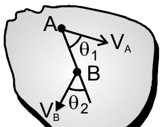
Rotational

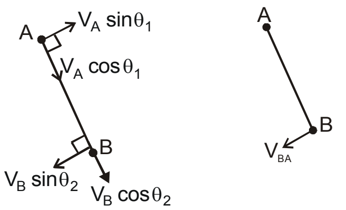
ROTATIONAL DYNAMICS

1. RIGID BODY :

Rigid body is defined as a system of particles in which distance between each pair of particles remains constant (with respect to time) that means the shape and size do not change, during the motion. Eg. Fan, Pen, Table, stone and so on.

Our body is not a rigid body, two blocks with a spring attached between them is also not a rigid body. For every pair of particles in a rigid body, there is no velocity of seperation or approach between the particles. In the figure shown velocities of A and B with respect to ground are $\vec{V}_A$ and $\vec{V}_B$ respectively





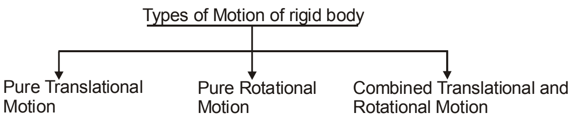
If the above body is rigid

$V_A \cos\theta_1 = V_B \cos\theta_2$

Note

With respect to any particle of rigid body the motion of any other particle of that rigid body is circular.

V_{BA} = relative velocity of B with respect to A.

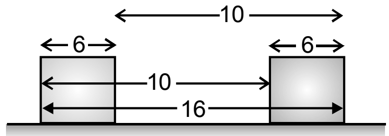


1.1. Pure Translational Motion :

A body is said to be in pure translational motion if the displacement of each particle is same during any time interval however small or large. In this motion all the particles have same $\vec{s}, \vec{v}$ & $\vec{a}$ at an instant.

example.

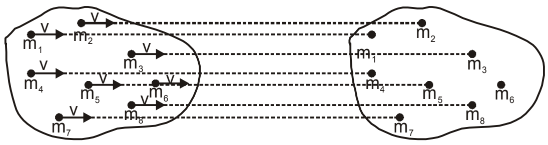
A box is being pushed on a horizontal surface.



$\vec{V}_{cm} = \vec{V}$ of any particle, $\vec{a}_{cm} = \vec{a}$ of any particle

$\Delta\vec{S}_{cm} = \Delta\vec{S}$ of any particle

For pure translational motion :-



$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$

Where m_1, m_2, m_3, ... are the masses of different particles of the body having accelerations $\vec{a}_1, \vec{a}_2, \vec{a}_3$,................ respectively.

But acceleration of all the particles are same So,

$\vec{a}_1 = \vec{a}_2 = \vec{a}_3 = ... = \vec{a}$

$$\vec{F}_{ext} = M\vec{a}$$

Where M = Total mass of the body

$\vec{a}$ = acceleration of any particle or of centre of mass of body

$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...$

Where m_1, m_2, m_3 ...... are the masses of different particles of the body having velocities $\vec{v}_1, \vec{v}_2, \vec{v}_3$............. respectively

But velocities of all the particles are same so

$\vec{v}_1 = \vec{v}_2 = \vec{v}_3 = ... = \vec{v}$

$\vec{P} = M\vec{v}$

Where $\vec{v}$ = velocity of any particle or of centre of mass of the body.

Total Kinetic Energy of body =

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + ... = \frac{1}{2}Mv^2$

1.2. Pure Rotational Motion :

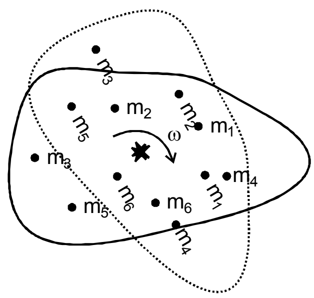
A body is said to be in pure rotational motion if the perpendicular distance of each particle remains constant from a fixed line or point and do not move parallel to the line, and that line is known as axis of rotation. In this motion all the particles have same $\vec{\theta}, \vec{\omega}$ and $\vec{\alpha}$ at an instant. Eg. : - a rotating ceiling fan, arms of a clock.

For pure rotation motion :-

$\theta = \frac{s}{r}$ Where θ = angle rotated by the particle

s = length of arc traced by the particle.

r = distance of particle from the axis of rotation.



$\omega = \frac{d\theta}{dt}$ Where ω = angular speed of the body.

$\alpha = \frac{d\omega}{dt}$ Where α = angular acceleration of the body.

All the parameters θ, ω and α are same for all the particles. Axis of rotation is perpendicular to the plane of rotation of particles.

Special case : If α = constant,

$\omega = \omega_0 + \alpha t$ Where ω_0 = initial angular speed

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ t = time interval

$\omega^2 = \omega_0^2 + 2\alpha\theta$

Total Kinetic Energy

$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + ...$

$= \frac{1}{2}[m_1r_1^2 + m_2r_2^2 + ...]\omega^2$

$= \frac{1}{2}I\omega^2$ Where I = Moment of Inertia

$= m_1r_1^2 + m_2r_2^2 + ...$

ω = angular speed of body.

1.3 Combined translation and rotational Motion

A body is said to be in translation and rotational motion if all the particles rotates about an axis of rotation and the axis of rotation moves with respect to the ground.

Section A - Moment of Inertia, Theorems of Moment of Inertia, Radius of gyration, Cavity problems

2. MOMENT OF INERTIA

Like the centre of mass, the moment of inertia is a property of an object that is related to its mass distribution. The moment of inertia (denoted by I) is an important quantity in the study of system of particles that are rotating. The role of the moment of inertia in the study of rotational motion is analogous to that of mass in the study of linear motion. Moment of inertia gives a measurement of the resistance of a body to a change in its rotaional motion. If a body is at rest, the larger the moment of inertia of a body the more difficuilt it is to put that body into rotational motion. Similarly, the larger the moment of inertia of a body, the more difficult to stop its rotational motion. The moment of inertia is calculated about some axis (usually the rotational axis).

Moment of inertia depends on :

(i) density of the material of body

(ii) shape & size of body

(iii) axis of rotation

In totality we can say that it depends upon distribution of mass relative to axis of rotation.

Note

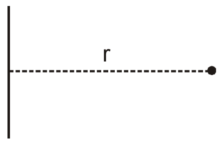
Moment of inertia does not change if the mass :

(i) is shifted parallel to the axis of the rotation

(ii) is rotated with constant radius about axis of rotation

2.1 Moment of Inertia of a Single Particle

For a very simple case the moment of inertia of a single particle about an axis is given by,



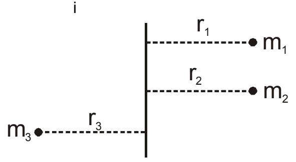
$$I = mr^2 \qquad ...(i)$$

Here, m is the mass of the particle and r its distance from the axis under consideration.

2.2 Moment of Inertia of a System of Particles

The moment of inertia of a system of particles about an axis is given by,

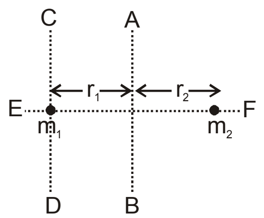
$$I = \sum_i m_i r_i^2 \qquad ...(ii)$$



where r_i is the perpendicular distance from the axis to the ith particle, which has a mass m_i.

EXAMPLE 1

Two heavy particles having masses m_1 & m_2 are situated in a plane perpendicular to line AB at a distance of r_1 and r_2 respectively.



(i) What is the moment of inertia of the system about axis AB?

(ii) What is the moment of inertia of the system about an axis passing through m_1 and perpendicular to the line joining m_1 and m_2 ?

(iii) What is the moment of inertia of the system about an axis passing through m_1 and m_2?

Sol. (i) Moment of inertia of particle on left is $I_1 = m_1r_1^2$.

Moment of Inertia of particle on right is $I_2 = m_2r_2^2$.

Moment of Inertia of the system about AB is

$I = I_1 + I_2 = m_1r_1^2 + m_2r_2^2$

(ii) Moment of inertia of particle on left is $I_1 = 0$

Moment of Inertia of the system about CD is

$I = I_1 + I_2 = 0 + m_2(r_1 + r_2)^2$

(iii) Moment of inertia of particle on left is $I_1 = 0$

Moment of inertia of particle on right is $I_2 = 0$

Moment of Inertia of the system about EF is

$I = I_1 + I_2 = 0 + 0$

EXAMPLE 2

Three light rods, each of length 2ℓ, are joined together to form a triangle. Three particles A, B, C of masses m, 2m, 3m are fixed to the vertices of the triangle. Find the moment of inertia of the resulting body about

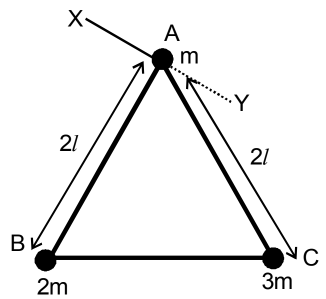
(a) an axis through A perpendicular to the plane ABC,

(b) an axis passing through A and the midpoint of BC.

Sol. (a) B is at a distant 2ℓ from the axis XY so the moment of

inertia of B (I_B) about XY is $2\ m\ (2\ell)^2$

Similarly I_c about XY is $3m\ (2\ell)^2$ and I_A about XY is $m(0)^2$



Therefore the moment of inertia of the body about XY is

$2m\ (2\ell)^2 + 3\ m(2\ell)^2 + m(0)^2 = 20\ m\ell^2$

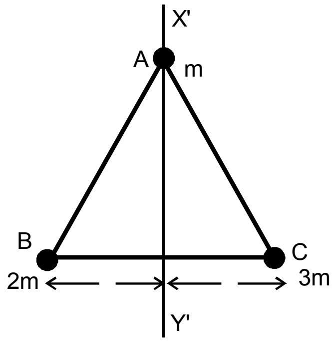
(b) I_A about X' Y' = $m(0)^2$

I_B about X' Y' = $2m\ (\ell)^2$

I_C about X' Y' = $3m\ (\ell)^2$

Therefore the moment of inertia of the body about X' Y' is

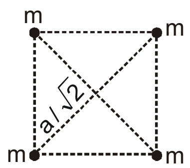
$m(0)^2 + 2m(\ell)^2 + 3m(\ell)^2 = 5\ m\ell^2$



EXAMPLE 3

Four particles each of mass m are kept at the four corners of a square of edge a. Find the moment of inertia of the system about a line perpendicular to the plane of the square and passing through the centre of the square.

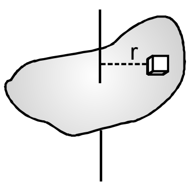
Sol. The perpendicular distance of every particle from the given line is $a/\sqrt{2}$. The moment of inertia of one particle is, therefore, $m(a/\sqrt{2})^2 = \frac{1}{2}ma^2$. The moment of inertia of the system is, therefore, $4 \times \frac{1}{2}ma^2 = 2\ ma^2$.



2.3 Moment of Inertia of Rigid Bodies

For a continuous mass distribution such as found in a rigid body, we replace the summation of $I = \sum_i m_i r_i^2$ by an integral. If the system is divided into infinitesimal element of mass dm and if r is the distance from a mass element to the axis of rotation, the moment of inertia is,

$$I = \int r^2\, dm$$



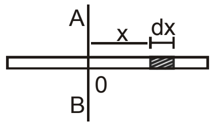
where the integral is taken over the system.

(A) Uniform rod about a perpendicular bisector

Consider a uniform rod of mass M and length l figure and suppose the moment of inertia is to be calculated about the bisector AB. Take the origin at the middle point O of the rod. Consider the element of the rod between a distance x and x + dx from the origin. As the rod is uniform,

Mass per unit length of the rod = M/l

so that the mass of the element = $(M/l)dx$.



The perpendicular distance of the element from the line AB is x. The moment of inertia of this element about AB is

$$dI = \frac{M}{l} dx\, x^2 .$$

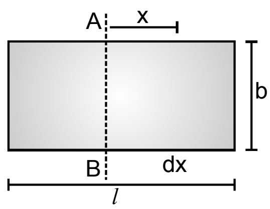
When $x = -l/2$, the element is at the left end of the rod. As x is changed from $-l/2$ to $l/2$, the elements cover the whole rod.

Thus, the moment of inertia of the entire rod about AB is

$$I = \int_{-l/2}^{l/2} \frac{M}{l} x^2 dx = \left[\frac{M x^3}{l\ 3}\right]_{-l/2}^{l/2} = \frac{Ml^2}{12}$$

(B) Moment of inertia of a rectangular plate about a line parallel to an edge and passing through the centre

The situation is shown in figure. Draw a line parallel to AB at a distance x from it and another at a distance x + dx. We can take the strip enclosed between the two lines as the small element.



It is "small" because the perpendiculars from different points of the strip to AB differ by not more than dx. As the plate is uniform,

$$\text{its mass per unit area} = \frac{M}{bl}$$

$$\text{Mass of the strip} = \frac{M}{bl} b\,dx = \frac{M}{l} dx .$$

The perpendicular distance of the strip from AB = x. The moment of inertia of the strip about AB = dI = $\frac{M}{l}dx\,x^2$. The moment of inertia of the given plate is, therefore,

$$I = \int_{-l/2}^{l/2} \frac{M}{l} x^2 dx = \frac{Ml^2}{12}$$

The moment of inertia of the plate about the line parallel to the other edge and passing through the centre may be obtained from the above formula by replacing *l* by b and thus,

$$I = \frac{Mb^2}{12}.$$

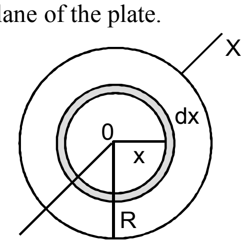
(C) *Moment of inertia of a circular ring about its axis (the line perpendicular to the plane of the ring through its centre)*

Suppose the radius of the ring is R and its mass is M. As all the elements of the ring are at the same perpendicular distance R from the axis, the moment of inertia of the ring is

$$I = \int r^2\,dm = \int R^2\,dm = R^2 \int dm = MR^2.$$

(D) *Moment of inertia of a uniform circular plate about its axis*

Let the mass of the plate be M and its radius R. The centre is at O and the axis OX is perpendicular to the plane of the plate.



Draw two concentric circles of radii x and x + dx, both centred at O and consider the area of the plate in between the two circles.

This part of the plate may be considered to be a circular ring of radius x. As the periphery of the ring is 2π x and its width is dx, the area of this elementary ring is 2πxdx. The area of the plate is π R^2. As the plate is uniform,

Its mass per unit area = $\frac{M}{\pi R^2}$

Mass of the ring $= \frac{M}{\pi R^2} 2\pi x dx = \frac{2Mxdx}{R^2}$

Using the result obtained above for a circular ring, the moment of inertia of the elementary ring about OX is

$$dI = \left[\frac{2Mxdx}{R^2}\right] x^2.$$

The moment of inertia of the plate about OX is

$$I = \int_0^R \frac{2M}{R^2} x^3 dx = \frac{MR^2}{2}.$$

(E) *Moment of inertia of a hollow cylinder about its axis*

Suppose the radius of the cylinder is R and its mass is M. As every element of this cylinder is at the same perpendicular distance R from the axis, the moment of inertia of the hollow cylinder about its axis is

$$I = \int r^2 dm = R^2 \int dm = MR^2$$

(F) *Moment of inertia of a uniform solid cylinder about its axis*

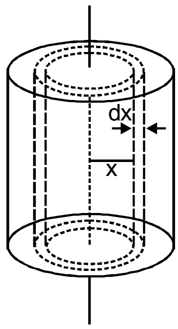
Let the mass of the cylinder be M and its radius R. Draw two cylindrical surface of radii x and x + dx coaxial with the given cylinder. Consider the part of the cylinder in between the two surface. This part of the cylinder may be considered to be a hollow cylinder of radius x. The area of cross-section of the wall of this hollow cylinder is 2π x dx. If the length of the cylinder is *l*, the volume of the material of this elementary hollow cylinder is 2π x dx*l*.

The volume of the solid cylinder is π R^2 *l* and it is uniform, hence its mass per unit volume is

$$\rho = \frac{M}{\pi R^2 l}$$

The mass of the hollow cylinder considered is

$$\frac{M}{\pi R^2 l} 2\pi x dx\, l = \frac{2M}{R^2} x dx.$$



As its radius is x, its moment of inertia about the given axis is

$$dI = \left[\frac{2M}{R^2} x\,dx\right] x^2 .$$

The moment of inertia of the solid cylinder is, therefore,

$$I = \int_0^R \frac{2M}{R^2} x^3 dx = \frac{MR^2}{2} .$$

Note that the formula does not depend on the length of the cylinder.

(G) *Moment of inertia of a uniform hollow sphere about a diameter*

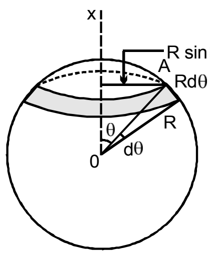
Let M and R be the mass and the radius of the sphere, O its centre and OX the given axis (figure). The mass is spread over the surface of the sphere and the inside is hollow.

Let us consider a radius OA of the sphere at an angle θ with the axis OX and rotate this radius about OX. The point A traces a circle on the sphere. Now change θ to θ + dθ and get another circle of somewhat larger radius on the sphere. The part of the sphere between these two circles, shown in the figure, forms a ring of radius R sinθ. The width of this ring is Rdθ and its periphery is 2πR sinθ. Hence, the area of the ring = (2πR sinθ) (Rdθ).

Mass per unit area of the sphere $= \frac{M}{4\pi R^2}$.

The mass of the ring

$$= \frac{M}{4\pi R^2}(2\pi R\sin\theta)(Rd\theta) = \frac{M}{2}\sin\theta\, d\theta.$$



The moment of inertia of this elemental ring about OX is

$$dI = \left(\frac{M}{2}\sin\theta\, d\theta.\right)(R\sin\theta)^2 = \frac{M}{2}R^2 \sin^3\theta\, d\theta$$

As θ increases from 0 to π, the elemental rings cover the whole spherical surface. The moment of inertia of the hollow sphere is, therefore,

$$I = \int_0^\pi \frac{M}{2}R^2\sin^3\theta\, d\theta = \frac{MR^2}{2}\left[\int_0^\pi (1-\cos^2\theta)\sin\theta\, d\theta\right]$$

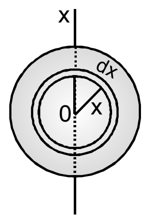
$$= \frac{MR^2}{2}\left[\int_{\theta=0}^{\pi} -(1-\cos^2\theta)\, d(\cos\theta)\right]$$

$$= \frac{-MR^2}{2}\left[\cos\theta - \frac{\cos^3\theta}{3}\right]_0^\pi = \frac{2}{3}MR^2$$

(H) *Moment of inertia of a uniform solid sphere about a diameter*

Let M and R be the mass and radius of the given solid sphere. Let O be centre and OX the given axis. Draw two spheres of radii x and x + dx concentric with the given solid sphere. The thin spherical shell trapped between these spheres may be treated as a hollow sphere of radius x.

The mass per unit volume of the solid sphere



$$= \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$$

The thin hollow sphere considered above has a surface area $4\pi x^2$ and thickness dx. Its volume is $4\pi x^2 dx$ and hence its mass is

$$= \left(\frac{3M}{4\pi R^3}\right)(4\pi x^2 dx) = \frac{3M}{R^3} x^2 dx$$

Its moment of inertia about the diameter OX is, therefore,

$$dI = \frac{2}{3}\left[\frac{3M}{R^3}x^2 dx\right]x^2 = \frac{2M}{R^3}x^4 dx$$

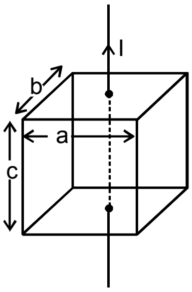
If x = 0, the shell is formed at the centre of the solid sphere. As x increases from 0 to R, the shells cover the whole solid sphere.

The moment of inertia of the solid sphere about OX is, therefore,

$$I = \int_0^R \frac{2M}{R^3}x^4 dx = \frac{2}{5}MR^2 .$$

EXAMPLE 4

Find the moment of Inertia of a cuboid along the axis as shown in the figure.



Sol. After compressing the cuboid parallel to the axis I

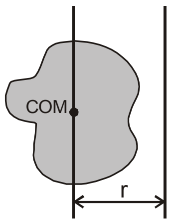
$$= \frac{M(a^2 + b^2)}{12}$$

3. THEOREMS OF MOMENT OF INERTIA

There are two important theorems on moment of inertia, which, in some cases enable the moment of inertia of a body to be determined about an axis, if its moment of inertia about some other axis is known. Let us now discuss both of them.

3.1 Theorem of parallel axes

A very useful theorem, called the parallel axes theorem relates the moment of inertia of a rigid body about two parallel axes, one of which passes through the centre of mass.



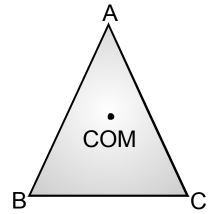
Two such axes are shown in figure for a body of mass M. If r is the distance between the axes and I_{COM} and I are the respective moments of inertia about them, these moments are related by,

$$I = I_{COM} + Mr^2$$

* Theorem of parallel axis is applicable for any type of rigid body whether it is a two dimensional or three dimensional

EXAMPLE 5

Three rods each of mass m and length l are joined together to form an equilateral triangle as shown in figure. Find the moment of inertia of the system about an axis passing through its centre of mass and perpendicular to the plane of triangle.

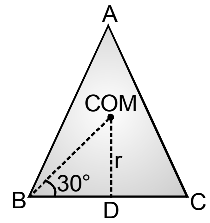


Sol. Moment of inertia of rod BC about an axis perpendicular to plane of triangle ABC and passing through the mid-point of rod BC (i.e., D) is

$$I_1 = \frac{ml^2}{12}$$

From theorem of parallel axes, moment of inertia of this rod about the asked axis is

$$I_2 = I_1 + mr^2 = \frac{ml^2}{12} + m\left(\frac{l}{2\sqrt{3}}\right)^2 = \frac{ml^2}{6}$$

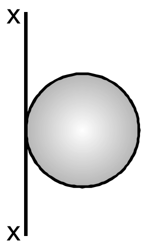


∴ Moment of inertia of all the three rod is

$$I = 3I_2 = 3\left(\frac{ml^2}{6}\right) = \frac{ml^2}{2}$$

EXAMPLE 6

Find the moment of inertia of a solid sphere of mass M and radius R about an axis XX shown in figure.

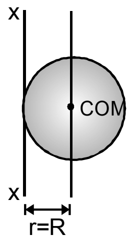


Sol. From theorem of parallel axis,

$I_{XX} = I_{COM} + Mr^2$

$= \frac{2}{5}MR^2 + MR^2$

$= \frac{7}{5}MR^2$



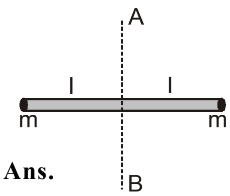
EXAMPLE 7

Consider a uniform rod of mass m and length 2l with two particles of mass m each at its ends. Let AB be a line perpendicular to the length of the rod passing through its centre. Find the moment of inertia of the system about AB.

Sol. $I_{AB} = I_{rod} + I_{both\ particles}$

$= \frac{m(2l)^2}{12} + 2(ml^2)$

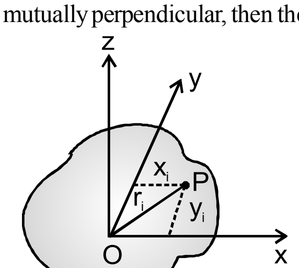
$= \frac{7}{3}ml^2$ **Ans.**



3.2 Theorem of perpendicular axes

The theorem states that the moment of inertia of a plane lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about two axes perpendicular to each other, in its own plane and intersecting each other, at the point where the perpendicular axis passes through it.

Let x and y axes be chosen in the plane of the body and z-axis perpendicular, to this plane, three axes being mutually perpendicular, then the theorem states that.



$I_z = I_x + I_y$

Important point in perpendicular axis theorem

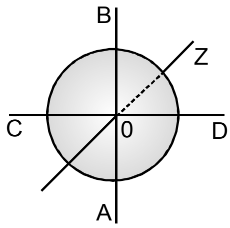
(i) This theorem is applicable only for the plane bodies (two dimensional).

(ii) In theorem of perpendicular axes, all the three axes (x, y and z) intersect each other and this point may be any point on the plane of the body (it may even lie outside the body).

(iii) Intersection point may or may not be the centre of mass of the body.

EXAMPLE 8

Find the moment of inertia of uniform ring of mass M and radius R about a diameter.



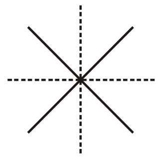
Sol. Let AB and CD be two mutually perpendicular diameters of the ring. Take them ax X and Y-axes and the line perpendicular to the plane of the ring through the centre as the Z-axis. The moment of inertia of the ring about the Z-axis is $I = MR^2$. As the ring is uniform, all of its diameter equivalent and so $I_x = I_y$, From perpendicular axes theorem,

$I_z = I_x + I_y$ Hence $I_x = \frac{I_z}{2} = \frac{MR^2}{2}$

Similarly, the moment of inertia of a uniform disc about a diameter is $MR^2/4$

EXAMPLE 9

Two uniform identical rods each of mass M and length ℓ are joined to form a cross as shown in figure. Find the moment of inertia of the cross about a bisector as shown dotted in the figure.

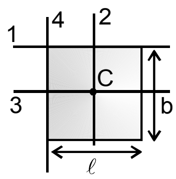


Sol. Consider the line perpendicular to the plane of the figure through the centre of the cross. The moment of inertia of each rod about this line is $\frac{M\ell^2}{12}$ and hence the moment of inertia of the cross is $\frac{M\ell^2}{6}$.

The moment of inertia of the cross about the two bisector are equal by symmetry and according to the theorem of perpendicular axes, the moment of inertia of the cross about the bisector is $\frac{M\ell^2}{12}$.

EXAMPLE 10

In the figure shown find moment of inertia of a plate having mass M, length ℓ and width b about axis 1,2,3 and 4. Assume that C is centre and mass is uniformly distributed



Sol. Moment of inertia of the plate about axis 1 (by taking rods perpendicular to axis 1)

$$I_1 = Mb^2/3$$

Moment of inertia of the plate about axis 2 (by taking rods perpendicular to axis 2)

$$I_2 = M\ell^2/12$$

Moment of inertia of the plate about axis 3 (by taking rods perpendicular to axis 3)

$$I_3 = \frac{Mb^2}{12}$$

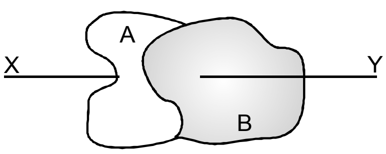
Moment of inertia of the plate about axis 4(by taking rods perpendicular to axis 4)

$$I_4 = M\ell^2/3$$

3.3 Moment of Inertia of Compound Bodies

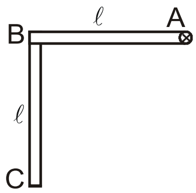
Consider two bodies A and B, rigidly joined together. The moment of inertia of this compound body, about an axis XY, is required. If I_A is the moment of inertia of body A about XY. I_B is the moment of inertia of body B about XY.Then, moment of Inertia of compound body $I = I_A + I_B$

Extending this argument to cover any number of bodies rigidly joined together, we see that the moment of inertia of the compound body, about a specified axis, is the sum of the moments of inertia of the separate parts of the body about the same axis.



EXAMPLE 11

Two rods each having length l and mass m joined together at point B as shown in figure.Then findout moment of inertia about axis passing thorugh A and perpendicular to the plane of page as shown in figure.



Sol. We find the resultant moment of inertia I by dividing in two parts such as

I = M.I of rod AB about A +

M.I of rod BC about A

$$I = I_1 + I_2 \quad \text{... (1)}$$

first calculate I_1 :

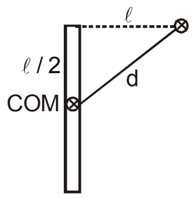
$$I_1 = \frac{m\ell^2}{3} \quad \text{...(2)}$$

Calculation of I_2 :

use parallel axis theorem

$$I_2 = I_{CM} + md^2$$

$$= \frac{m\ell^2}{12} + m\left(\frac{\ell^2}{4} + \ell^2\right) = \frac{m\ell^2}{12} + \frac{5\ell^2}{4}m \quad \text{...(3)}$$



Put value from eq. (2) & (3) into (1)

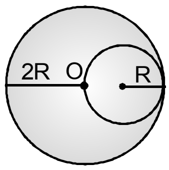
$$\Rightarrow I = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + \frac{5\ell^2 m}{4}$$

$$I = \frac{m\ell^2}{12}(4+1+15) \Rightarrow I = \frac{5m\ell^2}{3}$$

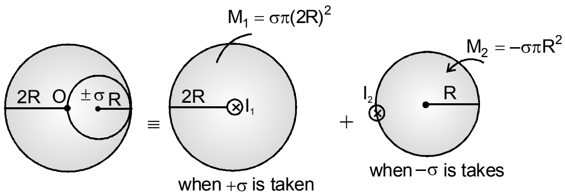
4. CAVITY PROBLEMS :

EXAMPLE 12

A uniform disc having radius 2R and mass density σ as shown in figure. If a small disc of radius R is cut from the disc as shown. Then find out the moment of inertia of remaining disc around the axis that passes through O and is perpendicular to the plane of the page.



Sol. We assume that in remaning part a disc of radius R and mass density ± σ is placed. Then



Total Moment of Inertia $I = I_1 + I_2$

$$I_1 = \frac{M_1(2R)^2}{2}$$

$$I_1 = \frac{\sigma\pi 4R^2.4R^2}{2} = 8\,\pi\,\sigma\,R^4$$

To calculate I_2 we use parallel axis theorem.

$$I_2 = I_{CM} + M_2R^2$$

$$I_2 = \frac{M_2R^2}{2} + M_2R^2$$

$$I_2 = \frac{3}{2}M_2R^2 = \frac{3}{2}(-\sigma\pi R^2)R^2 \qquad I_2$$

$$= -\frac{3}{2}\sigma\pi R^4$$

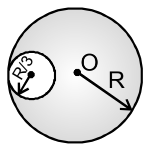
Now $I = I_1 + I_2$

$$I = 8\pi\sigma R^4 - \frac{3}{2}\sigma\pi R^4$$

$$I = \frac{13}{2}\sigma\pi R^4$$

EXAMPLE 13

A uniform disc of radius R has a round disc of radius R/3 cut as shown in Fig. The mass of the remaining (shaded) portion of the disc equals M. Find the moment of inertia of such a disc relative to the axis passing through geometrical centre of original disc and perpendicular to the plane of the disc.



Sol. Let the mass per unit area of the material of disc be σ. Now the empty space can be considered as having density – σ and σ.

Now $I_0 = I_\sigma + I_{-\sigma}$

$I_\sigma = (\sigma\pi R^2)R^2/2$ = M.I of σ about O

$$I_{-\sigma} = \frac{-\sigma\pi(R/3)^2(R/3)^2}{2} + [-\sigma\pi(R/3)^2](2R/3)^2$$

= M.I of – σ about 0

$$\therefore \qquad I_0 = \frac{4}{9}\sigma\pi R^4 \quad \textbf{Ans.}$$

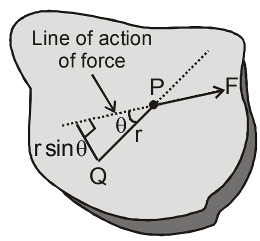
Note

The student can now attempt section A from exercise.

Section B - Torque (about point, about axis), Torque and angular Acceleration

5. TORQUE :

Torque represents the capability of a force to produce change in the rotational motion of the body



5.1 Torque about point :

Torque of force $\vec{F}$ about a point $\vec{\tau} = \vec{r} \times \vec{F}$

where $\vec{F}$ = force applied

P = point of application of force

Q = point about which we want to calculate the torque.

$\vec{r}$ = position vector of the point of application of force from the point about which we want to determine the torque.

$|\vec{\tau}| = rF\sin\theta \quad = r_{\perp}F \quad = rF_{\perp}$

where θ = angle between the direction of force and the position vector of P wrt. Q.

$r_{\perp}$ = perpendicular distance of line of action of force from point Q.

$F_{\perp}$ = force arm

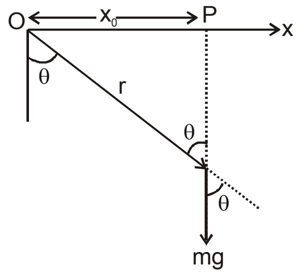
SI unit to torque is Nm

Torque is a vector quantity and its direction is determined using right hand thumb rule.

EXAMPLE 14

A particle of mass M is released in vertical plane from a point P at $x = x_0$ on the x-axis it falls vertically along the y-axis. Find the torque τ acting on the particle at a time t about origin?

Sol.



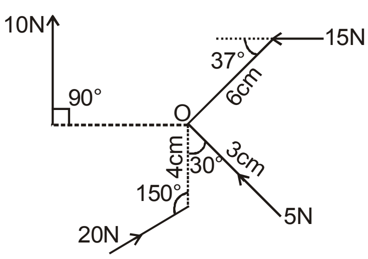
Torque is produced by the force of gravity

$\vec{\tau} = rF\sin\theta\,\hat{k}$

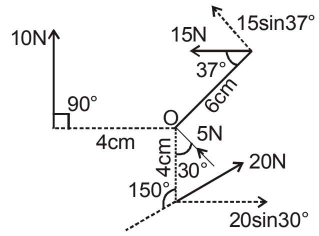
or $\tau = r_{\perp}F = x_0\,mg$

EXAMPLE 15

Calculate the total torque acting on the body shown in figure about the point O



Sol.



$\tau_0 = 15\sin37 \times 6\odot + 20\sin30° \times 4\odot - 10 \times 4\odot$

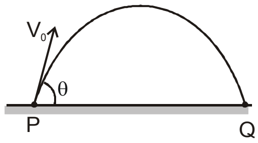
$= 54 + 40 - 40 = 54$ N-cm

$\tau_0 = 0.54$ N-m

EXAMPLE 16

A particle having mass m is projected with a velocity v_0 from a point P on a horizontal ground making an angle θ with horizontal. Find out the torque about the point of projection acting on the particle when

(a) it is at its maximum height ?

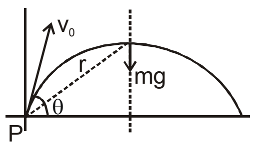


(b) It is just about to hit the ground back ?

Sol.

(a) Particle is at maximum height then τ about point P is $\tau_p = r_{\perp}F$

$F = mg\ ;\ r_{\perp} = \frac{R}{2}$



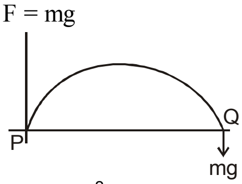
$\Rightarrow \tau_P = \frac{R}{2}mg = mg \times \frac{v_0^2\sin2\theta}{2g}$

$\tau_p = \frac{mv_0^2\sin2\theta}{2}$

(b) when particle is at point Q then τ about point P is

$\tau_p' = r_{\perp}F$

$r_{\perp} = R\ ;\ F = mg$



$\tau_p' = mgR = mg\frac{v_0^2\sin2\theta}{g}$

EXAMPLE 17

In the previous question, during the motion of particle from P to Q. Torque of gravitational force about P is :

(A) increasing

(B) decreasing

(C) remains constant

(D) first increasing then decreasing

Sol. Torque of gravitational force about P is increasing because $r_\perp$ is increasing from O to R. (Range)

5.2 Torque about axis :

$$\vec{\tau} = \vec{r} \times \vec{F}$$

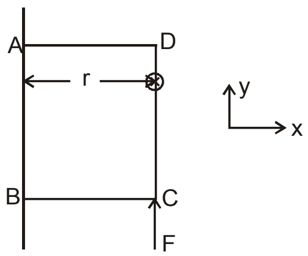
where $\vec{\tau}$ = torque acting on the body about the axis of rotation

$\vec{r}$ = position vector of the point of application of force about the axis of rotation.

$\vec{F}$ = force applied on the body.

$$\vec{\tau}_{net} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3 + ...$$

To understand the concept of torque about axis we take a general example which comes out in daily life. Figure shows a door ABCD. Which can rotate about
axis AB. Now if we apply force. F at point.



in inward direction then τ_{AB} = r F and direction of this τ_{AB} is along y axis from right hand thumb rule. Which
is parallel to AB so gives the resultant torque.

Now we apply force at point C in the direction as shown figure. At this time $\vec{r}$ & $\vec{F}$ are perpendicular to each other which gives

$$\tau_{AB} = rF$$

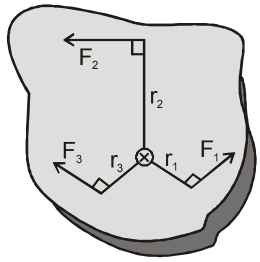
But door can't move when force is applied in this direction because the direction of τ_{AB} is perpendicular to AB according to right hand thumb rule.

So there is no component of τ along AB which gives

$\tau_{res} = 0$

Now conclude Torque about axis is the component of $\vec{r} \times \vec{F}$ parallel to axis of rotation.

Note : The direction of torque is calculated using right hand thumb rule and it is always perpendicular to the plane of rotation of the body.



If F_1 or F_2 is applied to body, body revolves in anti-clockwise direction and F_3 makes body revolve in clockwise direction. If all three are applied.

$\vec{\tau}_{resultant} = F_1r_1 + F_2r_2 - F_3r_3$ (in anti-clockwise direction)

6. BODY IS IN EQUILIBRIUM : -

We can say rigid body is in equillibrium when it is in

(a) Translational equilibrium

i.e. $\vec{F}_{net} = 0$

$F_{net\,x} = 0$ and $F_{net\,y} = 0$ and

(b) Rotational equillibrium

$\vec{\tau}_{net} = 0$

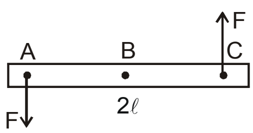
i.e., torque about any point is zero

Note

(i)If net force on the body is zero then net torque of the forces may or may not be zero.

example.

A pair of forces each of same magnitude and acting in opposite direction on the rod.



$\tau_A = 2F\ell \odot$

(2) If net force on the body is zero then torque of the forces about each and every point is same

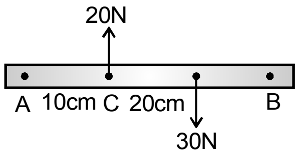
τ about B $\quad \tau_B = F\ell + F\ell \odot$

$\tau_B = 2F\ell \odot$

τ about C $\quad \tau_C = 2F\ell \odot$

EXAMPLE 18

Determine the point of application of third force for which body is in equillibrium when forces of 20 N & 30 N are acting on the rod as shown in figure



Sol. Let the magnitude of third force is F, is applied in upward direction then the body is in the equilibrium when

(i) $\vec{F}_{net} = 0$ (Translational Equillibrium)

$\Rightarrow$ 20 + F = 30 $\Rightarrow$ F = 10 N

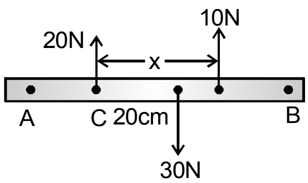
So the body is in translational equilibrium when 10 N force act on it in upward direction.

(ii) Let us assume that this 10 N force act.

Then keep the body in rotational equilibrium

So Torque about C = 0

i.e. $\tau_c = 0$



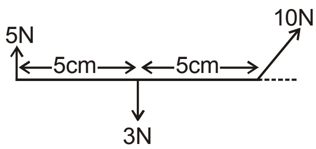
$\Rightarrow 30 \times 20 = 10\,x$

$x = 60$ cm

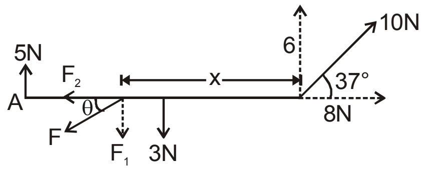
so 10 N force is applied at 70 cm from point A to keep the body in equilibrium.

EXAMPLE 19

Determine the point of application of force, when forces are acting on the rod as shown in figure.



Sol. ***Since the body is in equillibrium so we conclude*** $\vec{F}_{net} = 0$ ***and torque about any point is zero i.e.,*** $\vec{\tau}_{net} = 0$



Let us assume that we apply F force downward at A angle θ from the horizontal, at x distance from B

From $\vec{F}_{net} = 0$

$\Rightarrow$ $F_{net}\, x = 0$ which gives

$F_2 = 8$ N

From $F_{net\ y} = 0 \Rightarrow 5 + 6 = F_1 + 3$

$\Rightarrow$ $F_1 = 8$ N

If body is in equillibrium then torque about point B is zero.

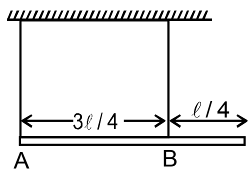
$\Rightarrow$ $3 \times 5 + F_1 .\, x - 5 \times 10 = 0$

$\Rightarrow$ $15 + 8x - 50 = 0$

$x = \dfrac{35}{9} \Rightarrow x = 4.375$ cm

EXAMPLE 20

A uniform rod length ℓ, mass m is hung from two strings of equal length from a ceiling as shown in figure. Determine the tensions in the strings ?

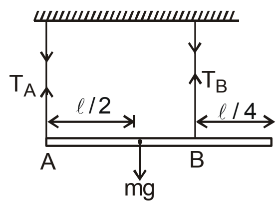


Sol. Let us assume that tension in left and right string is T_A and T_B respectively. Then

Rod is in equilibrium then $\vec{F}_{net} = 0$ & $\vec{\tau}_{net} = 0$

From $\vec{F}_{net} = 0$

$mg = T_A + T_B$ ...(1)



From $\tau_{net} = 0$ about A

$$mg\frac{\ell}{2} - \frac{3\ell}{4}T_B = 0$$

$$\Rightarrow \quad T_B = \frac{2mg}{3}$$

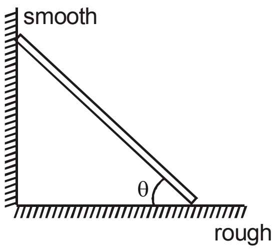
from eq. (1) $T_A + \frac{2mg}{3} = mg$

$$\Rightarrow \quad T_A = \frac{mg}{3}$$

Ladder Problems :

EXAMPLE 21

A stationary uniform rod of mass 'm', length 'ℓ' leans against a smooth vertical wall making an angle θ with rough horizontal floor. Find the normal force & frictional force that is exerted by the floor on the rod?

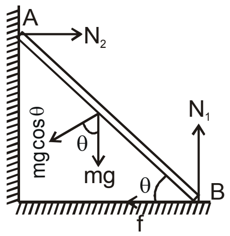


Sol. As the rod is stationary so the linear acceleration and angular acceleration of rod is zero.

i.e., $a_{cm} = 0$; $\alpha = 0$.

$$\left.\begin{matrix} N_2 = f \\ N_1 = mg \end{matrix}\right\} \because a_{cm} = 0$$

Torque about any point of the rod should also be zero



Free Body Diagram

$\therefore \alpha = 0$

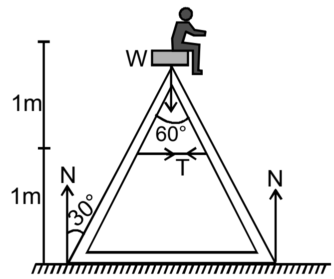
$$\tau_A = 0 \Rightarrow mg\cos\theta\,\frac{\ell}{2} + f\,\ell\sin\theta = N_1\cos\theta.\,\ell$$

$$N_1\cos\theta = \sin\theta\, f + \frac{mg\cos\theta}{2}$$

$$f = \frac{mg\cos\theta}{2\sin\theta} = \frac{mg\cot\theta}{2}$$

EXAMPLE 22

The ladder shown in figure has negligible mass and rests on a frictionless floor. The crossbar connects the two legs of the ladder at the middle. The angle between the two legs is 60°. The fat person sitting on the ladder has a mass of 80 kg. Find the contanct force exerted by the floor on each leg and the tension in the crossbar.



Sol. The forces acting on different parts are shown in figure. Consider the vertical equilibrium of "the ladder plus the person" system. The forces acting on this system are its weight (80 kg) g and the contact force N + N = 2 N due to the floor. Thus

2 N = (80 kg) g or N = (40 kg) (9.8 m/s^2) = 392 N

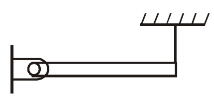
Next consider the equilibrium of the left leg of the ladder. Taking torques of the forces acting on it about the upper end,

$$N\,(2m)\tan 30° = T\,(1m)$$

or $$T = N\frac{2}{\sqrt{3}} = (392\text{ N}) \times \frac{2}{\sqrt{3}} = 450\text{ N}$$

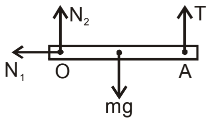
EXAMPLE 23

A thin plank of mass m and length ℓ is pivoted at one end and it is held stationary in horizontal position by means of a light thread as shown in the figure then find out the force on the pivot.



Sol. Free body diagram of the plank is shown in figure.

$\therefore$ Plank is in equilibrium condition



So F_{net} & τ_{net} on the plank is zero

(i) from $F_{net} = 0$

$\Rightarrow F_{net\ x} = 0$

$N_1 = 0$

Now $F_{net_y} = 0$

$\Rightarrow N_2 + T = mg$ ...(i)

from $\tau_{net} = 0$

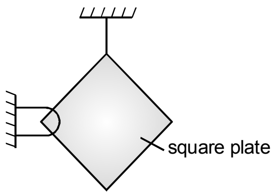
$\Rightarrow \tau_{net}$ about point A is zero

so $N_2 \cdot \ell = mg \cdot \ell/2$

$$\Rightarrow N_2 = \frac{mg}{2}$$

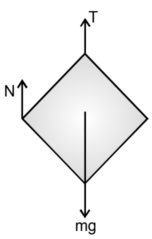
EXAMPLE 24

A square plate is hinged as shown in figure and it is held stationary by means of a light thread as shown in figure. Then find out force exerted by the hinge.



Sol. F.B.D.

$\because$ Body is in equilibrium and



T and mg force passing through one line so

from $\tau_{net} = 0$, N = 0

7. RELATION BETWEEN TORQUE AND ANGULAR ACCELERATION

The angular acceleration of a rigid body is directly proportional to the sum of the torque components along the axis of rotation. The proportionality constant is the inverse of the moment of inertia about that axis, or

$$\alpha = \frac{\Sigma\tau}{I}$$

Thus, for a rigid body we have the rotational analog of Newton's second law ;

$$\Sigma\tau = I\alpha \qquad ...(iii)$$

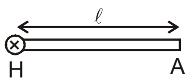
Following two points are important regarding the above equation.

(i) The above equation is valid only for rigid bodies. If the body is not rigid like a rotating tank of water, the angular acceleration α is different for different particles.

(ii) The sum $\Sigma\tau$ in the above equation includes only the torques of the external forces, because all the internal torques add to zero.

EXAMPLE 25

A uniform rod of mass m and length ℓ can rotate in vertical plane about a smooth horizontal axis hinged at point H.



(i) Find angular acceleration α of the rod just after it is released from initial horizontal position from rest?

(ii) Calculate the acceleration (tangential and radial) of point A at this moment.

Sol. (i) $\tau_H = I_H \alpha$

$$mg.\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha \Rightarrow \alpha = \frac{3g}{2\ell}$$

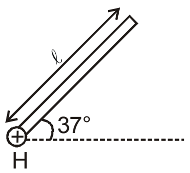
(ii) $a_{\tau A} = a\ell = \frac{3g}{2\ell}.\ell = \frac{3g}{2}$

$a_{CA} = \omega^2 r = 0.\ell = 0$

($\therefore$ $\omega = 0$ just after release)

EXAMPLE 26

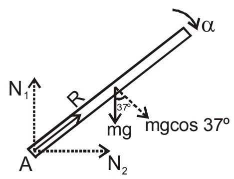
A uniform rod of mass m and length ℓ hinged at point H can rotate in vertical plane about a smooth horizontal axis. Find force exerted by the hinge just after the rod is released from rest, from an initial position making an angle of 37° with horizontal ?



Sol. Just After releasing at 37° from horizontal F.B.D. of plank

from $\tau_{net} = I\alpha$

τ about point A = τ_A = mg cos 37° $\frac{\ell}{2} = \frac{m\ell^2}{3}.\alpha$



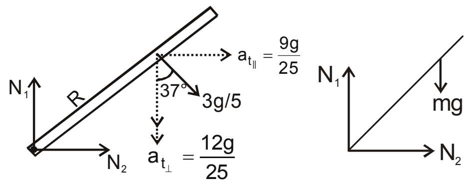
$\Rightarrow \alpha = \frac{6g}{5\ell}$ rad/sec^2

Now Tangential acceleration of centre of mass

$a_t = \alpha.\frac{\ell}{2} = \frac{3g}{5} m/s^2$

just after release $v_{cm} = 0 \Rightarrow \quad a_r = 0$

Now resolving of a_t in horizontal and vertical direction as shwon in figure



from F_{net} = ma in both horizontal and vertical direction

$$N_2 = m\left(\frac{9g}{25}\right) \quad \Rightarrow \quad N_1 = \frac{13mg}{25}$$

Now $\quad R = \sqrt{N_1^2 + N_2^2}$

$$R = \frac{mg\sqrt{10}}{5}$$

Note

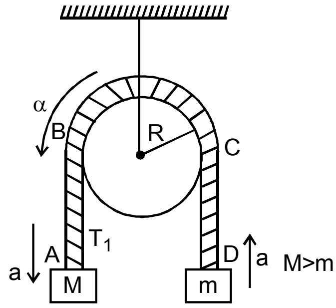
The student can now attempt section B from exercise.

Section C - Pulley Block system

PULLEY BLOCK SYSTEM

If there is friction between pulley and string and pulley have some mass then tension is different on two sides of the pulley.

Reason : To understand this concept we take a pulley block system as shown in figure.

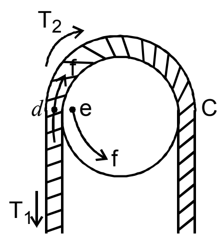


Let us assume that tension induced in part AB of the string is T_1 and block M move downward. If friction is present between pulley and string then it opposes the relative slipping between pulley and string, take two point e and *f* on pulley and string respectively. If friction is there then due to this, both points wants to move together. So friction force act on e and d in the direction as shown is figure

This friction force f acting on point d increases the tension T_1 by a small amount dT.

Then $\quad T_1 = T_2 + dT$

or we can say $T_2 = T_1 - f$



In this way the tension on two side of pulley is different If there is no relative slipping between pulley and string then $\alpha = \frac{a_t}{R} = \frac{a}{R}$

EXAMPLE 27

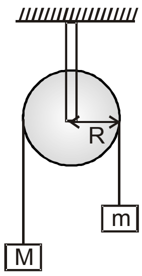
The pulley shown in figure has moment of inertia l about its axis and radius R. Find the acceleration of the two blocks. Assume that the string is light and does not slip on the pulley.

Sol. Suppose the tension in the left string is T_1 and that in the right string is T_2. Suppose the block of mass M goes down with an acceleration a and the other block moves up with the same acceleration. This is also the tangential acceleration of the rim of the wheel as the string does not slip over the rim.

The angular acceleration of the wheel $\alpha = \frac{a}{R}$.

The equations of motion for the mass M,

the mass m and the pulley are as follows ;



$Mg - T_1 = Ma$ ...(i)

$T_2 - mg = ma$ ...(ii)

$T_1R - T_2R = I\alpha = \frac{Ia}{R}$ ...(iii)

Substituting for T_1 and T_2 from equations (i) and (ii) in equation (iii)

$[M(g - a) - m(g + a)]R = \frac{Ia}{R}$

Solving, we get

$$a = \frac{(M-m)gR^2}{I+(M+m)R^2}$$

Note

The student can now attempt section C from exercise.

Section D - Angular Momentum (about point, about axis), Conservation of Angular Momentum

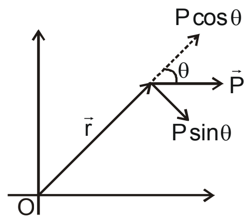
8. ANGULAR MOMENTUM

8.1 Angular momentum of a particle about a point.

$\vec{L} = \vec{r} \times \vec{P}$ $\Rightarrow$ $L = r\, p \sin\theta$

$|\vec{L}| = r_\perp \times P$ $\Rightarrow$ $|\vec{L}| = P_\perp \times r$

Where $\vec{P}$ = momentum of partilcle



$\vec{r}$ = position of vector of particle with respect to point about which

angular momentum is to be calculated.

θ = angle between vectors $\vec{r}$ & $\vec{p}$

$r_\perp$ = perpendicular distance of line of motion of particle from point O.

$P_\perp$ = perpendicular component of momentum.

SI unit of angular momentum is kgm^2/sec.

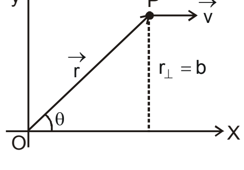
EXAMPLE 28

A particle of mass m is moving along the line y = b, z = 0 with constant speed v. State whether the angular momentum of particle about origin is increasing, decreasing or constant.

Sol. $|\vec{L}| = mvr\sin\theta$

$= mvr_\perp$

$= mvb$



$\therefore$ $|\vec{L}|$ = constant as m, v and b all are constants.

Direction of $\vec{r} \times \vec{v}$ also remains the same. Therefore, angular momentum of particle about origin remains constant with due course of time.

Note

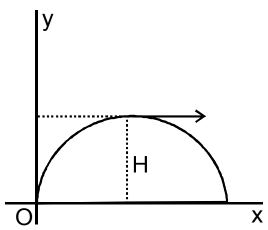
In this problem $|\vec{r}|$ is increasing, θ is decreasing but r sin θ, i.e., b remains constant. Hence, the angular momentum remains constant.

EXAMPLE 29

A particle of mass m is projected with velocity v at an angle θ with the horizontal. Find its angular momentum about the point of projection when it is at the highest point of its trajectory.

Sol. At the highest point it has only horizontal velocity v_x = v cos θ. Length of the perpendicular to the horizontal velocity from 'O' is the maximum height, where

$$H_{max} = \frac{v^2 \sin^2\theta}{2g}$$



$\Rightarrow$ Angular momentum $L = \dfrac{mv^3 \sin^2\theta \cos\theta}{2g}$

8.2 Angular Momentum of a rigid body rotating about a fixed axis

Suppose a particle P of mass m is going in a circle of radius r and at some instant the speed of the particle is v. For finding the angular momentum of the particle about the axis of rotation, the origin may be chosen anywhere on the axis. We choose it at the centre of the circle. In this case $\vec{r}$ and $\vec{P}$ are perpendicular to each other and $\vec{r} \times \vec{P}$ is along the axis. Thus, component of $\vec{r} \times \vec{P}$ along the axis is *mvr* itself. The angular momentum of the whole rigid body about AB is the sum of components of all particles, i.e.,

$$L = \sum_i m_i r_i v_i$$

Here, $v_i = r_i\,\omega$

$\therefore$ $L = \sum_i m_i r_i^2 \omega_i$ or $L = \omega \sum_i m_i r_i^2$

or $L = I\omega$

Here, I is the moment of inertia of the rigid body about AB.

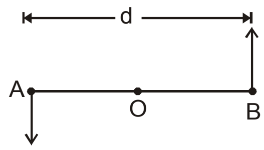
Note

Angular momentum about axis is the component of $I\vec{\omega}$ along the axis. In most of the cases angular momentum about axis is $I\omega$.

EXAMPLE 30

Two small balls A and B, each of mass m, are attached rigidly to the ends of a light rod of length d. The structure rotates about the perpendicular bisector of the rod at an angular speed ω. Calculate the angular momentum of the individual balls and of the system about the axis of rotation.

Sol.



Consider the situation shown in figure. The velocity of the ball A with respet to the centre O is $v = \frac{\omega d}{2}$.

The angular momentum of the ball with respect to the axis is

$L_1 = mvr = m\left(\frac{\omega d}{2}\right)\left(\frac{d}{2}\right) = \frac{1}{4}m\omega d^2$. The same the angular momentum L_2 of the second ball. The angular momentum of the system is equal to sum of these two angular momenta i.e., $L = 1/2\ m\omega d^2$.

9. CONSERVATION OF ANGULAR MOMENTUM :

The time rate of change of angular momentum of a particle about some referenence point in an inertial frame of reference is equal to the net torques acting on it.

or $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$ ....(i)

Now, suppose that $\vec{\tau}_{net} = 0$, then $\frac{d\vec{L}}{dt} = 0$, so that $\vec{L}$ = constant.

"When the resultant external torque acting on a system is zero, the total vector angular momentum of the system remains constant. This is the principle of the conservation of angular momentum.

For a rigid body rotating about an axis (the z-axis, say) that is fixed in an inertial reference frame, we have

$L_z = I\omega$

It is possible for the moment of inertia I of a rotating body to change by rearrangement of its parts. If no net external torque acts, then L_z must remains constant and if I does change, there must be a compensating change in ω. The principle of conservation of angular momentum in this case is expressed.

$I\omega$ = constant.

EXAMPLE 31

A wheel of moment of inertia I and radius R is rotating about its axis at an angular speed ω_0. It picks up a stationary particle of mass m at its edge. Find the new angular speed of the wheel.

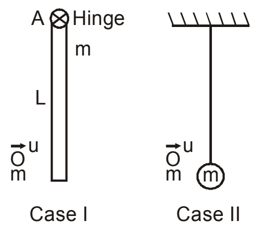
Sol. Net external torque on the system is zero. Therefore, angular momentum will remain conserved. Thus,

$I_1\omega_1 = I_2\omega_2$ or $\omega_2 = \frac{I_1\omega_1}{I_2}$

Here, $I_1 = I$, $\omega_1 = \omega_0$, $I_2 = I + mR^2$

$\therefore \quad \omega_2 = \frac{I\omega_0}{I + mR^2}$

Note :



Comments on Linear Momentum :

In case I : Linear momentum is not conserved just before and just after collision because during collision hinge force act as an external force.

In case II : Linear momentum is conserved just before and just after collision because no external force on the string.

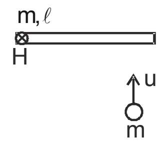
Comments on Angular Momentum :

In case I : Hinge force acts at an external force during collision but except point A all the other reference point given $\tau_{net} \neq 0$. So angular momentum is conserved only for point A.

In case II : angular momentum is conserved at all points in the world.

EXAMPLE 32

A uniform rod of mass m and length ℓ can rotate freely on a smooth horizontal plane about a vertical axis hinged at point H. A point mass having same mass m coming with an initial speed u perpendicular to the rod, strikes the rod in-elastically at its free end. Find out the angular velocity of the rod just after collision?

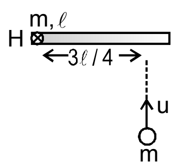


Sol. Angular momentum is conserved about H because no external force is present in horizontal plane which is producing torque about H.

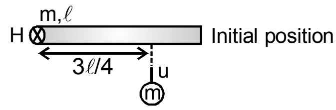
$$mu\ell = \left(\frac{m\ell^2}{3} + m\ell^2\right)\omega \Rightarrow w = \frac{3u}{4\ell}$$

EXAMPLE 33

A uniform rod of mass m and length ℓ can rotate freely on a smooth horizontal plane about a vertical axis hinged at point H. A point mass having same mass m coming with an initial speed u perpendicular to the rod, strikes the rod and sticks to it at a distance of 3ℓ/4 from hinge point. Find out the angular velocity of the rod just after collision?



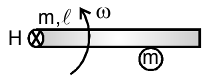
Sol.



from angular momentum conservation about H

initial angular momentum = final angular momentum

$$m.u\frac{3\ell}{4} = m\left(\frac{3\ell}{4}\right)^2 \omega + \frac{ml^2}{3}\omega$$



$$\Rightarrow \quad \frac{3mu\ell}{4} = m\ell^2\left[\frac{1}{3}+\frac{9}{16}\right]\omega$$

$$\frac{3u}{4\ell} = \left[\frac{16+27}{48}\right]\omega$$

$$\Rightarrow \quad \omega = \frac{36u}{43\ell}$$

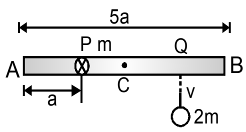
EXAMPLE 34

A uniform rod AB of mass m and length 5a is free to rotate on a smooth horizontal table about a pivot through P, a point on AB such that AP = a. A particle of mass 2m moving on the table strikes AB perpendicularly at the point 2a from P with speed v, the rod being at rest. If the coefficient of restitution between them is $\frac{1}{4}$, find their speeds immediately after impact.

Sol. Let the point of impact be Q so that

PQ = 2a

Let P be the point of pivot that AP = a



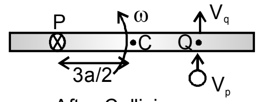
Before Collision

Let the velocities of point, Q and the particle after impact be v_q and v_p respectively then from momentum conservation about point P.

$$L_i = L_f$$

$$2a(2mv) = I_p\omega + (2a)(2mv_p) \qquad ...(i)$$

$$I_P = \frac{1}{3}m\left(\frac{5a}{2}\right)^2 + m\left(\frac{3a}{2}\right)^2 \qquad \left\{\begin{matrix}\text{use parallel}\\ \text{axis theorem}\end{matrix}\right\}$$



After Collision

$$= \frac{13ma^2}{3} \qquad ...(ii)$$

use equation (ii) in equation (i)

$$4ma(v - v_p) = \frac{13ma^2}{3}\omega$$

$$12(v - v_p) = 13a\omega \qquad ....(iii)$$

coefficient of restitution $e = \dfrac{\text{velocity of seperation}}{\text{velocity of approach}}$

$$\frac{1}{4} = \frac{v_q - v_p}{v}$$

$$\Rightarrow \quad v_q - v_p = \frac{v}{4} \qquad ...(iv)$$

$$v_q = 2a\omega \qquad ...(v)$$

Put value of ω from eq (iii) to equation (v)

$$v_q = 2\left(\frac{12}{13}\right)(v - v_p)$$

So now from equation (iv)

$$\frac{24}{13}(v - v_p) - v_p = \frac{v}{4} \quad \Rightarrow \quad v_p = \frac{83v}{148}$$

So in this way we get $\omega = \dfrac{15v}{37a}$

EXAMPLE 35

A person of mass m stands at the edge of a circular platform of radius R and moment of inertia. A platform is at rest initially. But the platform rotate when the person jumps off from the platform tangentially with velocity u with respect to platfrom. Determine the angular velocity of the platform.

Sol. Let the angular velocity of platform is ω. Then the velocity of person with respect to ground v.

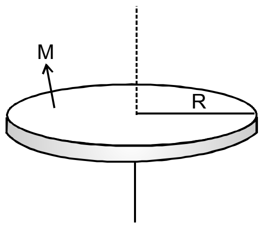
$$v_{mD} = v_{mG} - V_{DG}$$

$$u = v_m + \omega R$$

$$v_m = u - \omega R$$

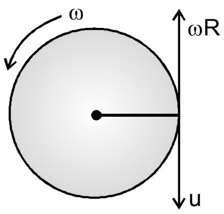
Now from angular momentum conservation

$$L_i = L_f$$



$0 = mv_m R - I\omega$

$\Rightarrow \quad I\omega = m(u - \omega R) \cdot R$

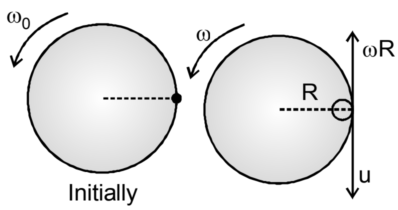


$\Rightarrow \quad \omega = \dfrac{muR}{I + mR^2}$

EXAMPLE 36

Consider the situation of previous example. If the platform is rotating intially with angular velocity ω_0 and then person jumps off tangentially. Determine the new angular velocity of the platform.

Sol. Let the angular velocity of platfrom after jumps off the mass is ω. Then velocity Of man.



$v_m = v_{mp} + v_p$

$v_m = u - \omega R$

From Angular momentum conservation

$(I + mR^2)\,\omega_0 = I\omega - m(u - \omega R)R$

$I\omega_0 + mR^2\omega_0 = I\omega - muR + m\omega R^2$

$\Rightarrow \quad \omega = \dfrac{(I + mR^2)\omega_0 + muR}{(I + mR^2)}$

Note

The student can now attempt section D from exercise.

Section E - Angular Impulse + Collision Of point Mass with Rigid Bodies

10. ANGULAR IMPULSE

The angular impulse of a torque in a given time interval is defined as $\int_{t_1}^{t_2} \vec{\tau}\, dt$

Here, $\vec{\tau}$ is the resultant torque acting on the body. Further, since

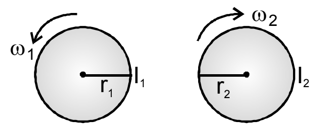
$$\vec{\tau} = \frac{d\vec{L}}{dt} \qquad \therefore \quad \vec{\tau}\, dt = d\vec{L}$$

or $\int_{t_1}^{t_2} \vec{\tau}\, dt$ = angular impulse = $\vec{L}_2 - \vec{L}_1$

Thus, the angular impulse of the resultant torque is equal to the change in angular momentum. Let us take few examples based on the angular impulse.

EXAMPLE 37

Figure shows two cylinders of radii r_1 and r_2 having moments of inertia I_1 and I_2 about their respective axes. Initially, the cylinders rotate about their axes with angular speeds ω_1 and ω_2 as shown in the figure. The cylinders are moved closer to touch each other keeping the axes parallel. The cylinders first slip over each other at the contact but the slipping finally ceases due to the friction between them. Find the angular speeds of the cylinders after the slipping ceases.



Sol. When slipping ceases, the linear speeds of the points of contact of the two cylinders will be equal. If ω_1' and ω_2' be the respective angular speeds, we have

$$\omega_1' r_1 = \omega_2' r_2 \qquad .....(i)$$

The change in the angular speed is brought about by the frictional force which acts as long as the slipping exists. If this force f acts for a time t, the torque on the first cylinder is fr_1 and that on the second is fr_2. Assuming $\omega_1 r_1 > \omega_2 r_2$, the corresponding angular impluses are $-fr_1 t$ and $fr_2 t$. We, therefore, have

$$-fr_1 t = I_1\,(\omega_1' = \omega_1)$$

and $\quad f r_2 t = I_2 (\omega_2' = \omega_2)$

or, $\quad -\frac{I_1}{r_1}(\omega_1' - \omega_1) = \frac{I_2}{r_2}(\omega_2' - \omega_2) \quad$ ...(ii)

Solving (i) and (ii),

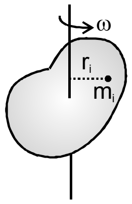
$\omega_1' = \frac{I_1\omega_1 r_2 + I_2\omega_2 r_1}{I_2 r_1^2 + I_1 r_2^2} r_2$ and $\omega_2' = \frac{I_1\omega_1 r_2 + I_2\omega_2 r_1}{I_2 r_1^2 + I_1 r_2^2} r_1$

Kinetic Energy of a rigid body rotating about a fixed axis.

Suppose a rigid body is rotating about a fixed axis with angular speed ω.

Then, kinetic energy of the rigid body will be :

$$K = \sum_i \frac{1}{2} m_i v_i^2 = \sum_i \frac{1}{2} m_i (\omega r_i)^2$$



$$= \frac{1}{2}\omega^2 \sum_i m_i r_i^2 = \frac{1}{2} I\omega^2 \qquad (\text{as } \sum_i m_i r_i^2 = I)$$

Thus, $\quad KE = \frac{1}{2} I\omega^2$

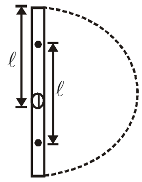
Sometimes it is called the rotational kinetic energy.

EXAMPLE 38

A uniform rod of mass m and length ℓ is kept vertical with the lower end clamped. It is slightly pushed to let it fall down under gravity. Find its angular speed when the rod is passing through its lowest position. Neglect any friction at the clamp. What will be the linear speed of the free end at this instant?

Sol. As the rod reaches its lowest position, the centre of mass is lowered by a distance ℓ. Its gravitational potential energy is decreased by $mg\ell$. As no energy is lost against friction, this should be equal to the increase in the kinetic energy. As the rotation occurs about the horizontal axis through the clamped end, the moment of inertia is $I = m\,\ell^2/3$. Thus,

$$\frac{1}{2} I\omega^2 = mg\ell \Rightarrow \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 = mg\,\ell$$



or $\quad \omega = \sqrt{\frac{6g}{\ell}}$

The linear speed of the free end is

$v = \ell\omega = \sqrt{6g\ell}$

Note

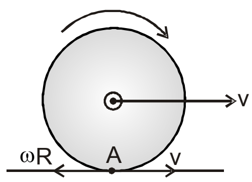
The student can now attempt section E from exercise.

Section F - Combined Translational and Rotational Motion of Rigid body, Pure Rolling, Slipping

11. COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY :

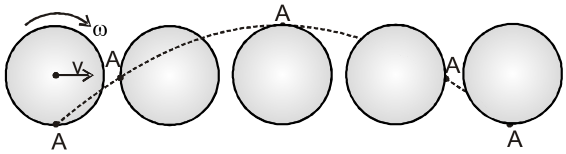
We have already learnt about translational motion caused by a force and rotational motion about a fixed axis caused by a torque. Now we are going to discuss a motion in which body undergoes translational as well as rotational motion. Rolling is an example of such motion. If the axis of rotation is moving then the motion is combined translational and rotational motion.

To understand the concept of combined translational and rotational motion we consider a uniform disc rolling on a horizontal surface. Velocity of its centre of mass is V_{com} and its angular speed is ω as shown in figure.



Let us take a point A on the disc and concentrate on its motion.

Path of point A with respect to ground will be a cycloid as shown in figure.



Motion of point A with respect to center of mass is pure rotational while center of mass itself is moving in a straight line. So for the analysis of rolling motion we deal translational motion seperately and rotational motion seperately and then we combine the result to analyses the over all motion.

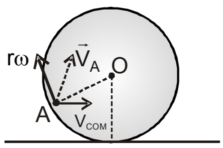
The velocity of any point A on the rigid body can be obtained as

$$\vec{V}_A = \vec{V}_{COM} + \vec{V}_{A\,COM}$$

$$|\vec{V}_{COM}| = V$$

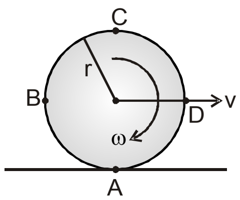
$|\vec{V}_{A.COM}| = r\omega$ in the direction $\perp$ to line OA

Thus, the velocity of point A is the vector sum of $\vec{V}_{COM}$ and $\vec{V}_{P.COM}$ as shown in figure



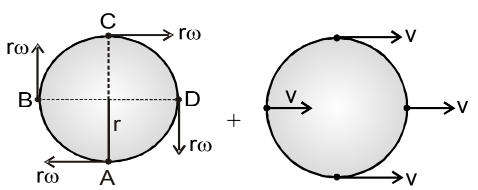
Important points in combined Rotational + translation motion :

1. **Velocity of any point of the rigid body in combined R + T motion is the vector sum of v(velocity of centre of mass) and rω**
for example
A disc of radius r has linear velocity v and angular velocity ω as shown in figure then find velocity of point A. B, C, D on the disc

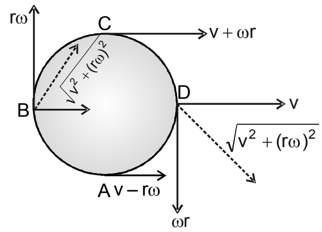


We divide our problem in two parts

(1) Pure Rotational + (2) Pure Translational
about centre of mass.

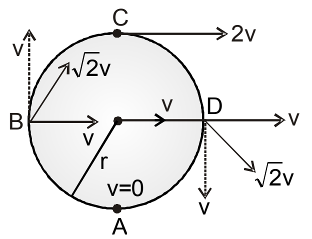


Then combine the result of above both

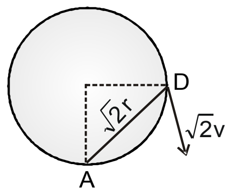


2. **In combined rotational and translational motion angular velocity of any point of a rigid body with respect to other point in the rigid body is always same.**

For example :

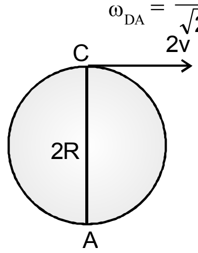


(v = rω)



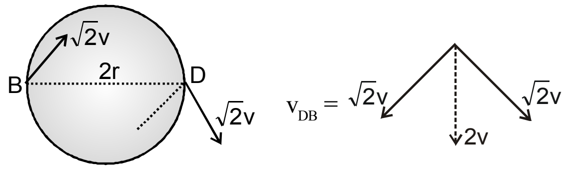
Now for ω_{DA} $\qquad \omega_{DA} = \dfrac{\sqrt{2}v}{\sqrt{2}r} = \dfrac{v}{r}$

For ω_{CA} :



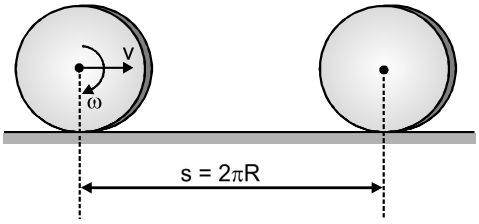
$$\omega_{CA} = \frac{2v}{2r} = \frac{v}{r}$$

For ω_{DB} :



$$\Rightarrow \qquad \omega_{DB} = \frac{2v}{2r} = \frac{v}{r}$$

3. Distance moved by the centre of mass of the rigid body in one full rotation is $2\pi R$.



This can be shown as under :

In one rotation angular displacement $\theta = 2\pi = \omega t$

$$s = v\,.\,T = (\omega\, R)\left(\frac{2\pi}{\omega}\right) = 2\pi R$$

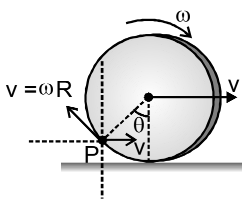
In forward slipping $\quad s > 2\pi R \quad$ (as $v > \omega R$)

and in backward slipping $\quad s < 2\pi R \quad$ (as $v < \omega R$)

4. The speed of a point on the circumference of the body at any instant t is $2R\omega \sin\frac{\omega t}{2}$

Proof :

$v_{xp} = v - v\cos\theta \qquad = v[1 - \cos\theta]$



$v_{yp} = v\sin\theta$

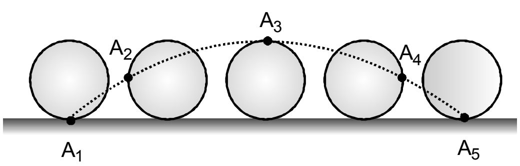
$$|\vec{v}_p| = \sqrt{v^2\sin^2\theta + v^2(1-\cos\theta)^2}$$

$$v = \sqrt{2v^2 - 2v^2\cos\theta}$$

$$= \sqrt{2v}(1-\cos\theta)^{1/2}$$

$$= 2v\sin\left(\frac{\theta}{2}\right) = 2v\sin\left(\frac{\omega t}{2}\right) = 2R\omega\sin\left(\frac{\omega t}{2}\right)$$

5. The path of a point on circumference is a cycloid and the distance moved by this point in one full rotation is 8R.



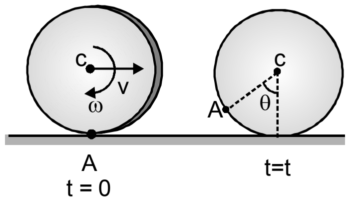
In the figure, the dotted line is a cycloid and the distance $A_1 A_2$ ......A_5 is 8R. This can be proved as under.

According to point (3), speed of point A at any moment is,

$$v_A = 2R\omega\sin\left(\frac{\omega t}{2}\right)$$

Distance moved by A in time dt is,

$$ds = v_A\, dt = 2R\omega\sin\left(\frac{\omega t}{2}\right) dt$$



Therefore, total distance moved in one full rotation is,

$$S = \int_0^{T=2\pi/\omega} ds$$

or $\qquad S = \displaystyle\int_0^{T=2\pi/\omega} 2R\omega\sin\left(\frac{\omega t}{2}\right)dt$

On integration we get, $\quad s = 8R$

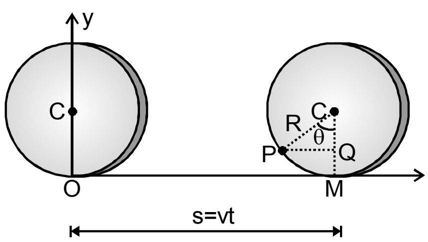
6. x and y coordinates of the bottommost point at any time t.

At time t the bottommost point will rotate an angle $\theta = \omega t$ with respect to the centre of the disc C. The centre C will travel a distance $s = vt$.

In the figure, $PQ = R\sin\theta = R\sin\omega t$

$CQ = R\cos\theta = R\cos\omega t$

Coordinates of point P at time t are,

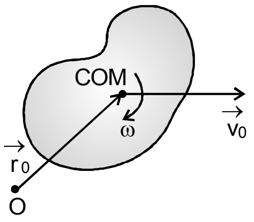


$x = OM - PQ = vt - R\sin\omega t$

and $y = CM - CQ = R - R\cos\omega t$

$\therefore\ (x, y) \equiv (vt - R\sin\omega t,\ R - R\cos\omega t)$

11.1 Angular momentum of a rigid body in combined rotation and translation



Let O be a fixed point in an inertial frame of reference. Angular momentum of the body about O is.

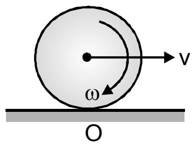
$$\vec{L} = \vec{L}_{cm} + M(\vec{r}_0 \times \vec{v}_0)$$

The first term $\vec{L}_{cm}$ represents the angular momentum of the body as seen from the centre of mass frame. The second term $M(\vec{r}_0 \times \vec{v}_0)$ equals the angular momentum of centre of mass about point O.

EXAMPLE 39

A circular disc of mass m and radius R is set into motion on a horizontal floor with a linear speed v in the forward direction and an angular speed $\omega = \frac{v}{R}$ in clockwise direction as shown in figure. Find the magnitude of the total angular momentum of the disc about bottommost point O of the disc.

Sol. $\vec{L} = \vec{L}_{cm} + m(\vec{r}_0 \times \vec{v}_0)$ ...(i)



Here, $\vec{L}_{cm} = I\omega$

(perpendicular to paper inwards)

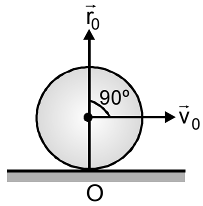
$$= \left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)$$

$$= \frac{1}{2}mvR$$

and $m(\vec{r}_0 \times \vec{v}_0) = mRv$

(perpendicular to paper inwards)

Since, both the terms of right hand side of Eq. (i) are in the same direction.



$\therefore$ $|\vec{L}| = \frac{1}{2}mvR + mvR$

or $|\vec{L}| = \frac{3}{2}mvR$ **Ans.**

11.2 Kinetic Energy of a Rolling Body

If a body of mass M is rolling on a plane such that velocity of its centre of mass is V and its angular speed is ω, its kinetic energy is given by

$$KE = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

I is moment of inertia of body about axis passing through centre of mass.

In case of rolling without slipping.

$$KE = \frac{1}{2}M\omega^2 R^2 + \frac{1}{2}I\omega^2 \quad [\therefore v = \omega R]$$

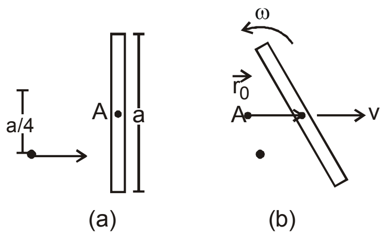
$$= \frac{1}{2}\left[MR^2 + I\right]\omega^2 \qquad = \frac{1}{2}I_c\,\omega^2$$

I_c is moment of inertia of the body about the axis passing through point of contact.

EXAMPLE 40

A uniform rod of mass M and length a lies on a smooth horizontal plane. A particle of mass m moving at a speed v perpendicular to the length of the rod strikes it at a distance a/4 from the centre and stops after the collision. Find (a) the velocity of the centre of the rod and (b) the angular velocity of the rod about its centre just after the collision.

Sol.



The situation is shown in figure. Consider the rod and the particle together as the system. As there is no external resultant force, the linear momentum of the system will remain constant. Also there is no resultant external torque on the system and so the angular momentum of the system about any line will remain constant.

Suppose the velocity of the centre of the rod is V and the angular velocity about the centre is ω.

(a) The linear momentum before the collision is mv and that after the collision is MV. Thus,

$$mv = MV, \text{ or } V = \frac{m}{M}v$$

(b) Let A be the centre of the rod when it is at rest. Let AB be the line perpendicular to the plane of the figure. Consider the angular momentum of "the rod plus the particle" system about AB. Initially the rod is at rest. The angular momentum of the particle about AB is

$$L = mv(a/4)$$

After the collision, the particle mass to rest. The angular momentum of the rod about A is

$$\vec{L} = \vec{L}_{cm} + M(\vec{r}_0 \times \vec{V})$$

As $\vec{r}_0 \parallel \vec{V}$, $\quad \vec{r}_0 \times \vec{V} = 0$

Thus, $\quad \vec{L} = \vec{L}_{cm}$

Hence the angular momentum of the rod about AB is

$$L = I\omega = \frac{Ma^2}{12}\omega$$

Thus, $\quad \frac{mva}{4} = \frac{Ma^2}{12}\omega \quad$ or, $\quad \omega = \frac{3mv}{Ma}$

EXAMPLE 41

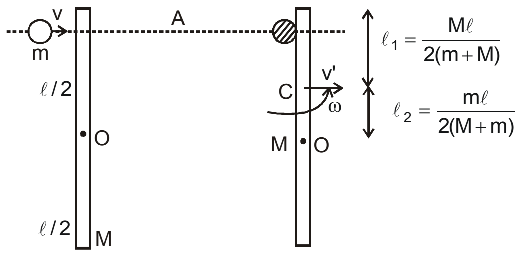
A uniform rod of length λ lies on a smooth horizontal table A particle moving on the table has a mass m and a speed v before the collision and it sticks to the rod after the collision. The rod has a mass M then find out.

(a) The moment of inertia of the system about the vertical axis passing through the centre of mass C after the collision.

(b) The velocity of the centre of mass C and the angular velocity of the system about the centre of mass after the collision.

Sol. Figure shows the situation of system just before and just after collision.

Initially the centre of mass of the rod is at point O. After collision when the particle sticks to the rod. Centre of mass is shifted from point O to C as shown in figure. Now the system is rotated about axis passing through C



Before collision $\qquad$ After collision

Now from linear momentum conservation

$mv = (M + m)\,v' \qquad \Rightarrow \qquad v' = \frac{mv}{M+m}$

(a) Let us assume that moment of inertia of the system about C is 1. Then $I = I_{(rod)C} + I_{(part)C}$

$$I = I_0 + M\ell_2^2 + m\ell_1^2$$

$$I = \frac{M\ell^2}{12} + \frac{Mm^2\ell^2}{4(m+M)^2} + \frac{mM^2\ell^2}{4(m+M)^2}$$

$$\Rightarrow \qquad I = \frac{M(M+4m)}{12(m+M)}\ell^2$$

(b) From Angular momentum conservation about A

$$L_i = L_f$$

$0 + 0 = I\,\omega - (m + M)\,v'\,\ell_1$

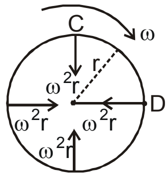
$\Rightarrow \qquad I\omega = (m + M)\,v'\,\ell_1$

Put the value of I, v', & ℓ_1 we get

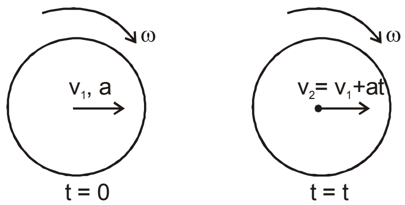
$$\omega = \frac{6mv}{(M+4m)\ell}$$

11.3 Acceleration of a point on the circumference of the body in R + T motion :

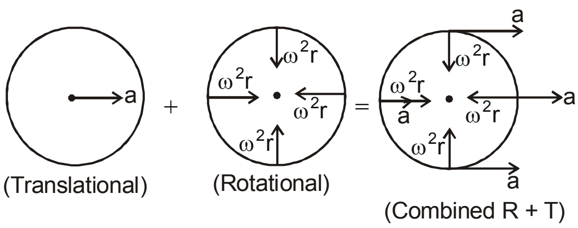
(A) Both ω & v are constant :



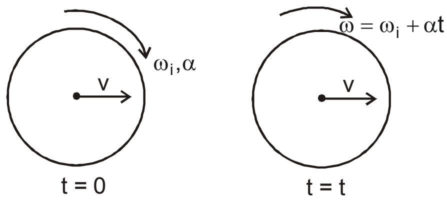
(B) **When ω is constant and v is variable**.



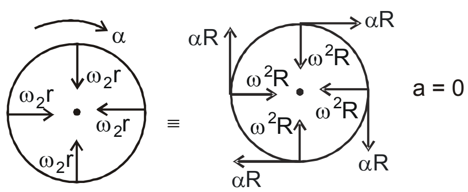
So acceleration of different point on the body is given by following figure.



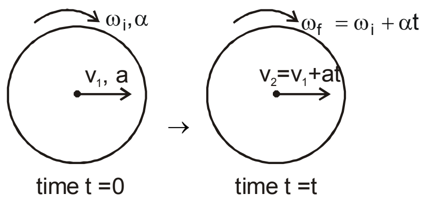
(C) When ω is variable and v is constant :



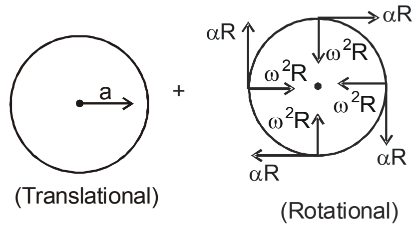
So acceleration of different point on the body is given by following way

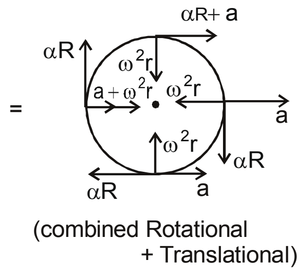


(D) When both ω & v are variable :



Now the net acceleration of different points on the rigid body is given by following way.





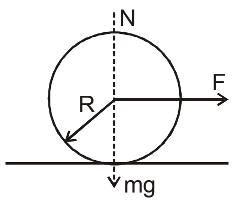
EXAMPLE 42

A force F acts at the centre of a thin spherical shell of mass m and radius R.

Find the acceleration of the shell if the surface is smooth.

Sol. ∵ Force F, mg & N passes through centre so

$\tau_{net} = 0$, i.e., body is in rotational equilibrium



But $\vec{F}_{net} = F$ so body moves with constant acceleration $a = \frac{F}{m}$

EXAMPLE 43

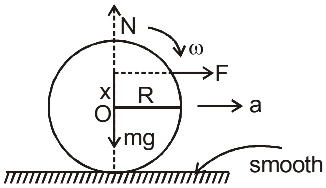
In a previous problem if force F applied at a distance x above the centre then find out linear and angular acceleation.

Sol. This force F translate the body linearly as well as rotate it. So,

Net toruqe about O it $\tau_0 = Fx$

From rotational motion $\tau_0 = I\alpha$

$$\alpha = \frac{\tau}{I} = \frac{Fx}{\frac{2MR^2}{3}} \Rightarrow \alpha = \frac{3Fx}{2MR^2}$$



From linear motion of sphere

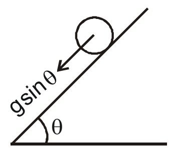
$$F = ma \Rightarrow a = \frac{F}{m}$$

EXAMPLE 44

A rigid body of mass m and radius r starts coming down an inclined plane of inclination θ. Then find out the acceleration of centre of mass if friciton is absent.

Sol. Friction is absent so body is moving down the incline with out

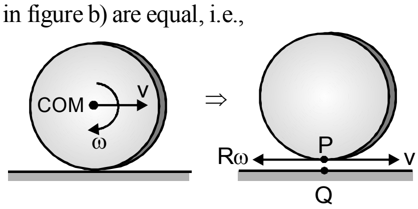
rolling so acceleration of centre of mass is $g \sin\theta$



12. UNIFORM PURE ROLLING

Pure rolling means no relative motion (or no slipping at point of contact between two bodies.)

For example, consider a disc of radius R moving with linear velocity v and angular velocity ω on a horizontal ground. The disc is said to be moving without slipping if velocities of points P and Q (shown in figure b) are equal, i.e.,



(a) (b)

$$v_p = v_Q$$

or $v - R\omega = 0$ or $v = R\omega$

If $v_p > v_Q$ or $v > R\omega$, the motion is said to be forward slipping and if $v_p < v_Q < R\omega$, the motion is said to be backward slipping.

Now, suppose an external force is applied to the rigid body, the motion will no longer remain uniform. The condition of pure rolling on a stationary ground is,

$$a = R\alpha$$

Thus, $v = R\omega$, $a = R\alpha$ is the condition of pure rolling on a stationary ground. Sometime it is simply said rolling.

Note : We can represent the moment of inertia of a different rigid body in a following way.

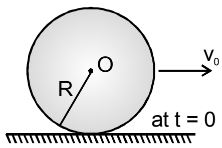
$$I = CMR^2$$

value of C = 1 for circular ring (R), $C = \frac{1}{2}$ for circular disc (D) and solid cylinder (S.C.)

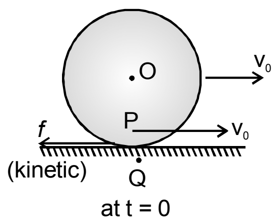
$C = \frac{2}{3}$ for Hollow sphere (H.S), $C = \frac{2}{5}$ for solid sphere (S.S)

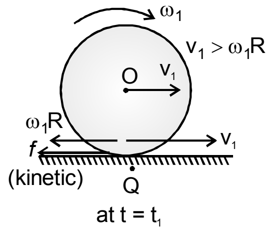
EXAMPLE 45

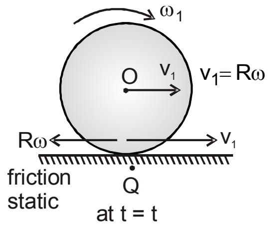
A rigid body $I = CMR^2$ is set into a motion on a rough horizontal surface with a linear speed v_0 in the forward direction at time t = 0 as shown in figure. After what time slipping finally stop and pure rolling starts. Find the linear speed of the body after it starts pure rolling on the surface.



Sol. According to the given condition in problem the point P in the body move with speed v_0 while the point Q on the ground is at rest. So the friciton acts on the body is in backward direction which gives the resultant torque on the body and increase the angualr speed ω as shown in figure.







As shown in above figure initially $v > R\omega$ so forward slipping takes place. After introducing the friciton speed decreases and ω increases and at time t = t the relation $v = r\omega$ is satisfied. Therefore pure rolling starts. Initially the friciton is kinetic untill the motion is in slipping condition. Afterwards at $v = r\omega$ fricition is static. We divide the above problem in two parts.

(1) Translational Motion :

Linear acceleration $a = -\mu g$

So after time t, $v = v_0 - \mu gt$ ...(1)

(2) Rotational Motion :

From $\tau_{net} = I\alpha$

Only friction force is responsible for providing torque. So torque about O is

$f.\ R = I\alpha$

$\mu mgR = CmR^2\alpha$ ...(2)

α is angular acceleration of the body

from eq. (2) $\alpha = \dfrac{\mu g}{CR}$

from $\omega_f = \omega_i + \alpha t$

$\omega = \alpha t \Rightarrow \omega = \dfrac{\mu g}{CR}.t$

$\therefore \omega = \dfrac{v}{R}$ at pure rolling condition.

So, $v = \dfrac{\mu gt}{C}$ ...(3)

from eq. (1) & (3)

$\Rightarrow v_0 - \mu gt = \dfrac{\mu gt}{C} \Rightarrow t = \dfrac{v_0 C}{\mu g(1+C)}$ ...(4)

Equation (4) gives the time after the pure rolling starts.

Put the value from eq. (4) to eq. (1)

$v = v_0 - \dfrac{v_0 C}{(1+C)} \Rightarrow v = \dfrac{v_0}{1+C}$ ...(5)

Equation (5) gives the linear speed at pure rolling situation.

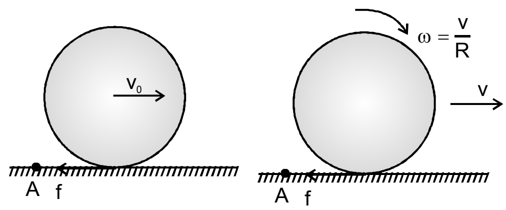
Alternate solution :

Net torque on the body about the bottom most point A is zero. Therefore angular momentum of the body will remain conserved about the bottom most point

Net torque about A $\tau_A = 0$

$\Rightarrow$ from Angular momentum conservation $L_i = L_f$

$mv_0R = I\omega + mvR$



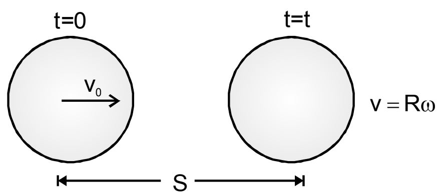
$mv_0R = CmR^2\dfrac{v}{R} + mvR$

$v_0 = Cv + v \quad \Rightarrow \quad v = \dfrac{v_0}{1+C}$

EXAMPLE 46

In the previous problem take rigid body a solid cylinder then find out the work done by friciton from time t = 0 to t = t (at v = rω)

Sol. Let us suppose that in between time t = 0 to t = t cylinder displaced s.



Translational work done by friciton + Rotational work Done by friciton

Now calculate each type of work done one by one

(A) Translational work done by friciton :

for solid cylinder $c = \dfrac{1}{2}$

from eq. (5) $v = \dfrac{v_0}{1+\frac{1}{2}} = \dfrac{2}{3}v_0$

from eq. $v_f^2 = u_i^2 + 2as \Rightarrow$

$$\left(\frac{2}{3}v_0\right)^2 = (v_0)^2 - 2\mu gs$$

$$s = \frac{5v_0^2}{18\mu g}$$

Translation W.D by friciton = – f.s

$$(w.D)_{fT} = -\mu mg.\frac{5v_0^2}{18\mu g} = -\frac{5mv_0^2}{18}$$

(B) Rotational W.D. by friciton :

We known that $\tau = I\alpha$

$$\Rightarrow \alpha = \frac{\tau}{I} \quad ...(a)$$

from $\omega_f^2 - \omega_i^2 = 2\alpha\theta$

$$\left(\frac{2v_0}{3R}\right)^2 = \frac{2\tau}{I}.\theta$$

Put $I = \frac{mR^2}{2} \Rightarrow \theta = \frac{v_0^2 m}{9\tau}$

Rotation W.D by friciton $W = \tau.\theta$

$$W_{fR} = \frac{v_0^2 m}{9}$$

(C) So total W.D. by friciton $W = W_{fT} + W_{fR}$

$$= -\frac{5}{18}mv_0^2 + \frac{v_0^2 m}{9}$$

$$W = -\frac{mv_0^2}{6}$$

Alternative Method :

from work – Energy Theorem

work done by friciton = change in kinetic energy

$(W.D)_f = \Delta K = k_f - k_i$

Now

$$k_f = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega^2$$

$$k_f = \frac{1}{2}m\left(\frac{2v_0}{3}\right)^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{2v_0}{3R}\right)^2 \quad \left\{\because v_f = \frac{2v_0}{3}\right\}$$

$$k_f = \frac{mv_0^2}{3}$$

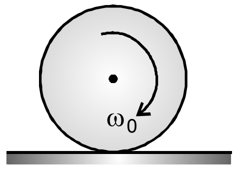
$$k_i = \frac{1}{2}mv_0^2$$

So., $(w.D)_f = \frac{mv_0^2}{3} - \frac{1}{2}mv_0^2 \Rightarrow (w.D)_f = -\frac{mv_0^2}{6}$

To calculate work done mostly prefer alternative method.

EXAMPLE 47

A solid sphere of radius r is gently placed on a rough horizontal ground with an initial angular speed ω_0 and no linear velocity. If the coefficient of friciton is μ, find the linear velocity v and angular velocity ω at the end of slipping.

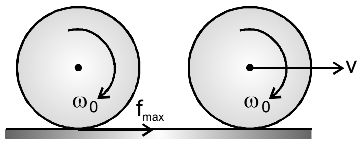


Sol. *m* be the mass of the sphere.

Since, it is a case of backward slipping, force of friction is in forward direction. Limiting friciton will act in this case.

Net torque on the sphere about the bottommost point is zero. Therefore, angular momentum of the sphere will remain conserved about the bottommost point.

$L_i = L_f$



$\therefore \quad I\omega_0 = I\omega + mrv$

or $\quad \frac{2}{5}mr^2\omega_0 = \frac{2}{5}mr^2\omega + mr(\omega r)$

$\therefore \quad \omega = \frac{2}{7}\omega_0$ and $v = r\omega = \frac{2}{7}r\omega_0$

12.1 Pure rolling when force F act on a body :

Suppose a force F is applied at a distance x above the centre of a rigid body of radius R, mass M and moment of inertia CMR^2 about an axis passing through the centre of mass. Now, the applied force F can produces by itself

(i) a linear acceleration a and

(ii) an angular acceleration α

If $a = R\alpha$, then there is no need of friction and force of friction f = 0,

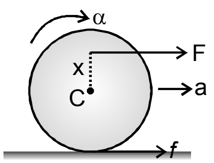
If $a < R\alpha$, then to support the linear momentum the force of friciton *f* will act in forward direction,

Similarly, if $a > R\alpha$, then no support the angular motion the force of friciton will act in backward direction.

So, in this case force of friction will be either backward, forward or even zero also. It all depends on M, I and R. For calculation you choose any direction of friction. Let we assume it in forward direction.

Let, a = linear acceleration,

α = angular acceleration



from linear motion

$F + f = Ma$ ...(1)

from rotational motion.

$Fx - fR = I\alpha$

$Fx - fR = CMR^2.\frac{a}{R}$

$Fx - fR = CMaR$ ...(2)

from eq. (1) and (2)

F(x+r) = MaR (C + 1)

$a = \frac{F(R+x)}{MR(C+1)}$ ...(3)

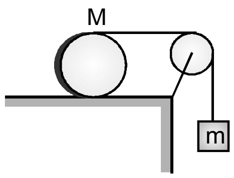
Put the value from eq. (3) to eq. (1)

$f = \frac{F(x-RC)}{R(C+1)}$

f should be $\leq \mu_s$ mg for pure rolling

EXAMPLE 48

Consider the arrangment shown in figure. The string is wrapped around a uniform cylinder which rolls without slipping. The other end of the string is passed over a massless, frictionless pulley to a falling weight. Determine the acceleration of the falling mass m in terms of only the mass of the cylinder M, the mass m and g.



Sol. Let T be the tension the string and f the force of (static) friction, between the cylinder and the surface

a_1 = acceleration of centre of mas of cylinder towards right

a_2 = downward acceleration of block m

α = angular acceleration of cylinder (clockwise)

Equations of motion are :

For block $mg - T = ma_2$ ...(i)

For cylinder, $T + f = Ma_1$ ...(ii)

$\alpha = \frac{(T-f)R}{\frac{1}{2}MR^2}$ ...(iii)

The string attaches the mass m to the highest point of the cylinder, hence

$v_m = v_{COM} + R\omega$

Differentiating, we get

$a_2 = a_1 + R\alpha$ ...(iv)

We also have (for rolling without slipping)

$a_1 = R\alpha$ ...(v)

Solving these equations, we get $a_2 = \frac{8mg}{3M+8m}$

Note

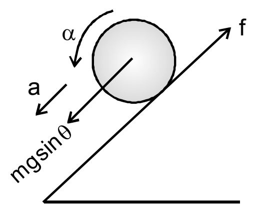
Work done by friction in pure rolling on a stationary ground is zero as the point of application of the force is at rest. Therefore, machanical energy can be conserved if all other dissipative forces are ignored.

12.2 Pure Rolling on an Inclined Plane:

A rigid body of radius R, and mass m is released at rest from height h on the incline whose inclination with horizontal is θ and assume that friciton is sufficient for pure rolling then.

$a = \alpha R$ and $v = R\omega$

From figure



$mg \sin\theta - f = ma$ ...(1)

$\{F_{net} = ma\}$

$f.R = cmR^2.\frac{a}{R}$ ...(2)

$\{F_{net} = I\alpha\}$

from eq. (1) & (2)

$a = \frac{g\sin\theta}{1+c}$

So body which have low value of C have greater acceleration.

value of C = 1 for circular ring (R)

$C = \frac{1}{2}$ for circular disc (D) and solid cylinder (S.C.)

$C = \frac{2}{3}$ for Hollow sphere (H.S)

$C = \frac{2}{5}$ for solid sphere (S.S)

So, descending order of a

$a_{S.S} > a_D = a_{S \cdot C} > a_{H.S.} > a_R$

and order of time of descend is

$t_{s.s} < t_D = t_{s.c} < t_{H.S} < t_R$

Kinetic Enegery : Work done by friction in pure rolling is zero. Therefore,

Increase in kinetic energy = change in potential energy $\Rightarrow$ K.E. = mgh

i.e., kinetic energy is constant for all rigid body rolling down the incline.

Requirement of Friction :

From eq. ...(2)

f = Cma

$$f = \frac{mg\sin\theta}{\left(1+\frac{1}{C}\right)} \quad ...(3)$$

from eq. (3) as the value of C increase requirement of friciton is increases.

EXAMPLE 49

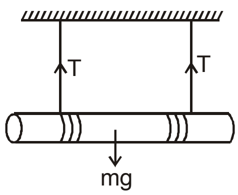
A cylinder of mass M is suspended through two strings wrapped around it as shown in figure. Find the tension in the string and the speed of the cylinder as it falls through a distance h.

Sol. The portion of the strings between ceiling and cylinder are at rest. Hence the points of the cylinder where the strings leave it are at rest also. The cylinder is thus rolling without slipping on the strings. Suppose the centre of cylinder falls with an acceleration a. The angular acceleration of cylinder about its axis given by

$$\alpha = \frac{a}{R} \quad ...(i)$$

as the cylinder does not slip over the strings.

The equation of motion for the centre of mass of cylinder is



Mg – 2T = Ma

and for the motion about the centre of mass it is

$$2T.R = \left(\frac{MR^2}{2}\right)\alpha, \text{ where } I = \frac{MR^2}{2}$$

$$2\,T\,R = \frac{MR^2}{2}\frac{a}{R} \quad \Rightarrow \quad 2\,T = \frac{Ma}{2} \quad ...(ii)$$

From (i) and (ii) on adding

$$Mg = \frac{Ma}{2} + Ma\,; \frac{3a}{2} = g$$

$$a = \frac{2g}{3}$$

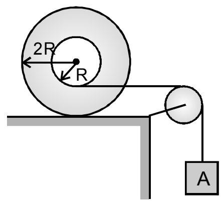
$$\therefore\ 2\,T = \frac{M}{2}\cdot\frac{2g}{3} \quad \Rightarrow \quad T = \frac{Mg}{6}$$

As the centre of cylinder starts moving from rest, the velocity after it has fallen a height h is given by

$$v^2 = 2\left[\frac{2g}{3}\right]h \quad \text{or } v = \sqrt{\frac{4gh}{3}}$$

EXAMPLE 50

A thin massless thread is wound on a reel of mass 3kg and moment of inertia 0.6 kg-m². The hub radius is R = 10 cm and peripheral radius is 2R = 20 cm. The reel is placed on a rough table and the friction is enough to prevent slipping. Find the acceleration of the centre of reel and of hanging mass of 1 kg.



Sol. Let, a_1 = acceleration of centre of mass of reel

a_2 = acceleration of 1 kg block

α = angular acceleration of reel (clockwise)

T = tension in the string

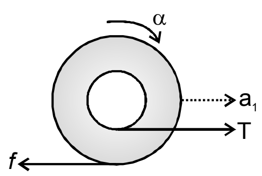
and *f* = force of friction

Free body diagram of reel is as shown below :

(only horizontal forces are shown).

Equations of motion are :

$$T - f = 3a_1 \quad ...(i)$$

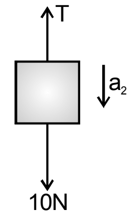


$$\alpha = \frac{\tau}{I} = \frac{f(2R) - T.R}{I} = \frac{0.2f - 0.1T}{0.6} = \frac{f}{3} - \frac{T}{6} \quad ...(ii)$$

Free body diagram of mass is,

Equation of motion is,

$$10 - T = a_2 \quad ...(iii)$$



For no slipping condition,

$$a_1 = 2R\alpha \text{ or } a_1 = 0.2\alpha \quad ...(iv)$$

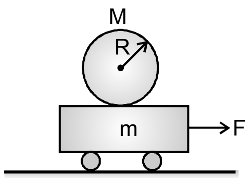
and $a_2 = a_1 - R\alpha$ or $a_2 = a_1 - 0.1\alpha$ ...(v)

Solving the above five equations, we get

$a_1 = 0.27$ m/s^2 and $a_2 = 0.135$ m/s^2

EXAMPLE 51

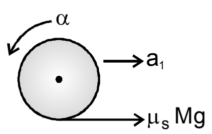
Determine the maximum horizontal force F that may be applied to the plank of mass m for which the solid sphere does not slip as it begins to roll on the plank. The sphere has a mass M and radius R. The coefficient of static and kinetic friction between the sphere and the plank are μ_s and μ_k respectively.



Sol. The free body diagrams of the sphere and the plank are as shown below :

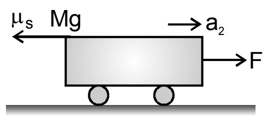
Writing equations of motion :

For sphere : Linear acceleration



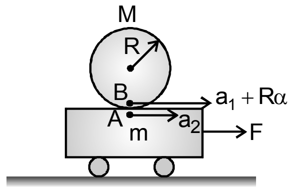
$$a_1 = \frac{\mu_s Mg}{M} = \mu_s g \quad ...(i)$$

Angular acceleration



$$\alpha = \frac{(\mu_s Mg)R}{\frac{2}{5}MR^2} = \frac{5}{2}\frac{\mu_s g}{R} \quad ...(ii)$$

For plank : Linear acceleration



$$a_2 = \frac{F - \mu_s Mg}{m} \quad ..(iii)$$

For no slipping acceleration of point B and A is same,

so : $a_2 = a_1 + R\alpha$

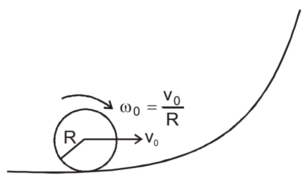
Solving the above four equation, we get

$$F = \mu_s g\left(M + \frac{7}{2}m\right)$$

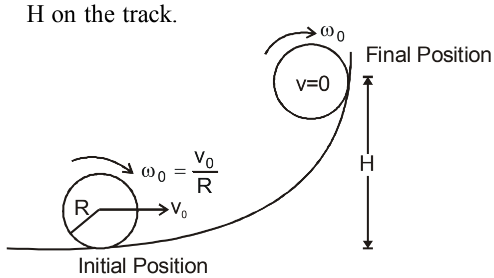
Thus, maximum value of F can be $\mu_s g\left(M + \frac{7}{2}m\right)$

EXAMPLE 52

Find out the maximum height attained by the solid sphere on a friciton less track as shown in figure.



Sol. Let us assume that sphere attain a maximum height H on the track.



As the sphere move upward speed is decreased due to gravity but there is no force to change the ω_0 (friction less track). So from energy conservation

$$\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0^2 = mg\,H_{max} + \frac{1}{2}I\omega_0^2,\ H_{max} = \frac{v_0^2}{2g}$$

Note

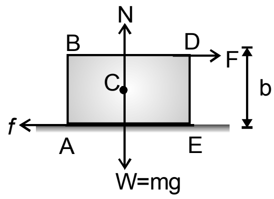
The student can now attempt section F from exercise.

Section G - Toppling + Direction of Friction

13. TOPPLING

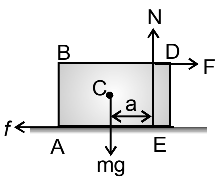
You might have seen in your practical life that if a force F is applied to a block A of smaller width it is more likely to topple down, before sliding while if the same force F is applied to an another block B of broader base, chances of its sliding are more compared to its toppling. Have you ever throught why it happens so. To understand it better let us take an example.

Suppose a force F is applied at a height b above the base AE of the block. Further, suppose the friction f is sufficient to prevent sliding. In this case, if the normal reaction N also passes through C, then despite the fact that the block is in translational equilibium ($F = f$ and $N = mg$), an unbalanced torque (due to the couple of forces F and f) is there. This torque has a tendency to topple the block about point E. To cancel the effect of this unbalanced torque the normal reaction N is shifted towards right a distance 'a' such that, net anticlockwise torque is equal to the net clockwise torque or

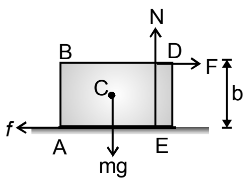


$$Fb = (mg)\, a$$

or $$a = \frac{Fb}{mg}$$



(a)

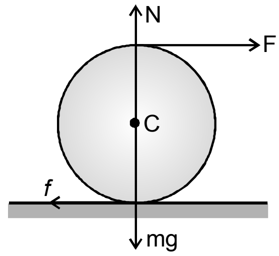


(b)

Now, as F or b (or both) are increased, distance a also increases. But it can not go beyond the right edge of the block. So, in extreme case (beyond which the block will topple down), the normal reaction passes through E as shown in figure.

Now, if F or b are further increased, the block will topple down. This is why the block having the broader base has less chances of toppling in comparison to a block of smaller base. Because the block of larger base has more margin for the normal reaction to shift.

Why the rolling is so easy on the ground.



Because in this case the normal reaction has zero margin to shift. so even if the body is in translational equilibrium ($F = f$, $N = mg$) an unbalanced torque is left behind and the body starts rolling clockwise. As soon as the body starts rolling the force of friction is so adjusted (both in magnitude and direction) that either the pure rolling starts (if friciton is sufficient enough) or the body starts sliding. Let us take few examples related to toppling.

EXAMPLE 53

A uniform block of height h and width a is placed on a rough inclined plane and the inclination of the plane to the horizontal is gradually increased. If μ is the coefficient of friction then under condition the block will

(A) slide before toppling :

The block will slide when

$mg \sin\theta > f$

$\Rightarrow mg \sin\theta > \mu\, mg \cos\theta$

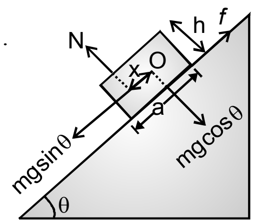
$\Rightarrow \tan\theta > \mu$

i.e., block is at rest when

$\tan\theta \le \mu$. ...(1)

(B) Now suppose the friction f is sufficient to prevent sliding. Then we assume that N is shifted towards downward a distance x to prevent toppling Therefore. torque about O is zero.

$\Rightarrow f.\frac{h}{2} = N\, x$



$mg \sin\theta.\ \frac{h}{2} = mg \cos\theta.x$

$x = \frac{\tan\theta.h}{2}$

Maximum value of x is a/2

so to prevent toppling $x \le \frac{a}{2}$

$\Rightarrow \frac{\tan\theta . h}{2} \le a/2$

$\Rightarrow \tan\theta \le \frac{a}{h}$ ...(2)

So, the block topple before sliding from (1) & (2)

$\mu_s > \frac{a}{h}$

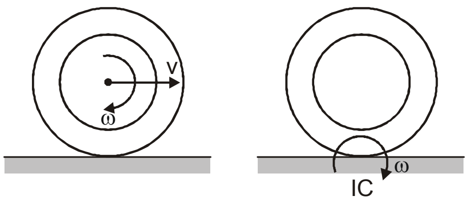
Note

The student can now attempt section G from exercise.

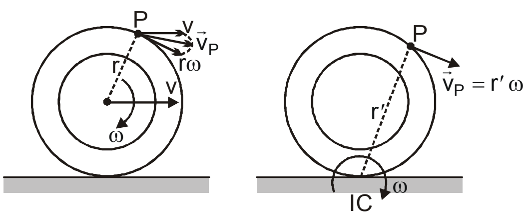
Section H - Instantaneous Axis of Rotation

14. INSTANTANEOUS AXIS OF ROTATION

The combined effects of translation of the centre of mass and rotation about an axis through the centre of mass are equivalent to a pure rotation with the same angular speed about an axis passing through a point of zero velocity. Such an axis is called the instantaneous axis of rotation. (IAOR). This axis is always perpendicular to the plane used to represent the motion and the intersection of the axis with this plane defines the location of instantaneous centre of zero velocity (IC).



For example consider a wheel which rolls without slipping. In this case the point of contact with the ground has zero velocity. Hence, this point represents the IC for the wheel. If it is imagined that the wheel is momentarily pinned at this point, the velocity of any point on the wheel can be found using v = rω. Here r is the distance of the point from IC. Similarly, the kinetic energy of the body can be assumed to be pure rotational about IAOR or,



$$K = \frac{1}{2} I_{IAOR}\omega^2$$

Rotation + Translation ⇒ Pure rotation about IAOR passing through IC

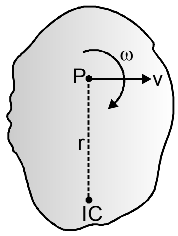
$$KE = \frac{1}{2}mv_{COM}^2 + \frac{1}{2}I_{COM}\omega^2 \Rightarrow KE = \frac{1}{2}I_{IAOR}\omega^2$$

14.1 Location of the IC

If the location of the *IC* is unknown, it may be determined by using the fact that the relative position vector extending from the IC to a point is always perpendicular to the velocity of the point. Following three possibilities exist.

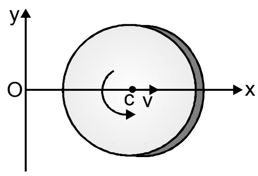
(i) Given the velocity of a point (normally the centre of mass) on the body and the angular velocity of the body

If v and ω are known, the IC is located along the line drawn perpendicular to $\vec{v}$ at P, such that the distance from P to IC is, $r = \frac{v}{\omega}$. Note that IC lie on that side of P which causes rotation about the IC, which is consistent with the direction of motion caused by $\vec{\omega}$ and $\vec{v}$.



EXAMPLE 54

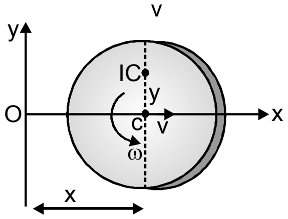
A rotating disc moves in the positive direction of the x-axis. Find the equation y(x) describing the position of the instantaneous axis of rotation if at the initial moment the centre c of the disc was located at the point O after which it moved with constant velocity v while the disc started rotating counter clockwise with a constant angular acceleration α. The initial angular velocity is equal to zero.



Sol. $t = \frac{x}{v}$ and $\omega = \alpha t = \frac{\alpha x}{v}$

The position of IAOR will be at a distance

$$y = \frac{v}{\omega} \quad \text{or} \quad y = \frac{v}{\frac{\alpha x}{v}}$$

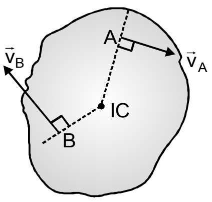


or $y = \frac{v^2}{\alpha x}$ or $xy = \frac{v^2}{\alpha} = \text{constant}$

This is the desired x-y equation. This equation represents a rectangular hyperbola.

(ii) Given the lines of action of two non-parallel velocities

Consider the body shown in figure where the line of action of the velocities $\vec{v}_A$ and $\vec{v}_B$ are known. Draw perpendiculars at A and B to these lines of action. The point of intersection of these perpendiculars as shown locates the IC at the instant considered.

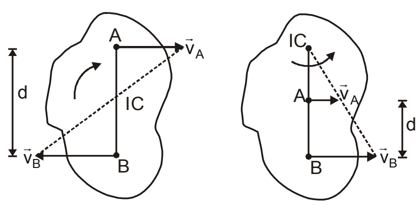


(iii) Given the magnitude and direction of two parallel velocities

When the velocities of points A and B are parallel and have known magnitudes v_A and v_B then the location of the IC is determined by proportional triangles as shown in figure.

In both the cases, $r_{A,IC} = \frac{v_A}{\omega}$

and $r_{B,IC} = \frac{v_B}{\omega}$



(a) (b)

In fig. (a) $r_{A,IC} + r_{B,IC} = d$

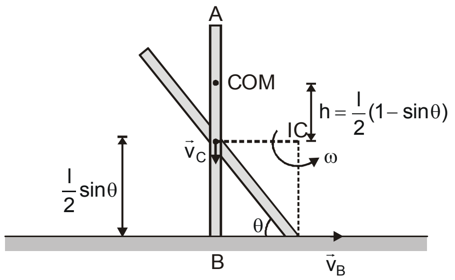
and in fig (b) $r_{B,IC} - r_{A,IC} = d$

As a special case, if the body is translating, $v_A = v_B$ and the IC would be located at infinity, in which case $\omega = 0$.

EXAMPLE 55

A uniform thin rod of mass m and length l is standing on a smooth horizontal surface. A slight disturbance causes the lower end to slip on the smooth surface and the rod starts falling. Find the velocity of centre of mass of the rod at the instant when it makes an angle θ with horizontal.

Sol. As the floor is smooth, mechanical energy of the rod will remain conserved. Further, no horizontal force acts on the rod, hence the centre of mass moves vertically downwards in a straight line. Thus velocities of COM and the lower end B are in the direction shown in figure. The location of IC at this instant can be found by drawing perpendiculars to $\vec{v}_C$ and $\vec{v}_B$ at respective points. Now, the rod may be assumed to be in pure rotational motion about IAOR passing through IC with angular speed ω.



Applying conservation of mechanical energy. Decrease in gravitational potential energy of the rod = increase in rotational kinetic energy about IAOR

$$\therefore \quad mgh = \frac{1}{2} I_{IAOR}\omega^2 \quad \text{or}$$

$$mg\frac{l}{2}(1 - \sin\theta) = \frac{1}{2}\left(\frac{ml^2}{12} + \frac{ml^2}{4}\cos^2\theta\right)\omega^2$$

Solving this equation, we get

$$\omega = \sqrt{\frac{12g(1 - \sin\theta)}{l(1 + 3\cos^2\theta)}}$$

Now, $|\vec{v}_C| = \left(\frac{l}{2}\cos\theta\right)\omega$

$$= \sqrt{\frac{3gl(1 - \sin\theta)\cos^2\theta}{(1 + 3\cos^2\theta)}} \quad \textbf{Ans.}$$

Note

The student can now attempt section H from exercise.

Exercise - 1 Objective Problems | JEE Main

Section A - Moment of Inertia, Theorems of Moment of Inertia, Radius of gyration, Cavity problems

1. The moment of inertia of a body depends upon -
(A) mass only
(B) angular velocity only
(C) distribution of particles only
(D) mass and distribution of mass about the axis

2. Two spheres of same mass and radius are in contact with each other. If the moment of inertia of a sphere about its diameter is I, then the moment of inertia of both the spheres about the tangent at their common point would be -
(A) 3I (B) 7I
(C) 4I (D) 5I

3. Mass M is distributed over the rod of length L. If linear mass density (λ) of the rod is linearly increasing with length as l = Kx, where x is measured from one end as shown in figure & K is contant. The moment of inertia. of the rod about the end perpendicular to rod where linear mass density is zero.

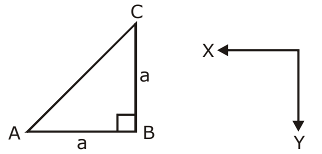
(A) $\frac{ML^2}{3}$ (B) $\frac{ML^2}{12}$

(C) $\frac{2}{3}ML^2$ (D) $\frac{KL^4}{4}$

4. A disc of metal is melted to recast in the form of a solid sphere. The moment of inertias about a vertical axis passing through the centre would -
(A) decrease
(B) increase
(C) remains same
(D) nothing can be said

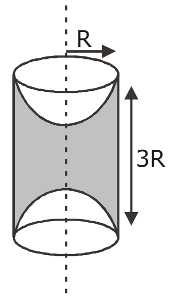
5. The M.I. of a disc about its diameter is 2 units. Its M.I. about axis through a point on its rim and in the plane of the disc is
(A) 4 units.
(B) 6 units
(C) 8 units
. (D) 10 units

6. A rigid body in shape of a triangle has v_A = 5 m/s down, v_B = 10 m/s down. Find velocity of point C.



(A) $5\hat{i}+10\hat{j}$ (B) $-5\hat{i}-10\hat{j}$
(C) $-5\hat{i}+10\hat{j}$ (D) $5\hat{i}-10\hat{j}$

7. A solid cylinder of radius R & length 3R is made of a metarial having density ρ. Now two hemispheres each of radius R is removed from two ends of cylinder as shown in figure. Determine the moment of inertia of this object about axis of cylinder?



(A) $\frac{11\pi R^5\rho}{18}$

(B) $\frac{26\pi R^5\rho}{3}$

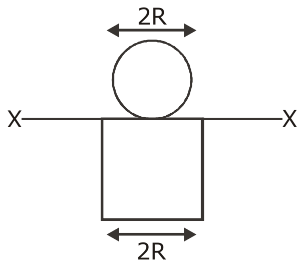
(C) $\frac{29\pi R^5\rho}{30}$

(D) $\frac{26\pi R^5\rho}{15}$

8. A solid sphere and a hollow sphere of the same mass have the same moments of inertia about their respective diameters, the ratio of their radii is
(A) $(5)^{1/2} : (3)^{1/2}$ (B) $(3)^{1/2} : (5)^{1/2}$
(C) 3 : 2 (D) 2 : 3

9. A stone of mass 4kg is whirled in a horizontal circle of radius 1m and makes 2 rev/sec. The moment of inertia of the stone about the axis of rotation is
(A) 64 kg × m^2 (B) 4 kg × m^2
(C) 16 kg × m^2 (D) 1 kg × m^2

10. A circle and a square (wire frame) each of mass m are arranged as shown in the diagram. The diameter of the circle and the edge of the square are 2R each. Find moment of inertia of this configuration about XX.



(A) $\frac{17mR^2}{6}$

(B) $\frac{11mR^2}{6}$

(C) $\frac{19mR^2}{6}$

(D) $\frac{13mR^2}{6}$

Section B - Torque (about point, about axis), Torque and angular Acceleration

11. A body is rotating uniformly about a vertical axis fixed in an inertial frame. The resultant force on a particle of the body not on the axis is
(A) vertical
(B) horizontal and skew with the axis
(C) horizontal and intersecting the axis
(D) none of these

12. One end of a uniform rod of mass m and length I is clamped. The rod lies on a smooth horizontal surface and rotates on it about the clamped end at a uniform angular velocity ω. The force exerted by the clamp on the rod has a horizontal component
(A) $m\omega^2 l$ (B) zero
(C) mg (D) $\frac{1}{2}m\omega^2\ell$

13. A rod of length 'L' is hinged from one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is
(A) $\sqrt{\frac{2g}{L}}$ (B) $\sqrt{\frac{3g}{L}}$
(C) $\sqrt{\frac{g}{2L}}$ (D) $\sqrt{\frac{g}{L}}$

14. A disc of radius 2m and mass 200kg is acted upon by a torque 100N-m. Its angular acceleration would be
(A) 1 rad/sec^2 (B) 0.25 rad/sec^2
(C) 0.5 rad/sec^2. (D) 2 rad/sec^2.

15. On applying a constant torque on a body-
(A) linear velocity may be increases
(B) angular velocity may be increases
(C) it will rotate with constant angular velocity
(D) it will move with constant velocity

16. A wheel starting with angular velocity of 10 radian/sec acquires angular velocity of 100 radian/sec in 15 seconds. If moment of inertia is 10kg-m^2, then applied torque (in newton-metre) is
(A) 900 (B) 100
(C) 90 (D) 60

17. An automobile engine develops 100H.P. when rotating at a speed of 1800 rad/min. The torque it delivers is
(A) 3.33 W-s (B) 200W-s
(C) 248.7 W-s (D) 2487 W-s

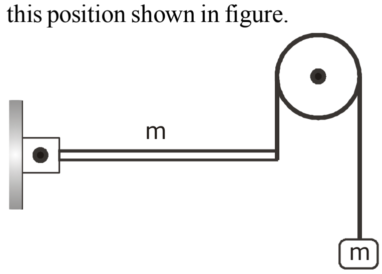
18. The moment of inertia and rotational kinetic energy of a fly wheel are 20kg-m^2 and 1000 joule respectively. Its angular frequency per minute would be –
(A) $\frac{600}{\pi}$ (B) $\frac{25}{\pi^2}$
(C) $\frac{5}{\pi}$ (D) $\frac{300}{\pi}$

19. The angular velocity of a body is $\vec{\omega} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and a torque $\vec{\tau} = \hat{i} + 2\hat{j} + 3\hat{k}$ acts on it. The rotational power will be
(A) 20 watt (B) 15 watt
(C) $\sqrt{17}$ watt (D) $\sqrt{14}$ watt

20. A torque of 2 newton-m produces an angular acceleration of 2 rad/sec^2 a body. If its radius of gyration is 2m, its mass will be:
(A) 2kg (B) 4 kg
(C) 1/2 kg (D) 1/4 kg

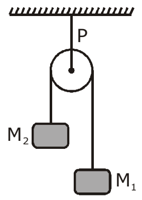
Section C - Pulley Block system

21. A rod and a block are of same mass. Initially rod is in horizontal position. What will be acceleration of tip of the rod just after the system is released from this position shown in figure.



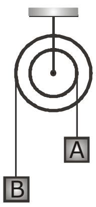
(A) zero (B) $\frac{3g}{4}$
(C) $\frac{3g}{8}$ (D) $\frac{3g}{2}$

22. In the figure, the blocks have masses M_1 and M_2 ($M_1 > M_2$) and acceleration a. The pulley P has a radius r and some mass the string not slip on the pulley.



(A) The two sections of the string have unequal tensions
(B) The two blocks have acceleration of unequal magnitude
(C) The angular acceleration of P is 'ar'
(D) angular acceleration of the pulley is zero.

23. A 0.6 m radius drum carrying the load A is rigidly attached to a 0.9m radius pulley carrying the load B as shown. At the time t=0, the load B moves with a velocity of 2m/s (downward) and a constant acceleration of 3m/s^2 (downward). Over the time interval $0 \le t \le 2$s, determine

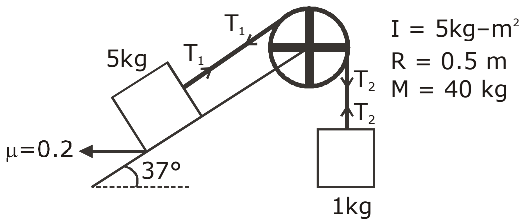


The number of revolution executed by the pulley and the displacement of the load A.

(A) $\frac{100}{3\pi}, \frac{20}{3}$m (B) $\frac{50}{9\pi}, \frac{20}{3}$m

(C) $\frac{50}{9\pi}, \frac{10}{3}$m (D) $\frac{25}{9\pi}, \frac{20}{3}$m

24. For the situation shwn in figure, if the system is released from rest. Determine tension T_1 and T_2. (Take g = 10 m/s^2)



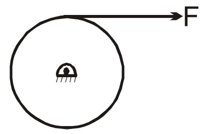
(A) $T_1 = \frac{256}{13}N; T_2 = \frac{136}{13}N$

(B) $T_1 = \frac{125}{13}N; T_2 = \frac{136}{13}N$

(C) $T_1 = \frac{256}{13}N; T_2 = \frac{272}{13}N$

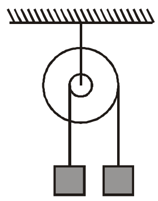
(D) $T_1 = \frac{128}{13}N; T_2 = \frac{136}{13}N$

25. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As the time increases,



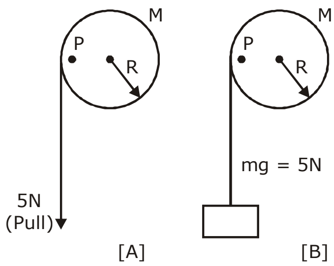
(A) its angular velocity increases, but force on hinge remains constant
(B) its angular velocity remains same, but force on hinge increases
(C) its angular velocity increases and force on hinge increases
(D) its angular velocity remains same and force on hinge is constant.

26. In the situation shown, a heavy wheel with a small drum attached is suspended by its frictionless axle from a ceiling. Attached to strings around the rims of the wheel and drum are two blocks of equal mass. The system is originally at rest. When the blocks are released.



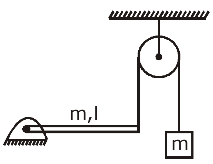
(A) Nothing happens, since the blocks have equal mass.
(B) The right hand block falls and the left hand one rises, with acceleration of the same magnitude.
(C) While the blocks are moving, the tension in the right hand string is less than that in the left hand string.
(D) Which block falls depends on the moment of inertia of the wheel–drum system.

27. A uniform disc of mass M = 2.50 kg and radius R = 0.20 m is mounted on an axle supoorted on fixed frictionless bearings. A light cord wrapped around the rim is pulled with a force 5N. On the same system of pulley and string, instead of pulling it down, a body of weight 5 N is suspended. If the first process is termed A and the second B, the tangential acceleration of point P will be



(A) equal in the processes A and B
(B) greater is process A than in B
(C) greater in process B than in A
(D) independent of the two processes

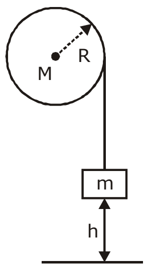
28. Uniform rod AB is hinged at end A in horizontal position as shown in the figure. The other end is connected to a block through a massless string m as shown. The pulley is smooth and massless. Masses of block and rod is same and is equal to m. Then find the reaction force at the hinge H. Just after release.



(A) $\frac{6mg}{13}$ (B) $\frac{g}{4}$

(C) $\frac{3mg}{8}$ (D) None

29. A solid uniform cylinder of mass M and radius R is pivoted at its centre free to rotate about horizontal axis. A massless inextensible string is wrapped around it, and attached to a block of mass m which is initially at a height h above the floor. The acceleration due to gravity is g, directed downward. The block is released from rest. By what total angle $\Delta\theta$ (in radians)



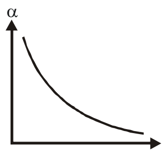
(A) $\sqrt{1+\frac{2M}{m}}$

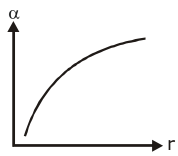
(B) $\frac{h}{R}$

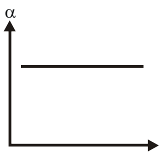
(C) $\sqrt{1+\frac{h}{2R}}$

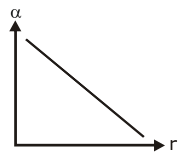
(D) $\frac{MR}{mh}$

30. A magnetic tape isbeing played on a cassette deck. The tension in the tape applies a torque to the supply reel. Assuming the tension remains constant during play, plot this angular acceleration ith reel radius r as the reel becomes empty. Neglect the moment of inertia of empty reel.

(A) 

(B) 

(C) 

(D) 

Section D - Angular Momentum (about point, about axis), Conservation of Angular Momentum

31. A particle moves with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin.
(A) is zero
(B) remains constant
(C) goes on increasing
(D) goes on decreasing.

32. A person sitting firmly over a rotating stool has his arms streatched. If he folds his arms, his angular momentum about the axis of rotation
(A) increases
(B) decreases
(C) remains unchanged
(D) doubles.

33. A man, sitting firmly over a rotating stool has his arms streched. If he folds his arms, the work done by the man is

(A) zero

(B) positive

(C) negative

(D) may be positive or negative.

34. A particle of mass 2 kg located at the position $(\hat{i}+\hat{j})$ m has a velocity $2(+\hat{i}-\hat{j}+\hat{k})$ m/s. Its angular momentum about z-axis in kg-m^2 /s is :

(A) zero (B) +8

(C) 12 (D) – 8

35. The angular momentum of a flywheel having a moment of inertia of 0.4 kg m^2 decreases from 30 to 20 kg m^2/s in a period of 2 second. The average torque acting on the flywheel during this period is :

(A) 10 N.m (B) 2.5 N.m

(C) 5 N.m (D) 1.5 N.m

36. The rotational kinetic energy of a rigid body of moment of inertia 5 kg-m^2 is 10 joules. The angular momentum about the axis of rotation would be -

(A) 100 joule-sec (B) 50 joule-sec

(C) 10 joule-sec (D) 2 joule -sec

37. The angular velocity of a body changes from one revolution per 9second to 1 revolution per second without applying any torque. The ratio of its radius of gyration in the two cases is

(A) 1 : 9 (B) 3 : 1

(C) 9 : 1 (D) 1 : 3

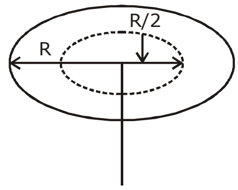
38. A dog of mass m is walking on a pivoted disc of radius R and mass M in a circle of radius R/2 with an angular frequency n: the disc will revolve in opposite direction with frequency -

(A) $\frac{mn}{M}$

(B) $\frac{mn}{2M}$

(C) $\frac{2mn}{M}$

(D) $\frac{2Mn}{M}$



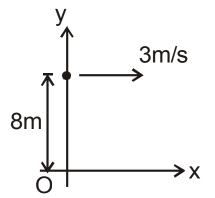
39. A particle starts from the point (0m, 8m) and moves with uniform velocity of 3 $\hat{i}$ m/s. After 5 seconds, the angular velocity of the particle about the origin will be :

(A) $\frac{8}{289}$ rad / s

(B) $\frac{3}{8}$ rad / s

(C) $\frac{24}{289}$ rad / s

(D) $\frac{8}{17}$ rad / s



40. A uniform rod kept vertically on the ground falls from rest. Its foot does not slip on the ground

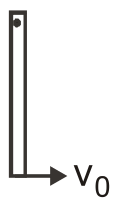
(A) No part of the rod can have acceleration greater than g in any positive

(B) For any one position of the rod, different points on it have different accelerations

(C) The maximum acceleration on any point on the rod, at any position, is 1.5 g

(D) None of these

41. A thin uniform rigid rod of legth/is hinged at once and so that it can move in a vertical plane by rotating about a horizontal axis through upper end. The lower and is given a sharp blow and made to acquire a linear velocity v_0. Find the maximum height to which the lower end can rise.



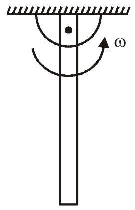
(A) 2l If $V_0 > \sqrt{5gl}$, $\frac{V_0^2}{3g}$ if $V_0 < \sqrt{6gl}$

(B) 2l If $V_0 > \sqrt{6gl}$, $\frac{V_0^2}{3g}$ if $V_0 < \sqrt{6gl}$

(C) 2l If $V_0 > \sqrt{6gl}$, $\frac{V_0^2}{3g}$ if $V_0 < \sqrt{5gl}$

(D) l If $V_0 > \sqrt{6gl}$, $\frac{V_0^2}{3g}$ if $V_0 < \sqrt{6gl}$

42. A thin uniform rod of mass m and length l is free to rotate about an horizontal axis as shown in figure. The minimum initial angular velocity imparted to rod so that it becomes horizontal is



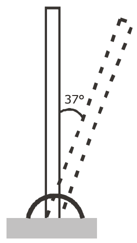
(A) $\sqrt{\frac{g}{l}}$

(B) $\sqrt{\frac{3g}{l}}$

(C) $\sqrt{\frac{2g}{l}}$

(D) $\sqrt{\frac{3g}{2l}}$

43. A uniform rod of mass M & length L is hinged about its one end as shown. Initially it is held vartical and then allowed to rotate, the angular velocity of rod when it makes an angle of 37° with the vertical is



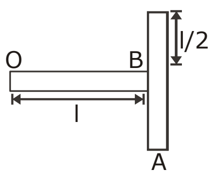
(A) $\sqrt{\frac{12g}{5L}}$

(B) $\sqrt{\frac{3g}{5L}}$

(C) $\sqrt{\frac{g}{5L}}$

(D) $\sqrt{\frac{g}{L}}$

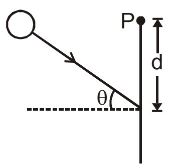
44. Two identical rode are joined as shown. The system is pivoted at point O and is released from rest from the horizontal positoin. The speed of point A when OB because vertical is



(A) $6\sqrt{\frac{gl}{17}}$ (B) $3\sqrt{\frac{5gl}{17}}$

(C) $4\sqrt{\frac{3gl}{17}}$ (D) $2\sqrt{\frac{15gl}{17}}$

45. A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is :



(A) 2 *mvd* (B) 2 *mvd* cosθ

(C) 2 *mvd* sinθ (D) zero

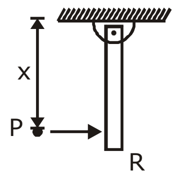
46. A uniform rod of mass M has an impulse applied at right angles to one end. If the other end begins to move with speed V, the magnitude of the impulse is

(A) MV (B) $\frac{MV}{2}$

(C) 2MV (D) $\frac{2MV}{3}$

Section E - **Angular Impulse + Collision Of point Mass with Rigid Bodies**

47. A particle P strikes the rod R perpendicularly as shown. The rod is suspended vertically with upper end hinged. ($x = \frac{l}{2}$, elastic collision) Then select correct statement :



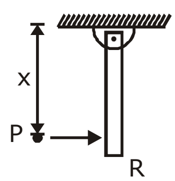
(A) Linear momentum of P,R system increases

(B) Linear momentum of P,R system decreases

(C) Linear momentum of P,R system remains constant

(D) Data is insufficient

48. A particle P strikes the rod R perpendicularly as shown. The rod is suspended vertically with upper end hinged. (x = l, elastic collision) Then select incorrect statement :



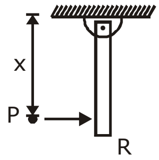
(A) Linear momentum of P,R system increases

(B) KE of the P decreases

(C) Angular momentum of the P,R system is conserved about image.

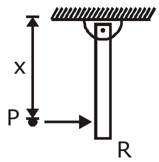
(D) None of these

49. A particle P strikes the rod R perpendicularly as shown. The rod is suspended vertically with upper end hinged. ($x = \frac{l}{2}$, P sticks to R) Then select correct statement :



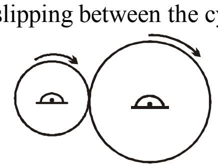
(A) Linear momentum of P,R system decreases
(B) KE of the P decreases
(C) Angular momentum of the P,R system is conserved about image.
(D) All of the above

50. A particle P strikes the rod R perpendicularly as shown. The rod is suspended vertically with upper end hinged. (x = l, P sticks to R) Then select correct statement :



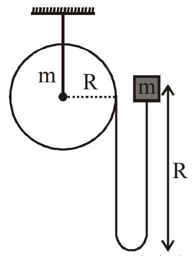
(A) Linear momentum of P,R system increases
(B) KE of the P decreases
(C) Angular momentum of the P,R system is conserved about image.
(D) All of the above

51. Two separate cylinders of masses m (= 1kg) and 4m and radii R(= 10 cm) and 2R rotating in clock-wise direction with ω_1 100 rad/sec and ω_2 = 200 rad/sec. Now they are held in contact with each other as in fig. Determine their angular velocities after the slipping between the cylinders stops.



(A) 150 rad/sec, 150 rad/sec
(B) 150 rad/sec, 300 rad/sec
(C) 300 rad/sec, 150 rad/sec
(D) 300 rad/sec, 300 rad/sec

52. Mass of the pulley is m & radius is R. Assume pulley to be disc. Block of mass m is released from the position shown. String is massless & inextensible. There is no slipping between rope and pulley. The impulse exerted by the string on the pulley at the moment string becomes taut is $\frac{2m\sqrt{gR}}{J}$ is equal to



(A) 5 (B) 3
(C) 1 (D) None of these

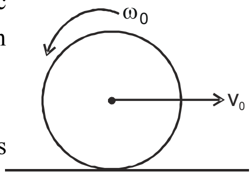
Section F - Combined Translational and Rotational Motion of Rigid body, Pure Rolling, Slipping

53. A ring of radius R rolls without sliding with a constant velocity. The radius of curvature of the path followed by any particle of the ring at the highest point of its path will be
(A) R (B) 2R
(C) 4R (D) none

54. The linear speed of a uniform spherical shell after rolling down an inclined plane of vertical height h from rest, is :
(A) $\sqrt{\frac{10gh}{7}}$ (B) $\sqrt{\frac{4gh}{5}}$
(C) $\sqrt{\frac{6gh}{5}}$ (D) $\sqrt{2gh}$

55. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :
(A) thin pipe
(B) uniform cylinder
(C) uniform sphere
(D) thin spherical shell

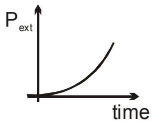
56. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then $\frac{v_0}{r\omega_0}$ is

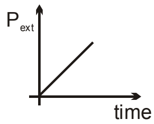


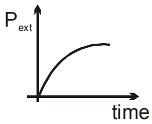
(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

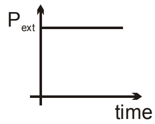
57. Consider the following statements
(A) both A and R are true and R is the correct explanation of A
(B) both A and R are true but R is not the correct explanation of A
(C) A is true but R is false
(D) A is false but R is true

58. A rod is hinged at its centre and rotated by applying a constant torque starting from rest. The power developed by the external torque as a function of time is :

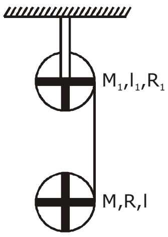
(A)


(B)


(C)


(D)


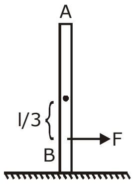
59. A string is would over a cylinder of mass M, radius R and moment of inertia I and then the string is wound over the pulley as shown in the figure. If the system is released from rest then determine the tension is string. Assume there is no slipping between string and pulley cylinder.



(A) $\frac{g}{M+\frac{I}{R^2}}$ (B) $\frac{g}{M+\frac{I}{R^2}+\frac{I_1}{R_1^2}}$

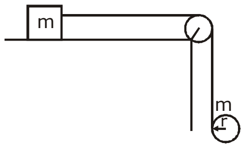
(C) $\frac{g}{M+\frac{I}{R^2}+\frac{2I_1}{R_1^2}}$ (D) None of these

60. A uniform rod ABC of mass M is placed vertically on a rough horizontal surface. The coefficient of kinetic friction between the rod and the surface is μ. A force F (> μ mg) is applied on the rod at point B at distance ℓ/3 below centre of the rod as shown in figure. The initial acceleration of point A is



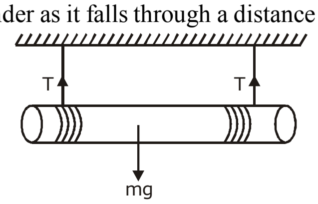
(A) $\mu g - \frac{F}{M}$
(B) $\frac{F}{M}$
(C) 4 μg
(D) $2\mu g - \frac{F}{M}$

61. A light thread is wound on a disk of mass m and other end of tjhread is connected to a block of mass m, which is placed on a rough ground as shown in diagram. Find the minimum value of coefficient of friction for which block remain at rest.



(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{1}{5}$

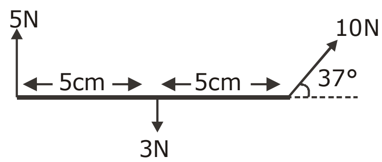
62. A cylinder of mass m is suspended through two strings wrapped around it as shown in figure. Find the tension T in the string and the speed of the cylinder as it falls through a distance h.



(A) $\frac{mg}{6},\sqrt{\frac{gh}{3}}$ (B) $\frac{mg}{6},\sqrt{\frac{2gh}{3}}$

(C) $\frac{mg}{3},\sqrt{\frac{4gh}{3}}$ (D) $\frac{mg}{6},\sqrt{\frac{4gh}{3}}$

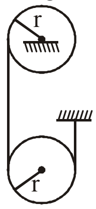
63. Determine the point of application of resultant force, when forces acting on the rod are as shown in figure.



(A) 2.5 cm right on the rod from the point where 5 N force is acting.
(B) 1.8 cm right on the rod from the point where 5 N force is acting.

(C) 5.625 cm right on the rod from the point where 5 N force is acting.
(D) None of these

64. Two uniform cylinders, each of mass m = 10 kg and radius r = 150 mm, are connected by a rough belt as shown. If the system is released from rest, determine
the velocity of the centre of cylinder A after it has moved through 1.2 m & the tension in the portion of the belt connecting the two cylinders.



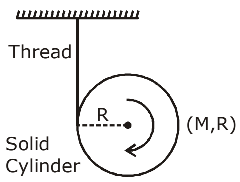
(A) $\sqrt{\frac{3}{7}}$ m/s ; $\frac{200}{7}$ N

(B) $4\sqrt{\frac{3}{7}}$ m/s ; $\frac{200}{7}$ N

(C) $4\sqrt{\frac{3}{7}}$ m/s ; $\frac{100}{7}$ N

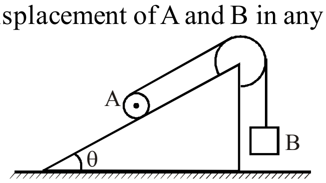
(D) $\sqrt{\frac{3}{7}}$ m/s ; $\frac{100}{7}$ N

65. Thread would around cylinder of mass M and radius R. It is allowed to fall as shown. Find its acceleration.



(A) $\frac{g}{3}$ (B) $\frac{4g}{3}$
(C) $\frac{2g}{3}$ (D) None

66. In the figure shown, a string is wound over a cylinder A. The other end of the string is attached to block B through a pulley. When the system is released the cylinder rolls down without slipping. The ratio of the magnitude of vertical component of displacement of A and B in any time interval t
(A) sin θ : 1
(B) sin θ : 2
(C) cos θ : 1
(D) cos θ : 2



67. A disc rolls on a table. The ratio of its K.E. of rotation to the total K.E. is -
(A) 2/5 (B) 1/3 (C) 5/6 (D) 2/3

68. A disk and a ring of the same mass are rolling to have the same kinetic energy. What is ratio of their velocities of centre of mass
(A) $(4:3)^{1/2}$ (B) $(3 : 4)^{1/2}$
(C) $(2)^{1/2} : (3)^{1/2}$ (D) $(3)^{1/2} : (2)^{1/2}$

69. A solid sphere, a hollow sphere and a disc, all having smooth incline and released. Least time will be taken in reaching the bottom by
(A) the solid sphere (B) the hollow sphere
(C) the disc (D)all will take same time.

70. The centre of a wheel rolling without slipping in a plane surface moves with speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed.
(A) zero (B) v_0
(C) $\sqrt{2}v_0$ (D) $2v_0$

71. A wheel of radius r rolling on a straight line, the velocity of its centre being v. At a certain instant the point of contact of the wheel with the grounds is M and N is the highest point on the wheel (diametrically opposite to M). The incorrect statement is :
(A) The velocity of any point P of the wheel is proportional to MP.
(B) Points of the wheel moving with velocity greater than v form a larger area of the wheel than points moving with velocity less than v.
(C) The point of contact M is instantaneously at rest.
(D) The velocities of any two parts of the wheel which are equidistant from centre are equal.

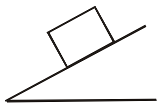
72. If the applied torque is directly proportional to the angular displacement θ, then the work done in rotating the body through an angle θ would be – (C is constant of proportionality)

(A) Cθ (B) $\frac{1}{2}$ Cθ (C) $\frac{1}{2}$ $C\theta^2$ (D) $C\theta^2$

Section G - Toppling + Direction of Friction

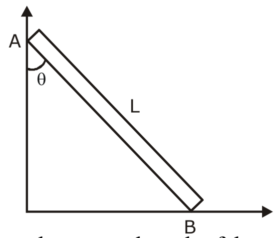
73. A Cubical bloc of mass M and edge a slides down a rough inclined plane of inclination θ with a uniform velocity. The torque of the normal force on the block about its centre has a magnitude.
(A) zero (B) Mga
(C) Mga sin θ (D) $\frac{1}{2}$Mga sin θ

74. A homogeneous cubical brick lies motionless on a rough inclined surface. The half of the brick which applies greater pressure on the plane is :



(A) left half
(B) right half
(C) both applies equal pressure
(D) the answer depend upon coefficient of friction

75. A rod AB of length L slides in the XY plane. If the rod makes an angle θ with the vertical, the angular velocity of the rod can be found by an expression which is



(A) dependent upon length of the rod and on linear velocity of end A of the rod at that instant.
(B) dependent upon L and θ at that instant.
(C) independent of velocity of end A of rod at that instant.
(D) dependent upon L, θ and velocity of end A of rod at that instant.

76. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. Find the minimum value of F for which the cube begins to tip about the edge? (Assume that the cube does not slide).

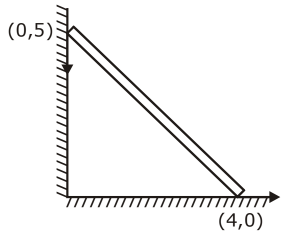
(A) $\frac{5mg}{3}$ (B) $\frac{2mg}{3}$
(C) $\frac{mg}{3}$ (D) None of these

Section H - Instantaneous Axis of Rotation

77. There is rod of length l. The velocities of its two ends are v_1 and v_2 is opposite directions normal to the rod. The distance of the instantaneous axis of rotation from v_1 is :

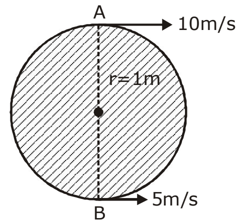
(A) zero (B) $\frac{v_2}{v_1 + v_2} l$
(C) $\frac{v_1 l}{v_1 + v_2}$ (D) l/2

78. Find the coordinates of IAOR



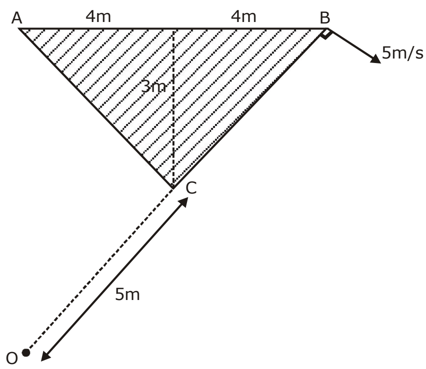
(A) (4,5) (B) (4,5)
(C) (5,4) (D) None of these

79. Find the distance of instantaneous point of rest, of the disc, from point B.



(A) 4 m (B) 2 m
(C) 1 m (D) 6 m

80. Find the angular speed of the triangular plate & speed of point A, if instantaneous axis of rotation passes through point O as shown in figure.



(A) $\frac{1}{2}$ rad / sec ; 3 m/sec
(B) $\frac{1}{4}$ rad / sec ; 6 m/sec
(C) $\frac{1}{2}$ rad / sec ; 6 m/sec
(D) $\frac{1}{4}$ rad / sec ; 3 m/sec

Section A - Moment of Inertia, Theorems of Moment of Inertia, Radius of gyration, Cavity problems

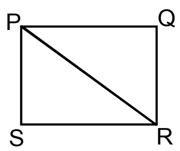
1. Three bodies have equal masses m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)

(A) A (B) B
(C) C (D) A and C both

2. Two rods of equal mass m and length *l* lie along the x axis and y axis with their centres origin. What is the moment of inertia of both about the line x = y :

(A) $\frac{ml^2}{3}$ (B) $\frac{ml^2}{4}$
(C) $\frac{ml^2}{12}$ (D) $\frac{ml^2}{6}$

3. Moment of inertia of a rectangular plate about an axis passing through P and perpendicular to the plate is I. Then moment of PQR about an axis perpendicular to the plane of the plate :

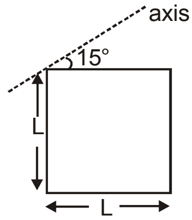


(A) about P = I/2 (B) about R = I/2
(C) about P > I/2 (D) about R > I/2

4. A thin uniform rod of mass M and length L has its moment of inertia I_1 about its perpendicular bisector. The rod is bend in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is I_2. The ratio of $I_1 : I_2$ will be __

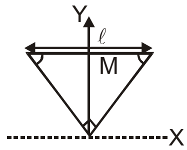
(A) < 1 (B) > 1
(C) = 1 (D) can't be said

5. A square plate of mass M and edge L is shown in figure. The moment of inertia of the plate about the axis in the plane of plate passing through one of its vertex making an angle 15° from horizontal is.



(A) $\frac{ML^2}{12}$ (B) $\frac{11ML^2}{24}$
(C) $\frac{7\,ML^2}{12}$ (D) none

Question No. 6 to 9 (4 questions)
The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to X - axis.



6. The moment of inertia of the plate about the z-axis is

(A) $\frac{ML^2}{12}$ (B) $\frac{ML^2}{24}$
(C) $\frac{ML^2}{6}$ (D) none of these

7. The moment of inertia of the plate about the x-axis is

(A) $\frac{ML^2}{8}$ (B) $\frac{ML^2}{32}$
(C) $\frac{ML^2}{24}$ (D) $\frac{ML^2}{6}$

8. The moment of inertia of the plate about its base parallel to the x-axis is

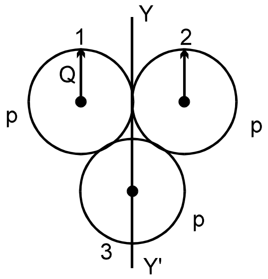
(A) $\frac{ML^2}{18}$ (B) $\frac{ML^2}{36}$
(C) $\frac{ML^2}{24}$ (D) none of these

9. The moment of inertia of the plate about the y-axis is

(A) $\frac{ML^2}{6}$ (B) $\frac{ML^2}{8}$

(C) $\frac{ML^2}{24}$ (D) none of these

10. Three rings, each of mass P and radius Q are arranged as shown in the figure. The moment of inertia of the arrangement about YY' axis will be



(A) $\frac{7}{2}PQ^2$ (B) $\frac{2}{7}PQ^2$

(C) $\frac{2}{5}PQ^2$ (D) $\frac{5}{2}PQ^2$

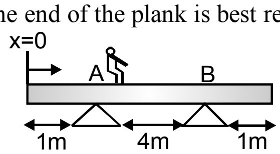
Section B - Torque (about point, about axis), Torque and angular Acceleration

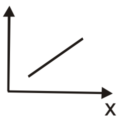
11. A horizontal force F = mg/3 is applied on the upper surface of a uniform cube of mass 'm' and side 'a' which is resting on a rough horizontal surface having μ_s = 1/2. The distance between lines of action of 'mg' and normal reaction 'N' is :

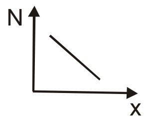
(A) a/2 (B) a/3

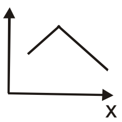
(C) a/4 (D) None

12. A man can move on a horizontal plank supported symmetrically as shown. The variation of normal reaction on support A with distance x of the man from the end of the plank is best represented by :



(A)


(B)


(C)


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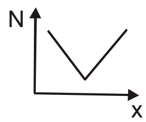
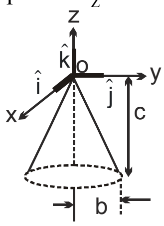


13. A solid cone hangs from a frictionless pivot at the origin O, as shown. If $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors, and a, b, and c are positive constants, which of the following forces F applied to the rim of the cone at a point P results in a torque τ on the cone with a negative component τ_Z ?



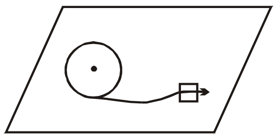
(A) F = $a\hat{k}$, P is (0, b, –c)

(B) F = $-a\hat{k}$, P is (0, –b, –c)

(C) F = $a\hat{j}$, P is (–b, 0, –c)

(D) None

14. A block of mass m is attached to a pulley disc of equal mass m, radius r by means of a slack string as shown. The pulley is hinged about its centre on a horizontal table and the block is projected with an initial velocity of 5 m/s. Its velocity when the string becomes taut will be

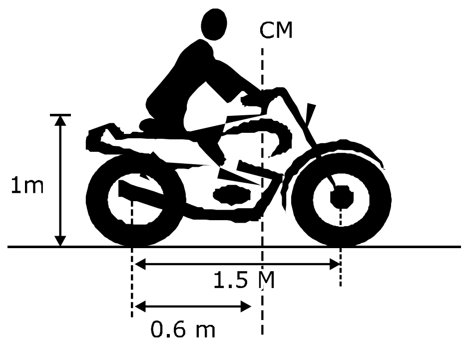


(A) 3 m/s (B) 2.5 m/s

(C) 5/3 m/s (D) 10/3 m/s

Paragraph for question(15-17)

There is a man of mass 100 kg sitting on a motorbike of mass 150 kg. The distance of the axles of the wheels (wheelbase) is 1.5 m, the common mass center of the man and the motorbike is at a 1 m height above the ground level, and at a 0.6 m distance from the vertical line going through the rear axle.

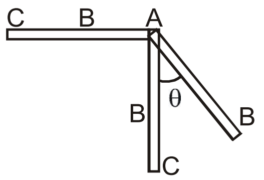


15. What normal force is exerted on the real wheel by the ground, when the motorbike starts off with an acceleration of a = 2 m/s^2?
(A) 1000 N (B) 1250 N
(C) $\frac{2000}{3}$N (D) $\frac{5500}{3}$N

16. What acceleration is needed to lift the front wheel?
(A) 10 m/s^2 (B) 5 m/s^2
(C) 6 m/s^2 (D) None of these

17. What minimum friction coefficient can ensure the above acceleration ?
(A) 1 (B) 0.6
(C) 0.5 (D) 0.8

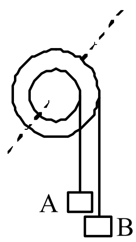
18. A rod hinged at one end is released from the horizontal position as shown in the figure. When it becomes vertical its lower half separates without exerting any reaction at the breaking point. Then the maximum angle 'θ' made by the hinged upper half with the vertical is :



(A) 30° (B) 45°
(C) 60° (D) 90°

Section C - Pulley Block system

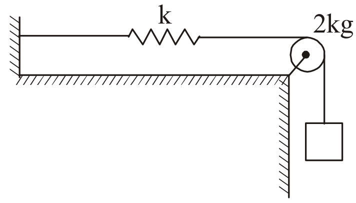
19. Figure shows a small wheel fixed coaxially on a bigger one of double the radius. The system rotates about the common axis. The strings supporting A and B do not slip on the wheels. If x and y be the distances travelled by A and B in the same time interval, then -



(A) x = 2y (B) x = y
(C) y = 2x (D) None of these

Paragraph Q. No. 20 to 21

A block of mass 4 kg suspended by a rope that passes over a pulley of. mass 2 kg radius 5cm. The rope is connected to a spring whose stiffness constant is 80 N/m.



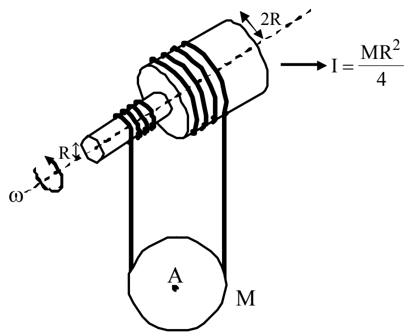
20. If the block is released from rest, what is the maximum extension of the spring?
(A) 0.98 m (B) 0.49 m
(C) 1.98 m (D) 0.24 m

21. What is the speed of the block after it has fallen 20 cm? Treat the pulley as a disc.
(A) 1.58 m/s (B) 2.58 m/s
(C) 1.98 m/s (D) 0.24 m/s

22. A pulley one meter in diameter rotating at 600 revolutions a minute is brought to rest in 80 see by a constant force of friction on its shaft. How many revolutions does it make before coming to rest
(A) 200 (B) 300
(C) 400 (D) 500

Passage (Ques. 23 to 24)

Figure shows a composite pulley of that consist of two cylinders of 'R' and '2R' stacked coaxially. Moment of inertia of composite pulley is $\frac{MR^2}{4}$. A light string is wound over pulley and another pulley 'A' of mass 'M' is hanged through the string as shown in figure.



23. The composite pulley is rotated with angular velocity 'ω' in the direction shown. The pulley 'A' will -

(A) Go up with velocity $\frac{R}{2}\omega$
(B) Go up with velocity $R\omega$
(C) Go down with velocity $R\omega$
(D) Will not move

24. If there were no friction between string and pulley 'A', then the acceleration of pulley 'A' will be -

(A) $\frac{2g}{3}$ (B) $\frac{g}{2}$ (C) $\frac{g}{3}$ (D) $\frac{4g}{5}$

25. If friction between pulley and string is sufficient to allow pure rolling then acceleration of pulley 'A' will be –

(A) $\frac{2g}{7}$ (B) $\frac{4g}{5}$ (C) $\frac{4g}{7}$ (D) $\frac{2g}{3}$

Section D - Angular Momentum (about point, about axis), Conservation of Angular Momentum

26. If a person sitting on a rotating stool with his hands outstretched, suddenly lowers his hands, then his
(A) Kinetic energy will decrease
(B) Moment of inertia will decrease
(C) Angular momentum will increase
(D) Angular velocity will remain constant

27. A thin circular ring of mass 'M' and radius 'R' is rotating about its axis with a constant angular velocity ω. Two objects each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velcoity.

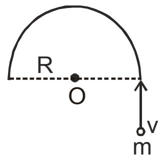
(A) $\frac{\omega M}{(M+m)}$ (B) $\frac{\omega M}{(M+2m)}$
(C) $\frac{\omega M}{(M-2m)}$ (D) $\frac{\omega(M+3m)}{M}$

28. A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ is 4 sec the magnitude of this torque is
(A) $4A_0$ (B) A_0
(C) $3A_0 / 4$ (D) $12\ A_0$

29. A rod of length L is hinged at one end. It is brought to a horizontal position and released. The angular velocity of the rod when it is in vertical position is

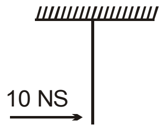
(A) $\sqrt{2g/L}$ (B) $\sqrt{3g/L}$
(C) $\sqrt{g/2L}$ (D) $\sqrt{g/L}$

30. A small bead of mass m moving with velocity v gets threaded on a stationary semicircular ring of mass m and radius R kept on a horizontal table. The ring can freely rotate about its centre. The bead comes to rest relative to the ring. What will be the final angular velocity of the system?



(A) v/R (B) 2v/R
(C) v/2R (D) 3v/R

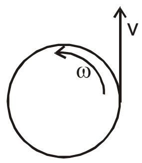
31. A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end when at rest. It receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. The kinetic energy of rod just after impact is



(A) 75 J (B) 100 J
(C) 200 J (D) none

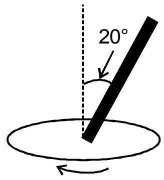
32. A child with mass m is standing at the edge of a disc with moment of inertia I, radius R, and initial angular velocity ω. See figure given below. The child jumps off the edge of the disc with tangential velocity v with respect to the ground. The new angular velocity of the disc is

(A) $\sqrt{\frac{I\omega^2 - mv^2}{I}}$
(B) $\sqrt{\frac{(I+mR^2)\omega^2 - mv^2}{I}}$
(C) $\frac{I\omega - mvR}{I}$
(D) $\frac{(I+mR^2)\omega - mvR}{I}$



Question No. (33) & (34) (2 questions)

A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of 20° to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



33. What is the direction of the rod's angular momentum vector (calculated about its lower end)
(A) vertically downwards
(B) down at 20° to the horizontal
(C) up at 20° to the horizontal
(D) vertically upwards

34. Is there a torque acting on it, and if so in what direction?
(A) yes, vertically
(B) yes, horizontally
(C) yes at 20° to the horizontal
(D) no

35. One ice skater of mass m moves with speed 2v to the right, while another of the same mass m moves with speed v toward the left, as shown in figure I. Their paths are separated by a distance b. At t = 0, when they are both at x = 0, they grasp a pole of length b and negligible mass. For t > 0, consider the system as a rigid body of two masses m separated by distance b, as shown in figure II. Which of the following is the correct formula for the motion after t = 0 of the skater initially at y = b/2 ?

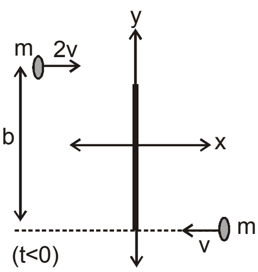


Figure 1

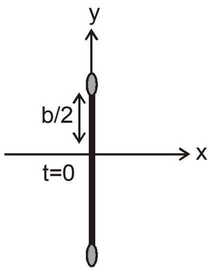
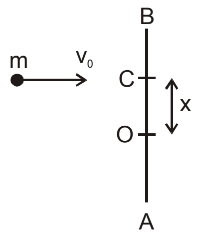


Figure II

(A) x = 2vt, y = b/2
(B) x = vt + 0.5b sin (3vt/b), y = 0.5b cos(3vt/b)
(C) x = 0.5vt + 0.5b sin (3vt/b), y = 0.5b cos(3vt/b)
(D) x = 0.5vt + 0.5b sin (6vt/b), y = 0.5b cos(6vt/b)

36. A uniform rod AB of length L and mass M is lying on a smooth table. A small particle of mass m strike the rod with a velocity v_0 at point C at a distance x from the centre O. The particle comes to rest after collision. The value of x, so that point A of the rod remains ststionary just after collision is :

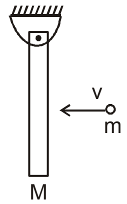


(A) L/3
(B) L/6
(C) L/4
(D) L/12

37. A uniform rod AB of mass m and length l is at rest on a smooth horizontal surface. An impulse J is applied to the end B, perpendicular to the rod in the horizontal direction. Speed of particle P at a distance $\frac{l}{6}$ from the centre towards A of the rod after time $t = \frac{\pi m l}{12J}$ is

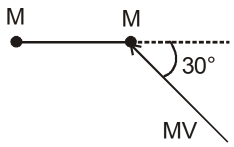
(A) $2\frac{J}{m}$
(B) $\frac{J}{\sqrt{2}m}$
(C) $\frac{J}{m}$
(D) $\sqrt{2}\frac{J}{m}$

38. A uniform rod of mass M is hinged at its upper end. A particle of mass m moving horizontally strikes the rod at its mid point elastically. If the particle comes to rest after collision find the value of M/m = ?



(A) 3/4
(B) 4/3
(C) 2/3
(D) none

39. Two equal masses each of mass M are joined by a massless rod of length L. Now an impulse MV is given to the mass M making an angle of 30° with the length of the rod. The angular velocity of the rod just after imparting the impulse is



(A) $\frac{v}{L}$
(B) $\frac{2v}{L}$
(C) $\frac{v}{2L}$
(D) none of these

40. Two particles of equal mass m at A and B are connected by a rigid light rod AB lying on a smooth horizontal table. An impulse J is applied at A in the plane of the table and perpendicular at AB. Then the velocity of particle at A is :

(A) $\frac{J}{2m}$
(B) $\frac{J}{m}$
(C) $\frac{2J}{m}$
(D) zero

Section E - Angular Impulse + Collision Of point Mass with Rigid Bodies

41. A body of mass *m* is moving with a constant velocity along a line parallel to the *x-axis,* away from the origin. Its angular momentum with respect to the origin

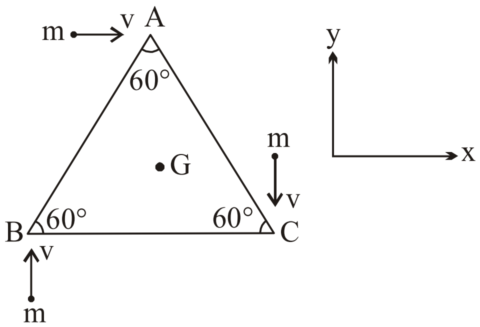
(A) is zero

(B) remains constant

(C) goes on increasing

(D) goes on decreasing

42. A triangular block *ABC* of mass *m* and sides *2a* lies on a smooth horizontal plane as shown. Three point masses of mass *m* each strikes the block at *A, B* and C with speeds v as shown. After the collision the particle come to rest. Select the correct altemative(s)



(A) The centre of mass of ABC remains stationary after collision

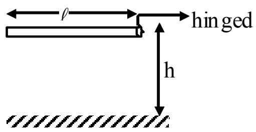
(B) The centre of mass of ABC moves with a velocity v along x-axis after collision

(C) The triangular block rotates with an angular velocity $\omega = \dfrac{2\sqrt{3}\,mva}{I}$ about its centre of mass

(here I is the moment of inertia of triangular block about its centroid axis perpendicular to its plane)

(D) A point lying at a distance of $\left(\dfrac{I}{2\sqrt{3}\,ma}\right)$ from G on perpendicular bisector of BC (below G) is at rest just after collision.

43. A thin rod of mass m and length ℓ is hinged at one end point which is at a distance h (h < ℓ) above the horizontal surface. The rod is released from rest from the horizontal position. If e is the co-efficient of restitution, the angular velocity of rod just after collision will be (h = 1m, ℓ = 2m, e = 1) -



(A) $\dfrac{3\sqrt{3g}}{8}$ (B) $\dfrac{6\sqrt{3g}}{8}$

(C) $\dfrac{5\sqrt{3g}}{8}$ (D) none of these

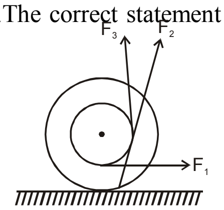
44. A recording disc rotates steadily at n_1 rps on a table. When a small mass m is dropped gently on the disc at a distance x from its axis, it sticks to the disc, the rate of revolution falls to n_2 rps. The original moment of inertia of the disc about a perpendicular axis through its centre is

(A) $I = \dfrac{mx^2}{n_1 - n_2}$ (B) $I = \dfrac{n_1 mx^2}{n_1 - n_2}$

(C) $I = \dfrac{n_2 mx^2}{n_1 - n_2}$ (D) $I = \dfrac{n_2 mx^2}{n_1}$

Section F - Combined Translational and Rotational Motion of Rigid body, Pure Rolling, Slipping

45. A yo-yo is resting on a perfectly rough horizontal table. Forces F_1, F_2 and F_3 are applied separately as shown.The correct statement is



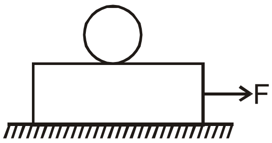
(A) when F_3 is applied the centre of mass will move to the right

(B) when F_2 is applied the centre of mass will move to the left

(C) when F_1 is applied the centre of mass will move to the right

(D) when F_2 is applied the centre of mass will move to the right

46. A plank with a uniform sphere placed on it, rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If the sphere does not slip over the plank.



(A) acceleration of centre of sphere is less than that of the plank

(B) acceleration of centre of sphere is greater than the plank because friction acts rightward on the sphere

(C) acceleration of the centre of sphere may be towards left

(D) acceleration of the centre of sphere relative to plank may be greater than that of the plank relative to floor

47. A hollow sphere of radius R and mass m is fully filled with water of mass m. It is rolled down a horizontal plane such that its centre of mass moves with a velocity v. If it purely rolls

(A) Kinetic energy of the sphere is $\frac{5}{6}mv^2$

(B) Kinetic energy of the sphere is $\frac{4}{5}mv^2$

(C) Angular momentum of the sphere about a fixed point on ground is $\frac{8}{3}mvR$

(D) Angular momentum of the sphere about a fixed point on ground is $\frac{14}{5}mvR$

48. A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. The smallest kinetic energy at the bottom of the incline will be achieved by

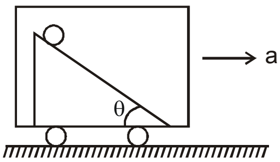
(A) the solid sphere

(B) the hollow sphere

(C) the disc

(D) all will achieve same kinetic energy.

49. Fig. shows a smooth inclined plane fixed in a car accelerating on a horizontal road. The angle of incline θ is related to the acceleration a of the car as a = g tan θ. If the sphere is set in pure rotation on the incline.



(A) it will continue pure rolling

(B) it will slip down the plane

(C) its linear velocity will increase

(D) its linear velocity will decrease.

50. A straight rod of length L is released on a frictionless horizontal floor in a vertical position. As it falls + slips, the distance of a point on the rod from the lower end, which follows a quarter circular locus is

(A) L/2 (B) L/4

(C) L/8 (D) None

51. A ladder of length L is slipping with its ends against a vertical wall and a horizontal floor. At a certain moment, the speed of the end in contact with the horizontal floor is v and the ladder makes an angle $\alpha = 30^\circ$ with the horizontal. Then the speed of the ladder's center must be

(A) $2v/\sqrt{3}$ (B) v/2

(C) v (D) none

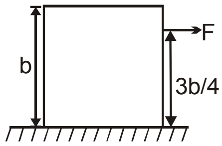
52. In the previous question, if dv/dt = 0, then the angular acceleration of the ladder when $\alpha = 45^\circ$ is

(A) $2v^2/L^2$ (B) $v^2/2L^2$

(C) $\sqrt{2}[v^2/L^2]$ (D) None

Section G - Toppling + Direction of Friction

53. A uniform cube of side 'b' and mass M rest on a rough horizontal table. A horizontal force F is applied normal to one of the face at a point, at a height 3b/4 above the base. What should be the coefficient of friction (μ) between cube and table so that is will tip about an edge before it starts slipping?



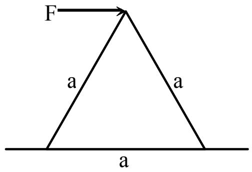
(A) $\mu > \frac{2}{3}$

(B) $\mu > \frac{1}{3}$

(C) $\mu > \frac{3}{2}$

(D) none

54. An equilateral prism of mass m rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the prism as shown in the figure. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, then the minimum force required to topple the prism is -

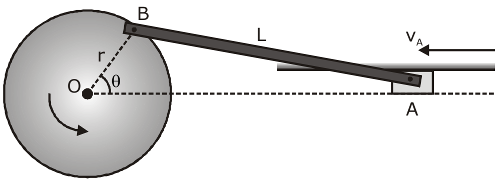


(A) $\frac{mg}{\sqrt{3}}$ (B) $\frac{mg}{4}$

(C) $\frac{\mu mg}{\sqrt{3}}$ (D) $\frac{\mu mg}{4}$

Section H - Instantaneous Axis of Rotation

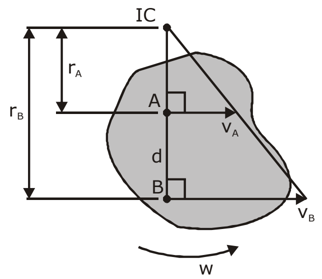
55. Find the velocity of point B in the crank mechanism if the velocity of point A is v_A, in the direction shown. The information is given in the figure.



(A) $v_A \frac{OA - 2r\cos\theta}{OA\sin\theta}$ (B) $v_A \frac{OA - r\sin\theta}{OA\sin\theta}$

(C) $v_A \frac{OA - r\cos\theta}{OA\sin\theta}$ (D) $v_A \frac{OA - r\sin\theta}{OA\cos\theta}$

56. Consider a rigid body rotating in a plane. We wish to determine the angular velocity of the rigid body given the known velocities of points A and B on the rigid body. These velocities are parallel and pointing in the same direction. The line joining points A and B is perpendicular to the direction of the velocities. The figure below illustrates the set up of the problem.



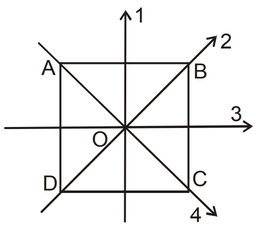
Note that IC is the intersection of the line passing through points A and B, and the line joining the tip of the vectors v_A and v_B.

(A) $\frac{v_B - v_A}{d}$ (B) $\frac{v_B + v_A}{d}$

(C) $\frac{v_B - v_A}{2d}$ (D) None of these

Section A - Moment of Inertia, Theorems of Moment of Inertia, Radius of gyration, Cavity problems

1. ABCD is a square plate with centre O. The moments of inertia of the plate about the perpendicular axis through O is I and about the axes 1, 2, 3 & 4 are I_1, I_2, I_3 & I_4 respectively. It follows that :



(A) $I_2 = I_3$ (B) $I = I_1 + I_4$
(C) $I = I_2 + I_4$ (D) $I_1 = I_3$

Section B - Torque (about point, about axis), Torque and angular Acceleration

2. A rod of weight w is supported by two parallel knife edges A and B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at a distance x from A.

(A) the normal reaction at A is $\frac{wx}{d}$

(B) the normal reaction at A is $\frac{w(d-x)}{d}$

(C) the normal reaction at B is $\frac{wx}{d}$

(D) the normal reaction at B is $\frac{w(d-x)}{d}$

3. A body is in equilibrium under the influence of a number of forces. Each force has a different line of action. The minimum number of forces required is

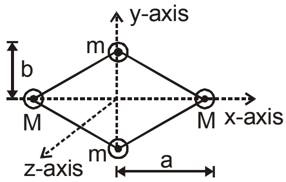
(A) 2, if their lines of action pass through the centre of mass of the body

(B) 3, if their lines of action are not parallel

(C) 3, if their lines of action are parallel

(D) 4, if their lines of action are parallel and all the forces have the same magnitude

4. Four point masses are fastened to the corners of a frame of negligible mass lying in the xy plane. Let w be the angular speed of rotation. Then



(A) rotational kinetic energy associated with a given angular speed depends on the axis of rotation.

(B) rotational kinetic energy about y-axis is independent of m and its value is $Ma^2\omega^2$

(C) rotational kinetic energy about z-axis depends on m and its value is $(Ma^2 + mb^2)\omega^2$

(D) rotational kinetic energy about z-axis is independent of m and its value is $Mb^2\omega^2$

5. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground.

(A) Angular momentum of the particle about O is increasing

(B) Torque of the gravitational force on the particle about O is decreasing

(C) The moment of inertia of the particle about O is decreasing

(D) The angular velocity of the particle about O is increasing

Section C - Pulley Block system

6. A uniform disc of mass 2kg and radius 1m is mounted on an axle supported on fixed frictionless bearings. A light chord is wrapped around the rim of the disc and mass of 1kg is tied to the free end. If it is released from rest-

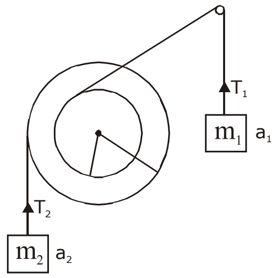
(A) the tension in the chord is 5N

(B) in first four seconds the angular displacement of the disc is 40 rad

(C) the work done by the torque on the disc in first four seconds is 200J

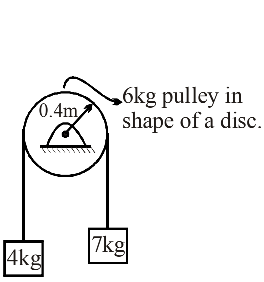
(D) the increase in the kinetic energy of the disc in the first four seconds in 200J

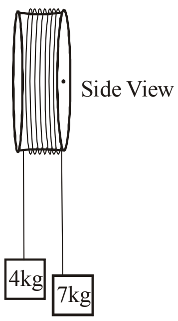
7. The pulley as shown in the figure, consists of two discs of different diameters attached to the same shaft. The rope connected to the block of mass m_1 = 1 kg passes over a smooth peg, while the block of mass m_2 = 3 kg hangs vertically from one disc. The moment of inertia of the pulley is 0.2 kg m^2; r_1 = 5 cm and r_2 = 10 cm. Find the tensions in the ropes and the accelerations of the blocks.



(A) $a_1 = 0.527$ m/s^2 (B) $a_2 = 1.05$ m/s^2
(C) $T_1 = 10.3$ N (D) $T_2 = 26.2$ N

8. In the figure, the massless and inextensible thread does not slip on the pulley. It is wound over pulley as shown. If the system is released from rest,



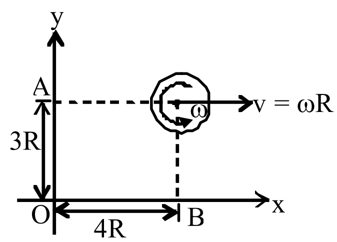


(A) The tension in right side of the string is greater than that in the left side.
(B) After the motion has set in the kinetic energy of pulley is lesser than kinetic energy of 4kg block.
(C) The force exerted by the hinge on pulley is less than 17g.
(D) If 7kg block suddenly strikes the ground and stops, the string on left hand side will remain taut.

Section D - Angular Momentum (about point, about axis), Conservation of Angular Momentum

9. A man spinning in free space changes the shape of his body, eg. by spreading his arms or curling up. By doing this, he can change his
(A) moment of inertia
(B) angular momentum
(C) angular velocity
(D) rotational kinetic energy

10. A disc of mass M and radius R moves in the x-y plane as shown in the figure. The angular momentum of the disc at the instant shown is –



(A) $\frac{5}{2}$ mR^2 ω about O

(B) $\frac{7}{2}$ mR^2 ω about O

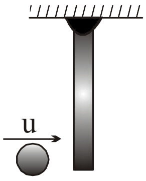
(C) $\frac{1}{2}$ mR^2 ω about A

(D) 4 mR^2 ω about A

Section E - Angular Impulse + Collision Of point Mass with Rigid Bodies

11. A thin uniform rod of mass m and length l is free to rotate about its upper end. When it is at rest, it receives an impulse J at its lowest point, normal to its length. Immediately after impact,
(A) the angular momentum of the rod is Jl
(B) the angular velocity of the rod is 3J/ml
(C) the kinetic energy of the rod is 3J^2/2m
(D) the linear velocity of the midpoint of the rod is 3J/2m

12. In the given figure a ball strikes a uniform rod of same mass elastically and rod is hinged at point A. Then which of the statement(s) is / are correct?

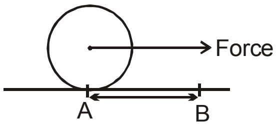


(A) linear momentum of system (ball + rod) is conserved.
(B) angular momentum of system (ball + rod) about hinged point A is conserved.
(C) kinetic energy of system (ball + rod) before the collision is equal to kinetic energy of system just after the collision
(D) linear momentum of ball is conserved.

Section F - Combined Translational and Rotational Motion of Rigid body, Pure Rolling, Slipping

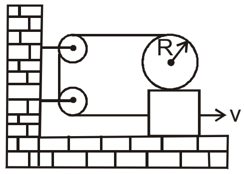
13. A ring rolls without slipping on the ground. Its centre C moves with a constant speed u. P is any point on the ring. The speed of P with respect to the ground is v.
(A) $0 \le v \le 2u$
(B) v = u, if CP is horizontal
(C) v = u, if CP makes an angle of 30° with the horizontal and P is below the horizontal level of C
(D) $v = \sqrt{2}u$, if CP is horizontal

14. A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. AB = s. The disc moves from A to B in time T. To the right of B,



(A) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged
(B) linear acceleration of the disc will increase
(C) the disc will make one rotation in time T/2
(D) the disc will cover a distance greater than s in further time T.

15. In the figure shown, the plank is being pulled to the right with a constant speed v. If the cylinder does not slip then :



(A) the speed of the centre of mass of the cylinder is 2v
(B) the speed of the centre of mass of the cylinder is zero
(C) the angular velocity of the cylinder is v/R
(D) the angular velocity of the cylinder is zero

16. If a cylinder is rolling down the incline with sliding
(A) after some time it may start pure rolling
(B) after sometime it will start pure rolling
(C) it may be possible that it will never start pure rolling
(D) none of these

17. Which of the following statements are correct
(A) friction acting on a cylinder without sliding on an inclined surface is always upward along the incline irrespective of any external force acting on it.
(B) friction acting on a cylinder without sliding on an inclined surface is may be upward may be downwards depending on the external force acting on it.
(C) friction acting on a cylinder rolling without sliding may be zero depending on the external force acting on it.
(D) nothing can be said exactly about it as it depends on the friction coefficient on inclined plane

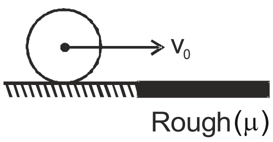
Question No. 18. to 20. (3 Questions)
A cylinder and a ring of same mass M and radius R are placed on the top of a rough inclined plane of inclination θ. Both are released simultaneously from the same height h.

18. Choose the correct statement(s) related to the motion of each body
(A) The friction force acting on each body opposes the motion of its centre of mass
(B) The friction force provides the necessary torque to rotate the body about its centre of mass
(C) without friction none of the two bodies can roll
(D) The friction force ensures that the point of contact must remain stationary

19. Identify the correct statement(s)
(A) The friction force acting on the cylinder may be more than that acting on the ring
(B) The friction force acting on the ring may be more than that acting on the cylinder
(C) If the friction is sufficient to roll the cylinder then the ring will also roll
(D) If the friction is sufficient to roll the ring then the cylinder will also roll

20. When these bodies roll down to the foot of the inclined plane, then
(A) the mechanical energy of each body is conserved
(B) the velocity of centre of mass of the cylinder is $2\sqrt{\frac{gh}{3}}$
(C) the velocity of centre of mass of the ring is $\sqrt{gh}$
(D) the velocity of centre of mass of each body is $\sqrt{2gh}$

Question No. 21. to 24. (4 Questions)
A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where the coefficient of friction is μ, as shown in figure.



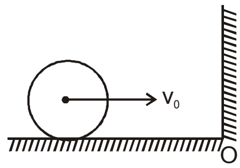
21. Choose the correct statement(s)
(A) As the ring enters on the rough surface, the Kinetic friction force acts on it
(B) The direction of friction is opposite to the direction of motion
(C) The friction force accelerates the ring in the clockwise sense about its centre of mass
(D) As the ring enters on the rough surface it starts rolling

22. Choose the correct statement(s)
(A) The momentum of the ring is conserved
(B) The angular momentum of the ring is conserved about its centre of mass
(C) The angular momentum of the ring conserved about any point on the horizontal surface
(D) The mechanical energy of the ring is conserved

23. Choose the correct statement(s)
(A) The ring starts its rolling motion when the centre of mass stationary
(B) The ring starts rolling motion when the point of contact becomes stationary
(C) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$
(D) The rolling velocity is $\frac{v_0}{2}$

24. Choose the correct alternative(s)
(A) The linear distance moved by the centre of mass before the ring starts rolling is $\frac{3v_0^2}{8\mu g}$
(B) The net work done by friction force is $-\frac{3}{8}mv_0^2$
(C) The loss is kinetic energy of the ring is $\frac{mv_0^2}{4}$
(D) The gain in rotational kinetic energy is $+\frac{mv_0^2}{8}$

25. Consider a sphere of mass 'm' radius 'R' doing pure rolling motion on a rough surface having velocity $\vec{v}_0$ as shown in the Figure. It makes an elastic impact with the smooth wall and moves back and starts pure rolling after some time again.



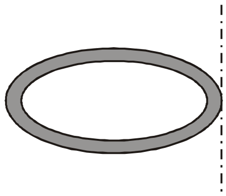
(A) Change in angular momentum about 'O' in the entire motion equals $2mv_0R$ in magnitude.
(B) Moment of impulse provided by wall during impact about O equals $2mv_0R$ in magnitude
(C) Final velocity of ball will be $\frac{3}{7}\vec{v}_0$
(D) Final velocity of ball will be $-\frac{3}{7}\vec{v}_0$

Section A - Moment of Inertia, Theorems of Moment of Inertia, Radius of gyration, Cavity problems

1. Find the moment of inertia of a uniform half-disc about an axis perpendicular to the plane and passing through its centre of mass. Mass of this disc is M and radius is R.

2. Find the moment of inertia of a pair of solid spheres, each having a mass m and radius r, kept in contact about the tangent passing through the point of contact.

3. Find the radius of gyration of a circular ring of radius r about a line perpendicular to the plane of this ring and tangent to the ring.

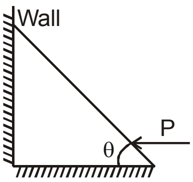


Section B - Torque (about point, about axis), Torque and angular Acceleration

4. A simple pendulum of length ℓ is pulled aside to made an angle θ with the vertical. Find the magnitude of the torque of the weight w of the bob about the point of suspension. When is the torque zero ?

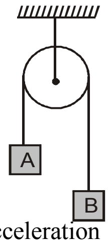
5. Two forces $\vec{F}_1 = 2\hat{i} - 5\hat{j} - 6\hat{k}$ and $\vec{F}_2 = -\hat{i} + 2\hat{j} - \hat{k}$ are acting on a body at the points (1, 1, 0) and (0, 1, 2). Find torque acting on the body about point (–1, 0, 1).

6. Assuming frictionless contacts, determine the magnitude of external horizontal force P applied at the lower end for equilibrium of the rod. The rod is uniform and its mass is 'm'.



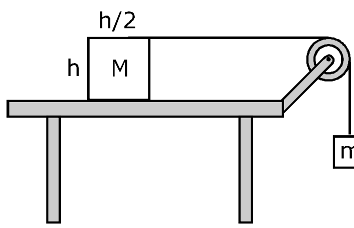
Section C - Pulley Block system

7. In the figure A & B are two blocks of mass 4 kg & 2 kg respectively attached to the two ends of a light string passing over a disc C of mass 40 kg and radius 0.1m. The disc is free to rotate about a fixed horizontal axes, coinciding with its own axis. The system is released from rest and the string does not slip over the disc. Find :

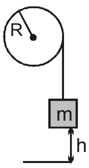


(i) the linear acceleration of mass B.
(ii) the number of revolutions made by the disc at the end of 10 sec. from the start.
(iii) the tension in the string segment supporting the block A.

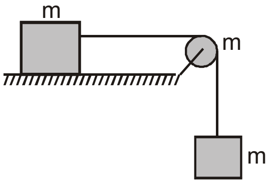
8. A cylinder of height h, diameter h/2 and mass M and with a homogeneous mass distribution is placed on a horizontal table. One end of a string running over a pulley is fastended to the top of the cylinder, a body of mass m is hung from the other end and the system is released. Friction is negligible every where. At what minimum ratio m/M will the cylinder tilt?



9. A mass m is attached to a pulley through a cord as shown in the fig. The pulley is a solid disk with radius R. The cord does not slip on the disk. The mass is released from rest at a height h from the ground and at the instant the mass reaches the ground, the disk is rotating with angular velocity ω. Find the mass of the disk.



10. Figure shows two blocks of mass m and m connected by a string passing over a pulley. The horizontal table over which the mass m slides is smooth. The pulley (uniform disc) has mass m and it can freely rotate about this axis. Find the acceleration of the mass m assuming that the string does not slip on the pulley.



Section D - Angular Momentum (about point, about axis), Conservation of Angular Momentum

11. A particle having mass 2 kg is moving along straight line 3x+ 4 y = 5 with speed 8m/s. Find angular momentum of the particle about origin, x and y are in meters.

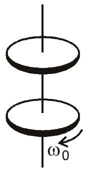
12. A particle having mass 2 kg is moving with velcoity $(2\hat{i} + 3\hat{j})m/s$. Find angular momentum of the particle about origin when it is at (1, 1, 0).

13. A uniform square plate of mass 2.0 kg and edge 10 cm rotates about one of its diagonals under the action of a constant torque of 0.10 N.m. Calculate the angular momentum and the kinetic energy of the plate at the end of the fifth second after the start.

14. A wheel of moment of inertia 0.500 kg-m^2 and radius 20.0 cm is rotating about its axis at an angular speed of 20.0 rad/s. It picks up a stationary particle of mass 200 g at its edge. Find new angular speed of the wheel.

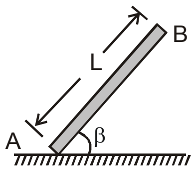
15. A uniform circular disc can rotate freely about a rigid vertical axis through its centre O. A man stands at rest at A on the edge due east of O. The mass of the disc is 22 times the mass of the man. The man starts walking anticlockwise. When he reaches the point A after completing one rotation relative to the disc he will be:

16. Two identical disks are positioned on a vertical axis. The bottom disk is rotating at angular velocity ω_0 and has rotational kinetic energy KE_0. The top disk is initially at rest. It is allowed to fall, and sticks to the bottom disk. What is the rotational kinetic energy of the system after the collision?



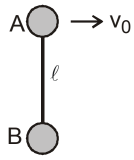
Section E - Angular Impulse + Collision Of point Mass with Rigid Bodies

17. The uniform rod AB of mass m is released from rest when $\beta = 60°$. Assuming that the friction force between end A and the surface is large enough to prevent sliding, determine (for the instant just after release)



(a) The angular acceleration of the rod
(b) The normal reaction and the friction force at A.
(c) The minimum value of μ, compatible with the described motion.

18. Two small spheres A & B respectively of mass m & 2m are connected by a rigid rod of length ℓ & negligible mass. The two spheres are resting on a horizontal, frictionless surface. When A is suddenly given the velocity v_0 as shown. Find velocities of A & B after the rod has rotated through 180°.

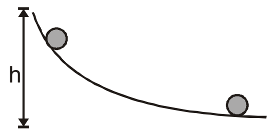


Section F - Combined Translational and Rotational Motion of Rigid body, Pure Rolling, Slipping

19. A sphere of mass m rolls on a plane surface. Find its kinetic energy at an instant when its centre moves with speed v.

20. A cylinder rolls on a horizontal plane surface. If the speed of the centre is 25 m/s, what is the speed of the highest point?

21. A small spherical ball is released from a point at a height h on a rough track shown in figure. Assuming that it does not slip anywhere, find its linear speed when it rolls on the horizontal part of the track.



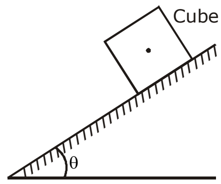
22. A sphere starts rolling down an incline of inclination θ. Find the speed of its centre when it has covered a distance ℓ.

23. A solid uniform sphere of mass m is released from rest from the rim of a hemispherical cup so that it rolls without sliding along the surface. If the rim of the hemisphere is kept horizotnal, find the normal force exerted by the cup on the ball when the ball reaches the bottom of the cup.

24. A uniform rod of mass m and length ℓ is struck at an end by a force F perpendicular to the rod for a short time interval t. Calculate
(a) the speed of the centre of mass,
(b) the angular speed of the rod about the centre of mass,
(c) the kinetic energy of the rod and
(d) the angular momentum of the rod about the centre of mass after the force has stopped to act. Assume that t is so small that the rod does not appreciably change its direction while the force acts.

Section G - Toppling + Direction of Friction

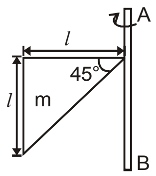
25. Coefficient of friction between block & incline is $\mu = \sqrt{3}$ & the angle of inclination of the plane is increased slowly with horizontal. Comment whether the block will topple of slip first.



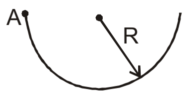
Exercise - 3 | Level-II   Subjective | JEE Advanced

Section A - Moment of Inertia, Theorems of Moment of Inertia, Radius of gyration, Cavity problems

1. Moment of inertial of a triangle plane of mass M shown in figure about vertical axis AB is :



2. A uniform rod of mass m is bent into the form of a semicircle of radius R. The moment of inertia of the rod about an axis passing through A and perpendicular to the plane of the paper is



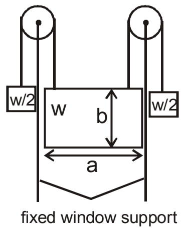
Section B - Torque (about point, about axis), Torque and angular Acceleration

3. A thin uniform rod of mass M and length L is hinged at its upper end, and released from rest in a horizontal position. The tension at a point located at a distance L/3 from the hinge point, when the rod becomes vertical, will be

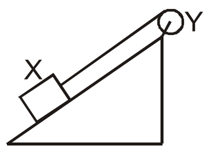
4. A rod of mass m and length L, lying horizontally, is free to rotate about a vertical axis through its centre. A horizontal force of constant magnitude F acts on the rod at a distance of L/4 from the centre. The force is always perpendicular to the rod. Find the angle rotated by the rod during the time t after the motion starts.

Section C - Pulley Block system

5. A slightly loosely fit window is balanced by two strings which are connected to weights w/2 each. The strings pass over the frictionless pulleys as shown in the figure. The strings are tied almost at the corner of the window. The string on the right is cut and then the window accelerates downwards. If the coefficients of friction between the window and the side supports is μ then calculate the acceleration of the window in terms of μ, a, b and g, where a is width and b is the length of the window.

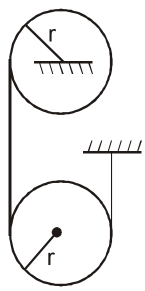


6. A block X of mass 0.5 kg is held by a long massless string on a frictionless inclined plane of inclination 30º to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2kg and of radius 0.2 m as shown in the figure. The drum is given an initial angular velocity such that the block X starts moving up the plane.



(i) Find the tension in the string during the motion
(ii) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad/sec. Calculate the distance travelled by X from that instant of time until it comes to rest.

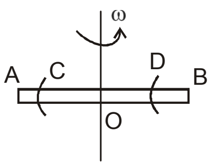
7. Two uniform cylinders, each of mass m = 10 kg and radius r = 150 mm, are connected by a rough belt as shown. If the system is released from rest, determine



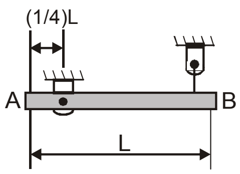
(a) the velocity of the centre of cylinder A after it has moved through 1.2 m & (b) the tension in the portion of the belt connecting the two cylinders.

Section D - Angular Momentum (about point, about axis), Conservation of Angular Momentum

8. A rigid horizontal smooth rod AB of mass 0.75 kg and length 40 cm can rotate freely about a fixed vertical axis through its mid point O. Two rings each of mass 1 kg are initially at rest a distance of 10 cm from O on either side of the rod. The rod is set in rotation with an angular velocity of 30 radians per second. The velocity of each ring along the length of the rod in m/s then they reach the ends of the rod is

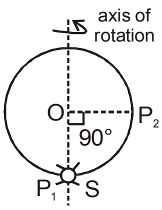


9. A uniform beam of length L and mass m is supported as shown. If the cable suddenly breaks, determine ;



(a) the acceleration of end B.
(b) the reaction at the pin support.

10. A uniform ring is rotating about vertical axis with angular velocity ω initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point P_1 and finally reaches the point P_2 (as shown in figure). The final angular velocity of the ring will be equal to



Section E - Angular Impulse + Collision Of point Mass with Rigid Bodies

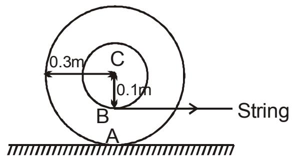
11. A thin rod AB of length a has variable mass per unit length $\rho_0\left(1+\frac{x}{a}\right)$ where x is the distance measured from A and ρ_0 is a constant.

(a) Find the mass M of the rod.
(b) Find the position of centre of mass of the rod.
(c) Find moment of inertia of the rod about an axis passing through A and perpendicular to AB. Rod is freely pivoted at A and is hanging in equilibrium when it is struck by a horizontal impulse of magnitude P at the point B.
(d) Find the angular velocity with which the rod begins to rotate.
(e) Find minimum value of impulse P if B passes through a point vertically above A.

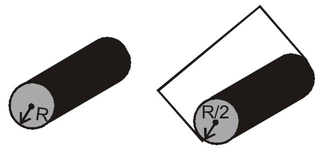
Section F - Combined Translational and Rotational Motion of Rigid body, Pure Rolling, Slipping

12. A straight rod AB of mass M and length L is placed on a frictionless horizontal surface. A horizontal force having constant magnitude F and a fixed direction starts acting at the end A. The rod is initially perpendicular to the force. The initial acceleration of end B is

13. A wheel is made to roll without slipping, towards right, by pulling a string wrapped around a coaxial spool as shown in figure. With what velocity the string should be pulled so that the centre of the wheel moves with a velocity of 3 m/s?



14. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. The horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2 will be :



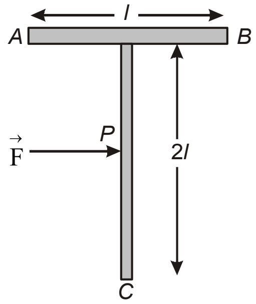
Section G - Toppling + Direction of Friction

15. A solid uniform disk of mass m rolls without slipping down a fixed inclined plane with an acceleration a. The frictional force on the disk due to surface of the plane is :

Exercise - 4 | Level-I   Previous Year | JEE Main

1. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force $\vec{F}$ is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C.

[AIEEE 2005]



(A) $\frac{2}{3}l$ (B) $\frac{3}{2}l$

(C) $\frac{4}{3}l$ (D) l

2. The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is

[AIEEE 2005]

(A) $\frac{1}{4}Mr^2$ (B) $\frac{2}{5}Mr^2$

(C) Mr^2 (D) $\frac{1}{2}Mr^2$

3. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' =

[AIEEE 2006]

(A) $\frac{\omega(m+2M)}{m}$ (B) $\frac{\omega(m-2M)}{(m+2M)}$

(C) $\frac{\omega m}{(m+M)}$ (D) $\frac{\omega m}{(m+2M)}$

4. Four point masses, each of value m, are placed at the corners of a square ABCD of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is **[AIEEE 2006]**

(A) $2ml^2$ (B) $\sqrt{3}ml^2$

(C) $3ml^2$ (D) ml^2

5. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation in gradually increased. The coin will leave contact with the platform for the first time

[AIEEE 2006]

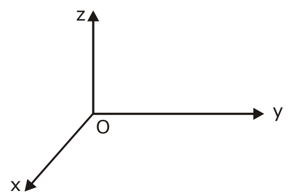
(A) at the mean position of the platform

(B) for an amplitude of g/ω^2

(C) for an amplitude of g^2/ω^2

(D) at the highest position of the platform

6. A force of $-F_k$ acts on O, the origin of the coordinate system. The torque about the point (1, –1) is



[AIEEE 2006]

(A) F(i – j) (B) –F(i + j)

(C) F(i + j) (D) –F(i – j)

7. Angular momentum of the particle rotating with a central force is constant due to **[AIEEE 2007]**

(A) constant force

(B) constant linear momentum

(C) zero torque

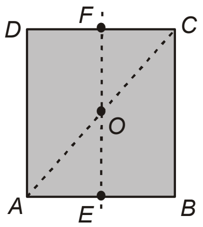
(D) constant torque

8. A round uniform body of radius R, mass M and moment of inertia I, rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is **[AIEEE 2007]**

(A) $\frac{g\sin\theta}{1+I/MR^2}$ (B) $\frac{g\sin\theta}{1+MR^2/I}$

(C) $\frac{g\sin\theta}{1-I/MR^2}$ (D) $\frac{g\sin\theta}{1-MR^2/I}$

9. For the given uniform square lamina ABCD, whose centre is O **[AIEEE 2007]**



(A) $\sqrt{2}I_{AC} = I_{EF}$ (B) $I_{AD} = 3I_{EF}$

(C) $I_{AD} = 4I_{EF}$ (D) $I_{AD} = \sqrt{2}I_{EF}$

10. A circular disc of radius R is removed from a bigger circular disc of radius 2R, such that the circumference of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is **[AIEEE 2007]**

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$

11. Consider a uniform square plate of side **a** and mass **m**. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[AIEEE 2008]**

(A) $\frac{5}{6}ma^2$ (B) $\frac{1}{12}ma^2$

(C) $\frac{7}{12}ma^2$ (D) $\frac{2}{3}ma^2$

12. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to maximum height of **[AIEEE 2009]**

(A) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (B) $\frac{1}{6}\frac{l\omega}{g}$

(C) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (D) $\frac{1}{6}\frac{l^2\omega^2}{g}$

13. A pulley of radius 2 m is rotated about its axis by a force F = (20t – 5t^2) N (where t is measured in seconds) applied tangentially. It the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2 the number of rotations made by the pulley before its direction of motion if reserved, is **[AIEEE 2011]**

(A) more than 3 but less than 6

(B) more than 6 but less than 9

(C) more than 9

(D) less than 3

14. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc. **[AIEEE 2011]**

(A) continuously decreases

(B) continuously increases

(C) first increases and then decreases

(D) remains unchanged

15. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it cases to slip ? **[JEE Mains 2013]**

(A) $\frac{r\omega_0}{2}$ (B) $r\omega_0$

(C) $\frac{r\omega_0}{4}$ (D) $\frac{r\omega_0}{3}$

16. A bob of mass m attached to an inextensible string of length ℓ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension : **[JEE Main 2014]**

(A) Angular momentum changes in direction but not in magnitude.

(B) Angular momentum changes both in direction and magnitude.

(C) Angular momentum is conserved.

(D) Angular momentum changes in magnitude but not in direction.

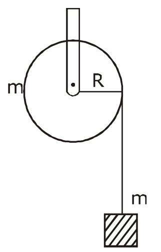
17. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release ? **[JEE Main 2014]**

(A) $\frac{5g}{6}$

(B) g

(C) $\frac{2g}{3}$

(D) $\frac{g}{2}$



18. A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is : **[JEE Main 2014]**

(A) $\frac{1}{3}$ m (B) $\frac{1}{2}$ m

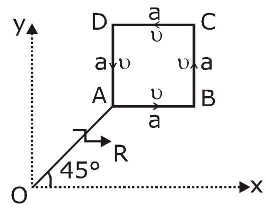
(C) $\frac{1}{6}$ m (D) $\frac{2}{3}$ m

19. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is : **[JEE Main 2015]**

(A) $\frac{4MR^2}{9\sqrt{3}\pi}$ (B) $\frac{4MR^2}{3\sqrt{3}\pi}$

(C) $\frac{MR^2}{32\sqrt{2}\pi}$ (D) $\frac{MR^2}{16\sqrt{2}\pi}$

20. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure: **[AIEEE-2016]**



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin ?

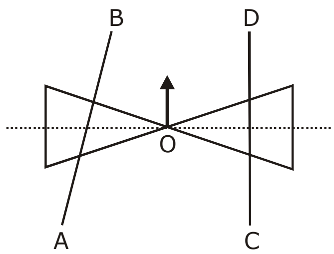
(A) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.

(B) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.

(C) $\vec{L} = \frac{m\upsilon}{\sqrt{2}}R\,\hat{k}$ when the particle is moving from D to A.

(D) $\vec{L} = -\frac{m\upsilon}{\sqrt{2}}R\,\hat{k}$ when the particle is moving from A to B.

21. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to : **[AIEEE-2016]**



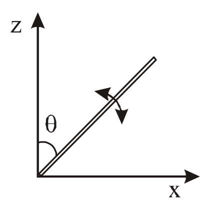
(A) turn right
(B) go straight
(C) turn left and right altemately.
(D) turn left.

22. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum ? **[AIEEE-2017]**

(A) $\frac{3}{\sqrt{2}}$ (B) $\sqrt{\frac{3}{2}}$

(C) $\frac{\sqrt{3}}{2}$ (D) 1

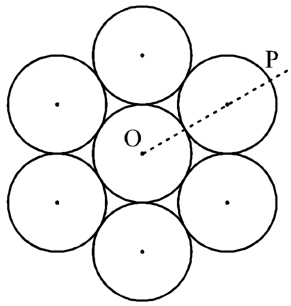
23. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically about the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is : **[AIEEE-2017]**



(A) $\frac{2g}{3l}\cos\theta$ (B) $\frac{3g}{2l}\sin\theta$

(C) $\frac{2g}{3l}\sin\theta$ (D) $\frac{3g}{2l}\cos\theta$

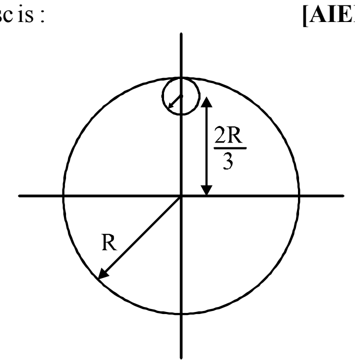
24. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : **[AIEEE-2018]**



(A) $\frac{181}{2}MR^2$ (B) $\frac{19}{2}MR^2$

(C) $\frac{55}{2}MR^2$ (D) $\frac{73}{2}MR^2$

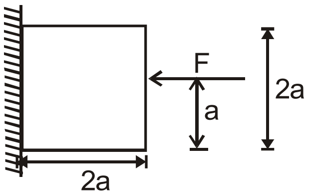
25. From a uniform circular disc of radiuis R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the fiigure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[AIEEE-2018]**



(A) $\frac{37}{9}MR^2$ (B) 4 MR^2

(C) $\frac{40}{9}MR^2$ (D) 10 MR^2

1. A block of mass m is held fixed against a wall by a applying a hor izontal force F. Which of the following option is incorrect: **[JEE'(Scr) 2005]**



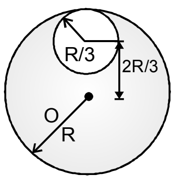
(A) friction force = mg

(B) F will not produce torque

(C) normal will not produce torque

(D) normal reaction = F

2. A disc has mass 9m. A hole of radius R/3 is cut from it as shown in the figure. The moment of inertia of remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is : **[JEE'(Scr) 2005]**



(A) 8 mR^2

(B) 4 mR^2

(C) $\frac{40}{9}mR^2$

(D) $\frac{37}{9}mR^2$

3. A particle moves in circular path with decreasing speed. Which of the following is correct

[JEE'(Scr) 2005]

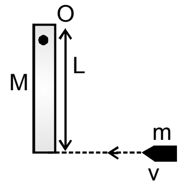
(A) $\vec{L}$ is constant

(B) only direction of $\vec{L}$ is constant

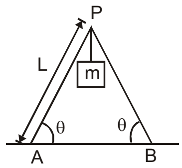
(C) acceleration $\vec{a}$ is towards the centre

(D) it will move in a spiral and finally reach the centre

4. A wooden log of mass M and length L is hinged by a frictionless nail at O. A bullet of mass m strikes with velocity v and sticks to it. Find angular velocity of the system immediately after the collision about O. **[JEE' 2005]**

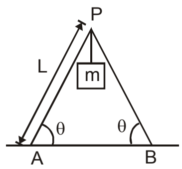


5. A cylinder of mass m and radius R rolls down an inclined plane of inclination θ. Calculate the linear acceleration of the axis of cylinder. **[JEE' 2005]**



6. Two identical ladders, each of mass M and length L are resting on the rough horizontal surface as shown in the figure. A block of mass m hangs from P. If the system is in equilibrium, find the magnitude and the direction of frictional force at A and B.

[JEE' 2005]



7. A solid sphere of mass M, radius R and having moment of inertia about an axis passing through the centre of mass as I, is recast into a disc of thickness t, whose moment of inertia about an axis passing through its edge and perpendicular to its plane remains I. Then, radius of the disc will be

[JEE' 2006]

(A) $2R/\sqrt{15}$ (B) $R\sqrt{2/15}$

(C) $4R/\sqrt{15}$ (D) R/4

8. A solid cylinder of mass m and radius r is rolling on a rough inclined plane of inclination θ. The coefficient of friction between the cylinder and incline is μ. Then **[JEE' 2006]**

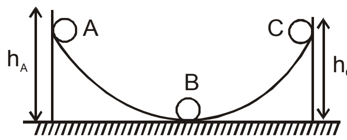
(A) frictional force is always μmg cos θ

(B) friction is a dissipative force

(C) by decreasing θ, frictional force decreases

(D) friction opposes translation and supports rotation

9. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respe0ctively. Then **[JEE' 2006]**



(A) $h_A > h_C$; $K_B > K_C$

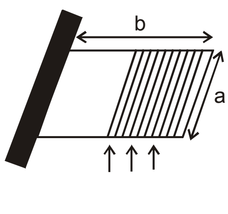
(B) $h_A > h_C$; $K_C > K_A$

(C) $h_A = h_C$; $K_B = K_C$

(D) $h_A < h_C$; $K_B > K_C$

10. There is a rectangular plate of mass M kg of dimensions (a × b). The plate is held in horizontal position by striking n small balls each of mass m per unit area per unit time. These are striking in the shaded half region of the plate. The balls are colliding elastically with velocity v. What is v ?

[JEE' 2006]



It is given n = 100, M = 3 kg, m = 0.01 kg; b = 2 m, a = 1m; g = 10 m/s^2.

Paragraph Q.11 to Q.13 (3 questions)

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia *I* and 2*I* respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction.

11. The ratio x_1/x_2 is **[JEE' 2007]**

(A) 2 (B) 1/2

(C) √ 2 (D) 1/√2

12. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is **[JEE' 2007]**

(A) 2Iω/(3t) (B) 9Iω/(2t)

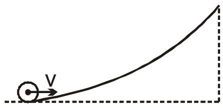
(C) 9Iω/(4t) (D) 3Iω/(2t)

13. The loss of kinetic energy during the above process is **[JEE' 2007]**

(A) $I\omega^2/2$ (B) $I\omega^2/3$

(C) $I\omega^2/4$ (D) $I\omega^2/6$

14. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $3v^2$ / (4g) with respect to the initial position. The object is **[JEE' 2007]**



(A) ring (B) solid sphere

(C) hollow sphere (D) disc

15. **STATEMENT-1** If there is no external torque on a body about its center of mass, then the velocity of the center of mass remains constant **[JEE 2007]**

because

STATEMENT-2

The linear momentum of an isolated system remains constant.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

16. **STATEMENT-1**

Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first. **[JEE-2008]**

STATEMENT-2

By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

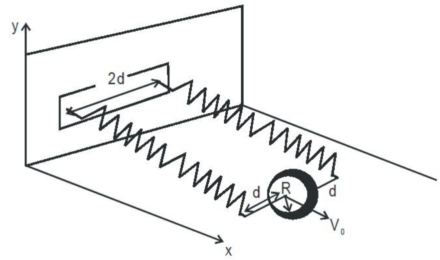
(D) STATEMENT-1 is False, STATEMENT-2 is True

Paragraph for Questions Nos. 17 to 19

A uniform thin cylindrical disk of mass M and radius R is attached to two id enthical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ

[JEE-2008]

Figure.



17. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) kx (B) –2kx

(C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

18. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to-

(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$

(D) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

19. The maximum value of V_0 for which the disk will roll without slipping is -

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$

(C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

20. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[JEE 2009]**

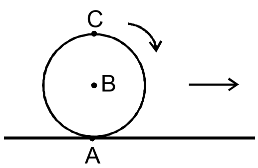
(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does

21. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure *A* is the point of contact, *B* is the centre of the sphere and *C* is its topmost point Then, **[JEE 2009]**



(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$

22. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0º. Then **[JEE-2009]**

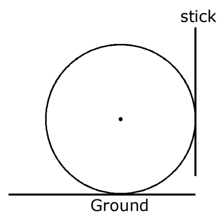
(A) at θ = 30º, the block will start sliding down the plane

(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60º, the block will start sliding down the plane and continue to do so at higher angles

(D) at θ = 60º, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

23. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is? **[JEE 2011]**



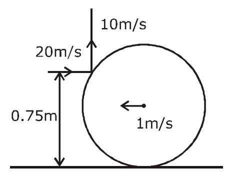
24. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. the moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg-m^2, then N is. **[JEE-2011]**

25. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[JEE 2011]**

(A) The ring has pure rotation about its stationary CM

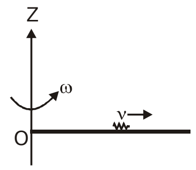
(B) The ring comes to a complete stop

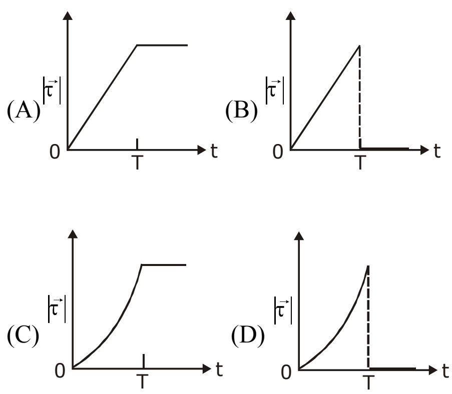
(C) Friction between the ring and the ground is to the left



(D) There is no friction between the ring and the ground

26. A thin uniform rod, pivoted at O is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time t = 0, small insect starts from O and moves with constant speed v with respect to the rod towards the other end. it reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\vec{\tau}|)$ on the system about O, as a function of time is best represented by which plot? **[JEE 2012]**





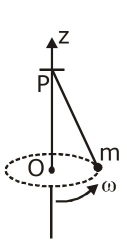
27. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_0$ and $\vec{L}_P$ respectively, then. **[JEE 2012]**

(A) $\vec{L}_0$ and $\vec{L}_P$ do not vary with time

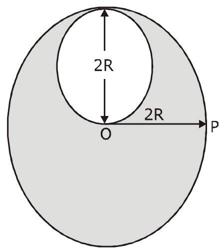
(B) $\vec{L}_0$ varies with time while $\vec{L}_P$ remains constant

(C) $\vec{L}_0$ remains constant while $\vec{L}_P$ varies with time

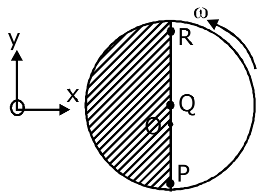
(D) $\vec{L}_0$ and $\vec{L}_P$ both vary with time.



28. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_0 and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_0}$ to the nearest integer is **[JEE 2012]**



29. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed 1/8 rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



(A) P lands in the shaded region and Q in the unshaded region

(B) P lands in the unshaded region and Q in the shaded region

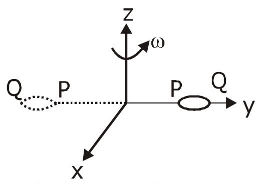
(C) Both P and Q land in the unshaded region

(D) Both P and Q land in the shaded region

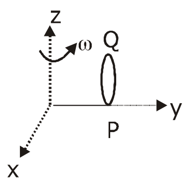
[JEE 2012]

Paragraph for Question Nos. 30 to 31

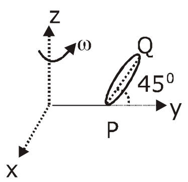
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to x-z plane; case (b) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

30. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct ? **[JEE 2012]**

(A) It is vertical for both the cases (a) and (b).

(B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(D) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

31. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct **[JEE 2012]**

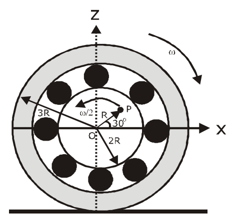
(A) It is $\sqrt{2}\omega$ for both the cases.

(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a); and $\sqrt{2}\omega$ for case (b).

(D) It is ω for both the cases.

32. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated by frictionless ball bearings. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface. **[JEE 2012]**



(A) the point O has a linear velocity $3R\omega\hat{i}$.

(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

33. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct? **[JEE 2012]**

(A) Both cylinders P and Q reach the ground at the same time.

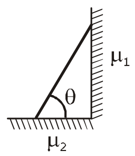
(B) Cylinder P has larger linear acceleration than cylinder Q.

(C) Both cylinders reach the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

34. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is : **[JEE 2013]**

35. In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then **[JEE 2014]**



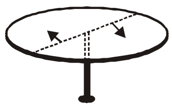
(A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$

(B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

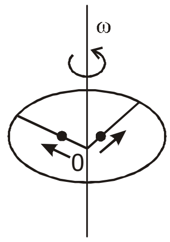
(C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$

(D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

36. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25b m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is **[JEE 2014]**



37. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is. **[JEE Advanced 2015]**



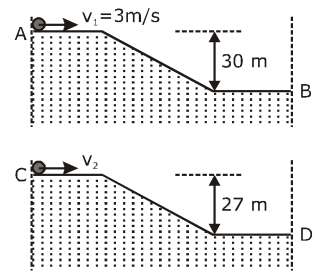
(A) $\frac{2}{3}R$

(B) $\frac{1}{3}R$

(C) $\frac{3}{5}R$

(D) $\frac{4}{5}R$

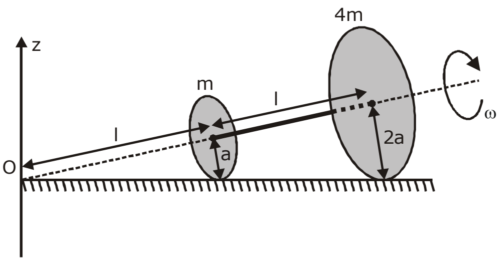
38. Two identical uniform discs roll without slipping on two different surface AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_2 in m/s is ($g = 10$ m/s^2) **[JEE Advanced 2015]**



39. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is - **[IIT-2015]**

40. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}$ a through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure) Which of the following is statement(s) is (are) true?

[JEE-2016]



(A) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5.

(B) The magnitude of angular momentum of center of mass of the assembly about the point O is 81 $ma^2\omega$

(C) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(D) The magnitude of the z-component of $\vec{L}$ is $55ma^2\omega$

41. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$, where $\alpha = 10/3\ ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement (s) is (are) true about the particle? **[JEE-2016]**

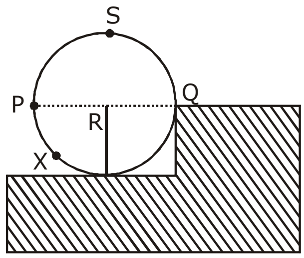
(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\ ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}$ N m s

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})N$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\hat{k}$ Nm

42. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just clims the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct? **[JEE-2017]**



(A) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs.

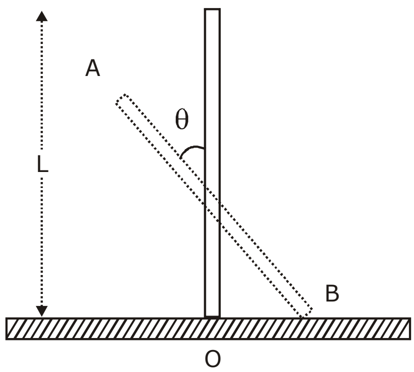
(B) If the force is applied tangentially at point S then τ ≠ 0 but the wheel never climbs the step.

(C) If the force is applied normal to the circumference at point P then τ is zero.

(D) If the force is applied normal to the circumeference at point X then τ is constant.

43. A right uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure.) At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?

[JEE-2017]



(A) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial posiiton is proportional to (1–cosθ)

(B) The midpoint of the bar will fal vertically downward

(C) Instantaneous torque about the point in contact with the floor is proportional to sin θ

(D) The trajectory of the point A is a parabola

Paragraph - (Q.44 to Q.45)

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g. **[JEE-2017]**

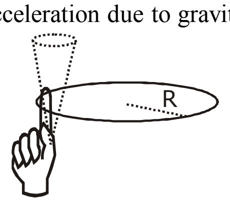


Figure 1

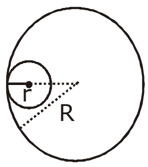


Figure 2

44. The minimum value of ω_0 below which the ring will drop down is -

(A) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (B) $\sqrt{\frac{g}{\mu(R-r)}}$

(C) $\sqrt{\frac{g}{2\mu(R-r)}}$ (D) $\sqrt{\frac{2g}{\mu(R-r)}}$

45. The total kinetic energy of the ring is

(A) $M\omega_0^2 (R-r)^2$ (B) $\frac{1}{2} M\omega_0^2 (R-r)^2$

(C) $\frac{3}{2} M\omega_0^2 (R-r)^2$ (D) $M\omega_0^2 R^2$

46. Consider a body of mass 1.0 kg at rest at the origin at time t=0. A force $\vec{F}=(\alpha t\hat{i}+\beta\hat{j})$ is applied on the body , where $\alpha = 1.0\ Ns^{-1}$ and $\beta = 1.0$ N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true?

(A) $|\vec{\tau}|=\frac{1}{3}$N m **[JEE-2018]**

(B) The torque $\vec{\tau}$ is in the direction oft he unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v}=\frac{1}{2}(\hat{i}+2\hat{j})$ m s^{-1}

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}$ m

47. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$s , then the height of the top of the inclined plane, in metres, is __________. Take g=10 ms^{-2}. **[JEE-2018]**

48. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq$ B. In each case, the force acting on the particle is either zero or conservative. In List–II, five physical quantities of the particle are mentioned; $\vec{p}$ is the linear momentum $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List–I with those quantities in List–II, which are conserved for that path **[JEE-2018]**

List–I	List–II
P. $\vec{r}(t)=\alpha t\hat{i}+\beta t\hat{j}$	1. $\vec{p}$
Q. $\vec{r}(t)=\alpha\cos\omega t\hat{i}+\beta\sin\omega t\hat{j}$	2. $\vec{L}$
R. $\vec{r}(t)=\alpha(\cos\omega t\hat{i}+\sin\omega t\ \hat{j})$	3. K
S. $\vec{r}(t)=\alpha t\hat{i}+\frac{\beta}{2}t^2\hat{j}$	4. U
	5. E

(A) P → 1,2,3,4,5 ; Q → 2,5 ; R → 2,3,4,5 ; S → 5
(B) P → 1,2,3,4,5 ; Q → 3,5 ; R → 2,3,4,5 ; S → 2,5
(C) P → 2,3,4 ; Q → 5; R → 1,2,4 ; S → 2,5
(D) P → 1,2,3,5 ; Q → 2,5 ; R → 2,3,4,5 ; S → 2,5

ANSWER KEYS

Exercise - 1 Objective Problems | JEE Main

1.	D	2.	B	3.	D	4.	A	5.	D	6.	C	7.	C
8.	A	9.	B	10.	C	11.	C	12.	D	13.	B	14.	B
15.	D	16.	D	17.	D	18.	D	19.	A	20.	D	21.	C
22.	A	23.	B	24.	A	25.	A	26.	C	27.	B	28.	D
29.	B	30.	A	31.	B	32.	C	33.	B	34.	D	35.	C
36.	C	37.	B	38.	B	39.	C	40.	B	41.	B	42.	B
43.	B	44.	B	45.	B	46.	B	47	B	48.	D	49.	D
50.	D	51.	C	52.	B	53.	C	54.	C	55.	A	56.	A
57.	B	58.	B	59.	B	60.	D	61.	A	62.	D	63.	C
64.	B	65	C	66.	B	67.	B	68.	A	69.	D	70.	C
71.	D	72.	C	73.	D	74.	A	75.	D	76.	B	77.	C
78.	B	79.	B	80.	A								

Exercise - 2 (Leve-I) Objective Problems | JEE Main

1.	C	2.	C	3.	C	4.	A	5.	B	6.	C	7.	A
8.	C	9.	C	10.	A	11.	B	12.	B	13.	C	14.	D
15.	D	16.	C	17.	B	18.	C	19.	C	20.	A	21.	A
22.	C	23.	A	24.	B	25.	A	26.	B	27.	B	28.	C
29.	B	30.	C	31.	A	32.	D	33.	B	34.	B	35.	C
36.	B	37.	D	38.	A	39.	C	40.	B	41	B	42.	B
43.	D	44.	C	45.	C	46.	A	47.	C	48.	B	49.	A
50.	B	51.	C	52.	A	53.	A	54.	A	55.	C	56.	A

Exercise - 2 (Level-II) Multiple Correct | JEE Advanced

1.	A,B,C,D	2.	BC	3.	BCD	4.	ABC	5.	ACD
6.	A,B,C,D	7.	A,B,C,D	8.	A,B,C,D	9.	ACD	10.	B,C
11.	A,B,C,D	12.	B,C	13.	A,B,D	14.	B,C,D	15.	B,C
16.	A,C	17.	B,C	18.	A,B,C,D	19.	B,D	20.	A,B,C
21.	A,B,C	22.	C	23.	B,C,D	24.	A,C,D	25.	A,B,D

Exercise - 3 | Level-I Subjective | JEE Advanced

1. $\frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2$ **2.** $\frac{14mr^2}{5}$ **3.** $\sqrt{2}r$

4. w ℓ sin θ, when the bob is at the lowest point **5.** $-14\hat{i}+10\hat{j}-9\hat{k}$

6. P = $\frac{mg}{2}\cot\theta$ **7.** (i) 10/13 m/s^2, (ii) 5000/26π, (iii) 480/13 N **8.** 1

9. M = $2m\left(\frac{2gh}{R^2\omega^2}-1\right)$ **10.** $\frac{2g}{5}\downarrow$ **11.** 16 kg m^2/s **12.** $2\hat{k}\,kg\ m^2/s$

13. 0.5 kg – m^2/s, 75 J **14.** 19.7 rad/s **15.** 60° east of south, 30° south of east.

16. (1/2)KE_0 **17.** (a) $\frac{3g}{4L}(cw)$ (b) $N=\frac{13mg}{16}\uparrow,\ F=\left(\frac{3\sqrt{3}}{16}\right)mg\rightarrow$ (c) $\frac{3\sqrt{3}}{16}$

18. $\frac{v_0}{3}(\leftarrow),\frac{2v_0}{3}(\rightarrow)$ **19.** $\frac{7}{10}mv^2$ **20.** 50m/s **21.** $\sqrt{\frac{10gh}{7}}$

22. $\sqrt{\frac{10}{7} g\ell \sin\theta}$ 23. $\frac{17}{7}mg$ 24. (a) $\frac{Ft}{m}$ (b) $\frac{6Ft}{m\ell}$ (c) $\frac{2F^2t^2}{m}$ (d) $\frac{F\ell t}{2}$

25. **Topple first**

Exercise - 3 | Level-II   Subjective | JEE Advanced

1. $\frac{Ml^2}{2}$ 2. $2mR^2$ 3. 2mg 4. $\frac{3Ft^2}{2m\ell}$ 5. $a = \left[\frac{b - \mu a}{3b + \mu a}\right] g$

6. 1.63 N, 1.224 m 7. (a) $4\sqrt{\frac{3}{7}} m/s$, (b) $\frac{200}{7}N$ 8. 3 9. (a) $\frac{9g}{7}\downarrow$ (b) $\frac{4mg}{7}\uparrow$

10. $\frac{\omega}{3}$ 11. (a) $\frac{3\rho_0 a}{2}$ (b) $\frac{5a}{9}$, (c) $\frac{7a^3\rho_0}{12}$, (d) $\frac{18P}{7Ma}$, (e) $\frac{M}{9}\sqrt{70ag}$ 12. 2F/M

13. 2m/s 14. $v = \sqrt{\frac{14gR}{3}}$ 15. 1/2 ma

Exercise - 4 | Level-I   Previous Year | JEE Main

1.	**C**	**2.**	**D**	**3.**	**D**	**4.**	**C**	**5.**	**B**	**6.**	**C**	**7.**	**C**
8.	**A**	**9.**	**C**	**10.**	**A**	**11.**	**D**	**12.**	**D**	**13.**	**A**	**14.**	**C**
15.	**A**	**16.**	**A**	**17.**	**D**	**18.**	**C**	**19.**	**A**	**20.**	**A or C**	**21.**	**D**
22.	**B**	**23.**	**B**	**24.**	**A**	**25.**	**B**						

Exercise - 4 | Level-II   Previous Year | JEE Advanced

1. **C** **2.** **B** **3.** **B** **4.** $\omega = \frac{3mv}{(3m+M)L}$ **5.** $a_{axis} = \frac{2g\sin\theta}{3}$

6. $f = (M+m)g\frac{\cot\theta}{2}$ **7.** **A** **8.** **C,D** **9.** **A,B** **10.** **10 m/s**

11.	**C**	**12.**	**A**	**13.**	**B**	**14.**	**D**	**15.**	**D**	**16.**	**D**	**17.**	**D**
18.	**D**	**19.**	**C**	**20.**	**A**	**21.**	**B,C**	**22.**	**B**	**23.**	**4**	**24.**	**0009**
25.	**C**	**26.**	**B**	**27.**	**C**	**28.**	**3**	**29.**	**C**	**30.**	**A**	**31.**	**D**
32.	**A,B**	**33.**	**D**	**34.**	**8**	**35.**	**C,D**	**36.**	**4**	**37.**	**D**	**38.**	**7**
39.	**6**	**40.**	**A,C**	**41.**	**ABD**	**42.**	**C, D**	**43.**	**A,B,C**	**44.**	**B**	**45.**	**A**
46.	**AC**	**47.**	**0.75 m**			**48.**	**A**						