontal plane. A is given a velocity of v m/s along the ground perpendicular to line AB as shown in figure. Find the tension in string during their sub sequent motion.



Section C - Spring block system

6. Mass m_1 hits & sticks with m_2 while sliding horizontally with velocity v along the common line of centres of the three equal masses ($m_1 = m_2 = m_3 = m$). Initially masses m_2 and m_3 are stationary and the spring is unstretched. Find



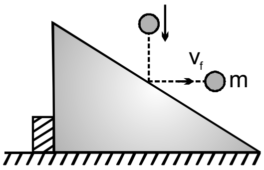
(a) the velocities of m_1, m_2 and m_3 immediately after impact.
(b) the maximum kinetic energy of m_3.
(c) the minimum kinetic energy of m_2.
(d) the maximum compression of the spring.

Section D - Impulse

7. A particle A of mass 2 kg lies on the edge of a table of height 1m. It is connected by a light inelastic string of length 0.7 m to a second particle B of mass 3 kg which is lying on the table 0.25 m from the edge (line joining A & B is perpendicular to the edge). If A is pushed gently so that it start falling from table then, find the speed of B when it starts to move. Also find the imulsive tension in the string at that moment.

Section E - Collision in 1D, Oblique collision

8. As shown in the figure a body of mass m moving vertically with speed 3 m/s hits a smooth fixed inclined plane and rebounds with a velocity v_f in the horizontal direction. If ∠ of inclined is 30°, the velocity v_f will be



(A) 3 m/s (B) $\sqrt{3}$ m/s
(C) $1/\sqrt{3}$ m/s (D) this is not possible

9. A particle is projected from point O on level ground towards a smooth vertical wall 50m from O and hits the wall. The initial velocity of the particle is 30m/s at 45° to the horizontal and the coefficient of restitution between the particle and the wall is e. Find the distance from O of the point at which the particle hits the ground again if (a) e = 0, (b) e = 1, (c) e = 1/2

1. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}$ M and, a body C of mass $\frac{2}{3}$ M. The centre of mass of bodies B and C taken together shifts compared to that of body A towards **[AIEEE 2005]**

(A) depends on height of breaking
(B) does not shift
(C) body C
(D) body B

2. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision. **[AIEEE 2005]**

(A) v (B) $\sqrt{3}v$
(C) $\frac{2}{\sqrt{3}}v$ (D) $\frac{v}{\sqrt{3}}$

3. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to **[AIEEE 2006]**

(A) 150 N (B) 3 N
(C) 30 N (D) 300 N

4. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position? **[AIEEE 2006]**

(A) $\frac{m_2}{m_1}d$ (B) $\frac{m_1}{m_1+m_2}d$
(C) $\frac{m_1}{m_2}d$ (D) d

5. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4 ms^{-1}. The kinetic energy of the other mass is **[AIEEE 2006]**

(A) 144 J (B) 288 J
(C) 192 J (D) 96 J

6. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to **[AIEEE 2006]**

(A) 150 N (B) 3 N
(C) 30 N (D) 300 N

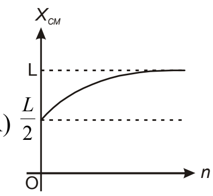
7. A circular disc of radius R is removed from a bigger circular disc of radius 2 R, such that the circumference of the discs coincide. The centre of mass of the new disc is αR from the centre of the bigger disc. The value of α is **[AIEEE 2007]**

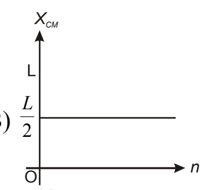
(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$

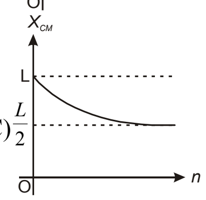
8. A block of mass 'm' is connected to another block of mass 'M' by a spring (mass less) of spring constant 'k'. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is stretched. Then a constant force 'F' starts acting on the block of mass 'M' to pull it. Find the force on the block of mass 'm'. **[AIEEE-2007]**

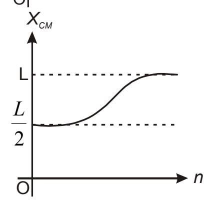
(A) $\frac{mF}{M}$ (B) $\frac{(M+m)F}{m}$
(C) $\frac{mF}{(m+M)}$ (D) $\frac{MF}{(m+M)}$

9. A thin rod of length L is lying along the x-axis with its ends at x = 0 and x = L. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n? **[AIEEE 2008]**

(A) 

(B) 

(C) 

(D) 

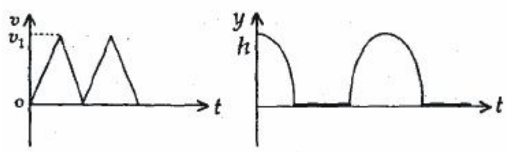
10. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[AIEEE 2008]**

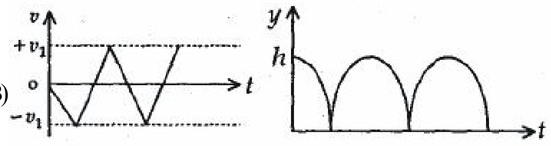
(A) 0.16 J (B) 1.00 J
(C) 0.67 J (D) 0.34 J

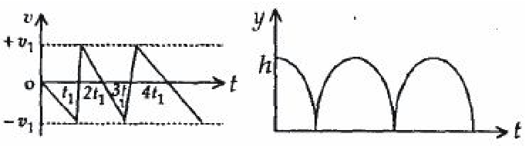
11. A body of mass m = 3.513 kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as **[AIEEE-2008]**

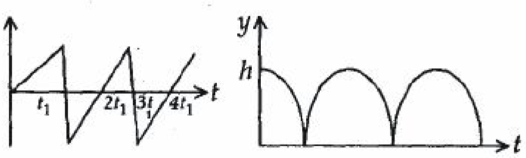
(A) 17.6 kg ms^{-1} (B) 17.565 kg ms^{-1}
(C) 17.56 kg ms^{-1} (D) 17.57 kg ms^{-1}

12. Consider a rubber ball freely falling from a height h = 4.9 m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then the velocity as a function of time the height as function of time will be **[AIEEE-2009]**

(A) 

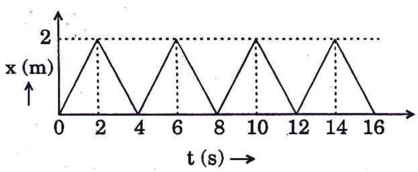
(B) 

(C) 

(D) 

13. The figure shows the position – time (x – t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[AIEEE-2010]**

(A) 0.4 Ns (B) 0.8 Ns
(C) 1.6 Ns (D) 0.2 Ns



14. **Statement I :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.
Statement II : Principle of conservation of momentum holds true for all kinds of collisions. **[AIEEE 2010]**

(A) Statement I is true, Statement II is ture; Statement II is the correct explanation of Statement I.
(B) Statement I is true, Statement II is true; Statement II is not correct explanation of Statement I.
(C) Statement I is false, Statement II is true.
(D) Statement I is the true, Statement II is false.

15. **Statement-I** : A point particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}m\upsilon^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.
Statement-II : Maximum energy loss occurs when the particles get stuck together as a result of the collision. **[JEE MAIN 2013]**

(A) Statement-I is true, Statement-II is false
(B) Statement-I is false, Statement-II is true.
(C) Statement-I is true, Statement-II is a correct explanation of Statement-I
(D) Statement-I is true, Statement-II is ture, Statement-II is not a correct explanation of Statement-I.

16. Distance of the centre of mass of a solid uniform cone from its vertex if z_0. If the radius of its base is R and its height is h then z_0 is equal to : **[JEE MAIN 2015]**

(A) $\frac{5h}{8}$ (B) $\frac{3h^2}{8R}$ (C) $\frac{h^2}{4R}$ (D) $\frac{3h}{4}$

17. A particle of mass m moving in the x direction with speed 2υ is hit by another particle of mass 2m moving in the y direction with speed υ. if the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to : **[JEE MAIN 2015]**

(A) 56% (B) 62% (C) 44% (C) 50%

18. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is : **[AIEEE-2018]**

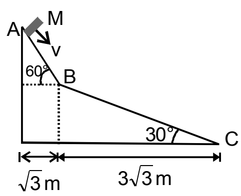
(A) $\frac{v_0}{\sqrt{2}}$ (B) $\frac{v_0}{4}$ (C) $\sqrt{2}v_0$ (D) $\frac{v_0}{2}$

1. **STATEMENT-1**
In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.
because
STATEMENT-2
In an elastic collision, the linear momentum of the system is conserved
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

2. **A particle moves in the X-Y plane under the influence of a force such that its linear momentum is** $p(t) = A\left[\hat{i}\cos(kt) - \hat{j}\sin(kt)\right]$**, where A and k are constants. The angle between the force and the momentum is - [JEE-2007]**
(A) 0° (B) 30° (C) 45° (D) 90°

3. The balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}'_1$ and $\vec{p}'_2$ be their final momenta. The following options(s) is (are) **NOT ALLOWED** for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2. **[JEE 2008]**
(A) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$; $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j}$
(B) $\vec{p}'_1 = c_1\hat{k}$; $\vec{p}'_2 = c_2\hat{k}$
(C) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$; $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$
(D) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j}$; $\vec{p}'_2 = a_2\hat{i} + b_1\hat{j}$

Paragraph for Question No. 4 to 6
A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ m/s^2$). Figure :
[JEE 2008]



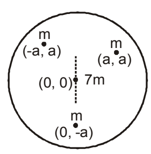
4. The speed of the block at point B immediately after it strikes the second incline is
(A) $\sqrt{60}\ m/s$ (B) $\sqrt{45}\ m/s$
(C) $\sqrt{30}\ m/s$ (D) $\sqrt{15}\ m/s$

5. The speed of the block at point C, immediately before it leaves the second incline is
(A) $\sqrt{120}\ m/s$ (B) $\sqrt{105}\ m/s$
(C) $\sqrt{90}\ m/s$ (D) $\sqrt{75}\ m/s$

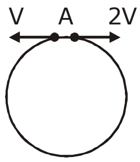
6. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is
(A) $\sqrt{30}\ m/s$ (B) $\sqrt{15}\ m/s$
(C) 0 (D) $-\sqrt{15}\ m/s$

7. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[JEE-2009]**
(A) linear momentum of the system does not change in time
(B) kinetic energy of the system does not change in time
(C) angular momentum of the system does not change in time
(D) potential energy of the system does not change in time

8. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink unsed to draw the outer circle is 6m. The coordinates of the centres of the different parts are outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, – a). The y-coordinate of the centre of mass of the ink in this drawing is
[JEE 2009]

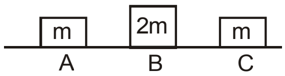


9. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circle orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particulars will again reach the point A ? **[JEE 2009]**



(A) 4 (B) 3
(C) 2 (D) 1

10. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, $2m$ and m, respectively. The object A moves towards B with a speed 9 ms^{-1} and makess an elastic collision with it. There after, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in ms^{-1}) of the object C. **[JEE 2009]**



11. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverse its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses ? **[JEE 2010]**

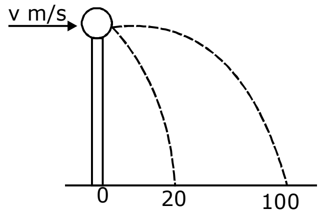
(A) Total momentum of the system is 3 kg ms^{-1}

(B) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(C) Kinetic energy of the centre of mass is 0.75 J

(D) Total kinetic energy of the system is 4 J

12. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is **[JEE 2011]**



(A) 250 m/s (B) $250\sqrt{2}$ m/s
(C) 400 m/s (D) 500 m/s

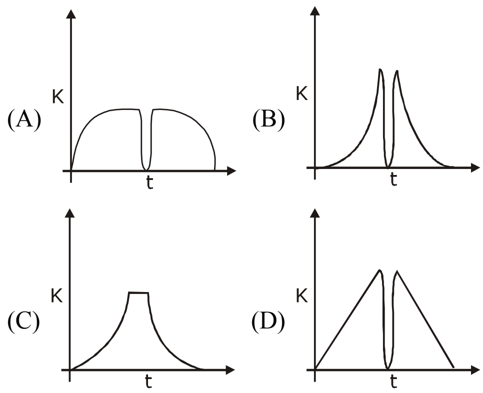
13. A particle of mass m is projected from the ground with an intial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same intial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **[IIT 2013]**

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{4}+\alpha$ (C) $\frac{\pi}{2}-\alpha$ (D) $\frac{\pi}{2}$

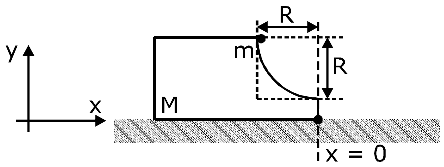
14. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is **[IIT 2013]**

(A) 0.3×10^{-17} kg ms^{-1}
(B) 1.0×10^{-17} kg ms^{-1}
(C) 3.0×10^{-17} kg ms^{-1}
(D) 9.0×10^{-17} kg ms^{-1}

15. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale. **[JEE Advanced 2014]**



16. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a co-ordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are correct? **[JEE Advanced 2017]**



(A) The velocity of the point mass m is : $v = \sqrt{\dfrac{2gR}{1+\dfrac{m}{M}}}$

(B) The x component of displacement of the center of mass of the block M is ; $-\dfrac{mR}{M+m}$

(C) The position of the point mass is : $x = -\sqrt{2}\dfrac{mR}{M+m}$

(D) The velocity of the block M is : $V = -\dfrac{m}{M}\sqrt{2gR}$

17. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true? **[JEE Advanced 2017]**

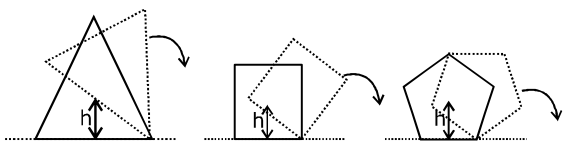
(A) The pressure difference between the leading and trailing faces of the plate is proportional to uv

(B) At a later time the external force F balances the resistive force

(C) The resistive force experienced by the plate is proportional to v

(D) The plate will continue to move with constant non-zero acceleration, at all times

18. Consider regular polygons with number of sides n = 3, 4, 5 ...... as shown in the figure, The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted,. The maximum increase in height of the locus of the center of mass for each polygton is Δ. Then Δ depends on n and h as **[JEE Advanced 2017]**



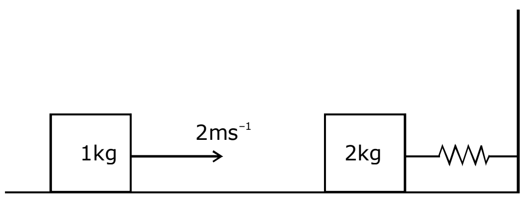
(A) $\Delta = h\sin^2\left(\dfrac{\pi}{n}\right)$

(B) $\Delta = h\tan^2\left(\dfrac{\pi}{2n}\right)$

(C) $\Delta = h\left(\dfrac{1}{\cos\left(\dfrac{\pi}{n}\right)} - 1\right)$

(D) $\Delta = h\sin\left(\dfrac{2\pi}{n}\right)$

19. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 $N\,m^{-1}$ and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 m s^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _________. **[JEE Advanced 2018]**



ANSWER KEYS

Exercise - 1 Objective Problems | JEE Main

1.	**C**	**2.**	**C**	**3.**	**C**	**4.**	**D**	**5.**	**B**	**6.**	**D**	**7.**	**A**
8.	**C**	**9.**	**A**	**10.**	**B**	**11.**	**A**	**12.**	**A**	**13.**	**B**	**14.**	**A**
15.	**C**	**16.**	**C**	**17.**	**C**	**18.**	**B**	**19.**	**D**	**20.**	**D**	**21.**	**D**
22.	**A**	**23.**	**A**	**24.**	**C**	**25.**	**A**	**26.**	**A**	**27.**	**C**	**28.**	**(a)B (b)C**
29.	**C**	**30.**	**A**	**31.**	**C**	**32.**	**D**	**33.**	**D**	**34.**	**A**	**35.**	**A**
36.	**C**	**37.**	**C**	**38.**	**D**	**39.**	**B**	**40.**	**D**	**41.**	**A**	**42.**	**C**
43.	**B**	**44.**	**B**	**45.**	**B**	**46.**	**C**	**47.**	**D**	**48.**	**B**	**49.**	**B**
50.	**A**	**51.**	**C**	**52.**	**B**	**53.**	**A**	**54.**	**C**	**55.**	**C**	**56.**	**C**
57.	**C**	**58.**	**B**	**59.**	**D**	**60.**	**A**	**61.**	**A**	**62.**	**A**	**63.**	**B**
64.	**C**	**65.**	**A**	**66.**	**C**	**67.**	**A**	**68.**	**C**	**69.**	**D**	**70.**	**D**
71.	**A**												

Exercise - 2 (Leve-I) Objective Problems | JEE Main

1.	**C**	**2.**	**D**	**3.**	**C**	**4.**	**B**	**5.**	**D**	**6.**	**B**	**7.**	**D**
8.	**D**	**9.**	**A**	**10.**	**A**	**11.**	**D**	**12.**	**A**	**13.**	**D**	**14.**	**A**
15.	**B**	**16.**	**A**	**17.**	**C**	**18.**	**A**	**19.**	**B**	**20.**	D	**21.**	**A**
22.	**D**	**23.**	**C**	**24.**	**D**	**25.**	**B**	**26.**	**D**	**27.**	**D**	**28.**	**B**
29.	**C**	**30.**	**C**	**31.**	**D**	**32.**	**B**	**33.**	**D**	**34.**	**D**	**35.**	**A**
36.	**B**	**37.**	**C**	**38.**	**A**	**39.**	**A**	**40.**	**B**	**41.**	**D**	**42.**	**C**

Exercise - 2 (Level-II) Multiple Correct | JEE Advanced

1.	**C,D**	**2.**	**B,D**	**3.**	**A,B**	**4.**	**B,C**	**5.**	**C,D**
6.	**A, B**	**7.**	**B**	**8.**	**B,D**	**9.**	**C**	**10.**	**C**
11.	**B,C**	**12.**	**B**	**13.**	**A,B,C,D**	**14.**	**A,C**	**15.**	**B,C**
16.	B,C	17.	B,D	18.	A,B,D	19.	A,B,C	20.	A,B,C
21.	AB	22.	B,C	23.	A,B,D	24.	B,C,D	25.	A,B,C
26.	B,C	27.	A,B,D	28.	A,B,C,D				

Exercise - 3 | Level-I Subjective | JEE Advanced

1. **(a)** $\lambda(x) = \lambda + \frac{\lambda x}{L}$**, (b)** $\frac{5}{9}L$ **2.** $y = \frac{4}{3\pi}\left[\frac{b^3 - a^3}{b^2 - a^2}\right]$ **3.** $\frac{3}{4}a$ **4.** **0016**

5. At R/5 from the centre of the bigger disc towards the centre of the smaller disc. **6.** (5a/6, 5a/6)

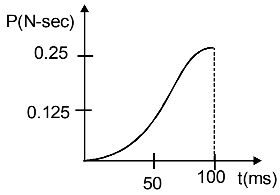
7. $\sqrt{\frac{13}{4}}$ **8.** **100 m**

9. **(i) zero; (ii) right ; (iii) 20 cm ; (iv) 2.2 m ; (v) 1.8 m**

10. **g/9 downwards** **11.** $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$ **12.** **9 m/s, 9 m**

13. $v = \frac{m_B v_0}{m_A + m_B}, h = \left[\frac{m_B v_0}{m_A + m_B}\right]\frac{v_0^2}{2g}$ **14.** 40 cm **15.** 2 **16.** **(a)** $\sqrt{\frac{kx_0^2}{2m}}$ **, (b)** $\frac{Kx_0^2}{4}$

17. $\frac{2}{5\sqrt{3}}$ m **18.** **(i) 20Ns, (ii) 50 N** **19.**



20. **187.5 N, 625 kPa** **21.** $m \times \sqrt{u^2 - uv + v^2}$

22. **(a)** $\frac{p_1 + p_2}{m_p} = 12.3\,m/sec$, **(b)** $\frac{\sqrt{p_1{}^2 + p_2{}^2}}{m_p}$ **=9.4 m/s**

23. **(a)** $4\sqrt{5}\,N$, **(b)** $2000\sqrt{5}\,N$ **24.** **0.2**

Exercise - 3 | Level-II Subjective | JEE Advanced

1. $0 \le b \le a$ **2.** (a) $\lambda(x) = \lambda_0 + \frac{\lambda_0 x}{L}$, (b) $\frac{5}{9}L$

3. **(a) 360 m, (b) 10800 J** **4.** $v_B = \frac{m_A}{m_B}\sqrt{\frac{2gl}{1 + m_A/m_B}}$ **; T = 3m$_A$g +** $\frac{2m_A^2 g}{m_B}$

5. **2mv²/3*l*** **6.** **(a) v/2, v/2, 0; (b) 2mv²/9, (c) mv²/72, (d)** $x = \sqrt{m/6k}\,v$

7. **1.5 m/s, 3.6 Ns** **8.** **B** **9.** **(a) 50 m, (b) 10 m, (c) 30 m**

Exercise - 4 | Level-I Previous Year | JEE Main

1.	**B**	**2.**	**C**	**3.**	**C**	**4.**	**C**	**5.**	**B**
6.	**C**	**7.**	**A**	**8.**	**C**	**9.**	**A**	**10.**	**C**
11.	**A**	**12.**	**C**	**13.**	**B**	**14.**	**A**	**15.**	**B**
16.	**D**	**17.**	**A**	**18.**	**C**				

Exercise - 4 | Level-II Previous Year | JEE Advanced

1.	**B**	**2.**	**D**	**3.**	**A,D**	**4.**	**B**	**5.**	**B**
6.	**C**	**7.**	**A**	**8.**	**a/10**	**9.**	**C**	**10.**	**4**
11.	**A,C**	**12.**	**D**	**13.**	**A**	**14.**	**B**	**15.**	**B**
16.	**A,B**	**17.**	**A,B,C**	**18.**	**C**	**19.**	**2**		